Chapter 2
AC to DC CONVERSION (RECTIFIER)

- Single-phase, half wave rectifier
  - Uncontrolled
  - R load
  - R-L load
  - R-C load
  - Controlled
  - Free wheeling diode

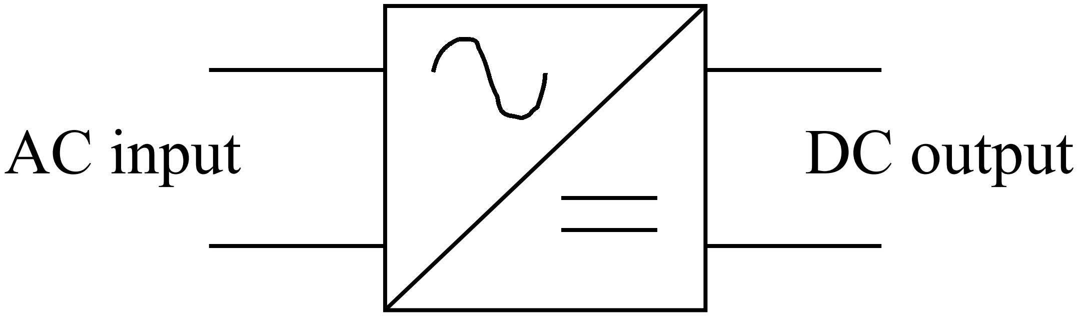
- Single-phase, full wave rectifier
  - R load
  - R-L load,
  - Controlled R, R-L load
  - continuous and discontinuous current mode

- Three-phase rectifier
  - uncontrolled
  - controlled

Rectifiers

- DEFINITION: Converting AC (from mains or other AC source) to DC power by using power diodes or by controlling the firing angles of thyristors/controllable switches.

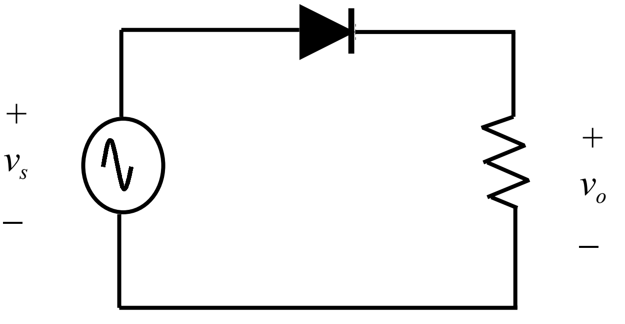
- Basic block diagram

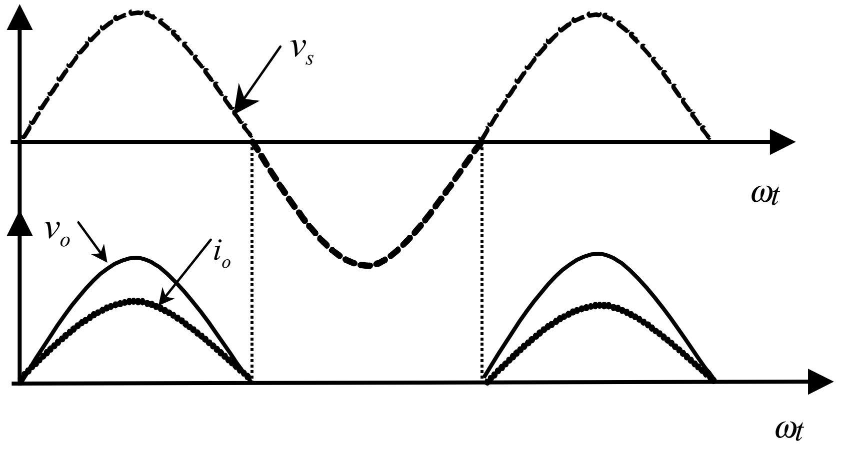


- Input can be single or multi-phase (e.g. 3-phase).
- Output can be made fixed or variable

- Applications: DC welder, DC motor drive, Battery charger,DC power supply, HVDC

Single-phase, half-wave with R-load





Output voltage (average),

$$V_o = V_{avg} = \int_0^{\pi} V_m \sin(\omega t) d\omega t$$

$$= \frac{V_m}{\pi} = 0.318 V_m$$

RMS voltage

Output voltage (RMS)

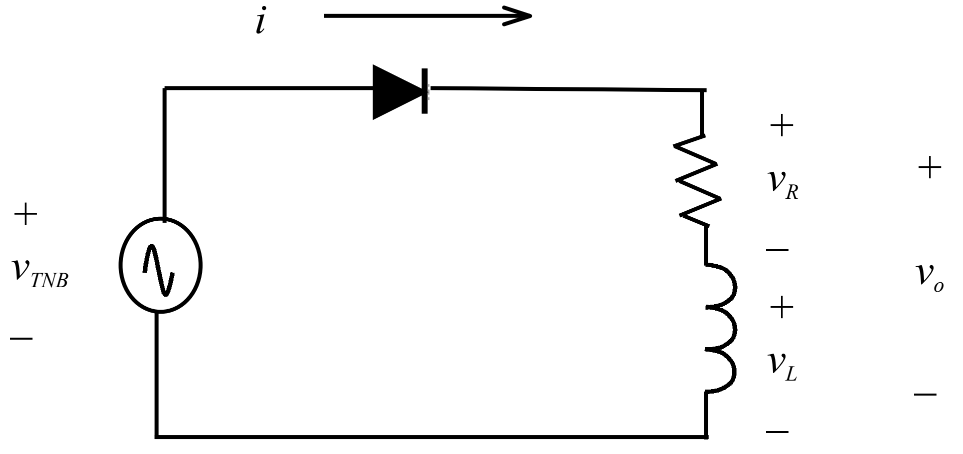
$$V_{o,RMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(V_m \sin(\omega t)\right)^2 d\omega t} = \frac{V_m}{2}$$

Output current (DC),

$$I_o = \frac{V_o}{R} = \frac{0.318V_m}{R}$$

- DC voltage is fixed at 0.318 or 31.8% of the peak value
- RMS voltage is reduced from 0.707 (normal sinusoidal RMS) to 0.5 or 50% of peak value.
- Half wave is not practical because of high distortion supply current. The supply current contains DC component that may saturate the input transformer

Half-wave with R-L load



$$v_s = v_R + v_L$$

$$V_m \sin(\omega t) = i(t)R + L\frac{di(t)}{dt}$$

This is a first order differential equation.
Solution is in the form of :

$$i(t) = i_f(t) + i_n(t)$$

where : i_f, i_n are known as "forced" and "natural" response, respectively.
From diagram, forced response is :

$$i_f(t) = \left(\frac{V_m}{Z}\right)\cdot\sin(\omega t - \theta)$$

where :

$$Z = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

R-L load

Natural response is when source = 0,

$$i(t)R + L\frac{di(t)}{dt} = 0$$

which results in :

$$i_n(t) = Ae^{t/\tau} \quad ;\tau = L/R$$

Hence

$$i(t) = i_f(t) + i_n(t) = \left(\frac{V_m}{Z}\right)\cdot\sin(\omega t - \theta) + Ae^{-t/\tau}$$

A can be solved by realising inductor current is zero before the diode starts conducting, i.e :

$$i(0) = \left(\frac{V_m}{Z}\right)\cdot\sin(0 - \theta) + Ae^{-0/\tau}$$

$$\Rightarrow A = \left(\frac{V_m}{Z}\right)\cdot\sin(-\theta) = \left(\frac{V_m}{Z}\right)\cdot\sin(\theta)$$

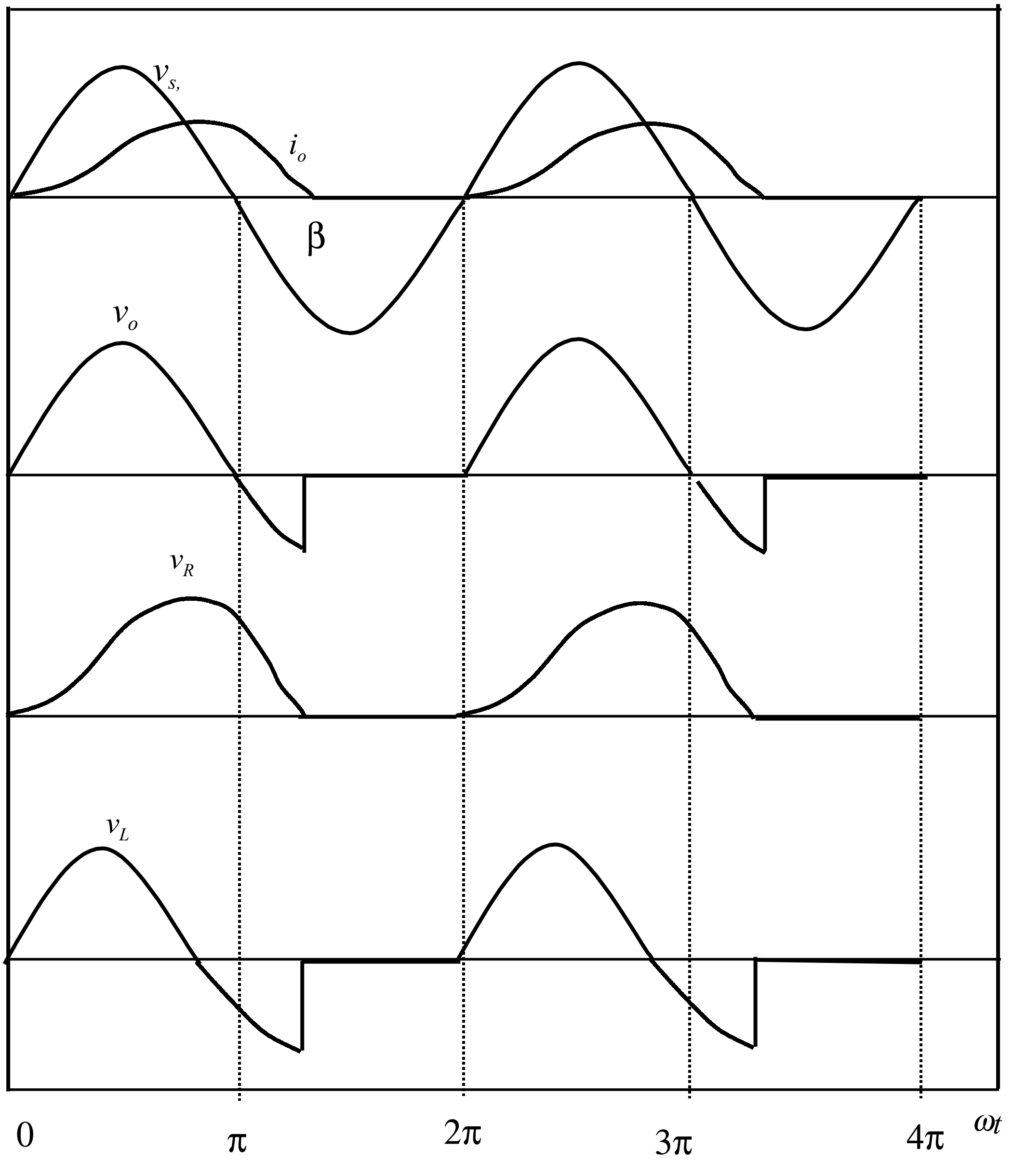
Therefore the current is given as,

$$i(t) = \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\omega t - \theta) + \sin(\theta)e^{-t/\tau}\right]$$

or

$$i(\omega t) = \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right]$$

R-L waveform



Note:
v_L is negative because the current is decreasing, i.e:

$$v_L = L\frac{di}{dt}$$

Extinction angle

Note that the diode remains in forward biased longer than π radians (although the source is negative during that duration)

The point when current reaches zero is when diode turns OFF. This point is known as the extinction angle, β.

$$i(\beta) = \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau}\right] = 0$$

which reduces to :

$$\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau} = 0$$

β can only be solved numerically.
Therefore, the diode conducts between 0 and β

To summarise the rectfier with R - L load,

$$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] \\ \text{for } 0 \le \omega t \le \beta \\ 0 \\ \text{otherwise} \end{cases}$$

RMS current, Power Factor

The average (DC) current is :

$$I_o = \frac{1}{2\pi}\int_0^{2\pi} i(\omega t)\,d\omega t = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d\omega t$$

The RMS current is :

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d\omega t}$$

Power absorbed by the load is :

$$\mathrm{P_o} = (I_{RMS})^2 \cdot R$$

Power Factor is computed from definition :

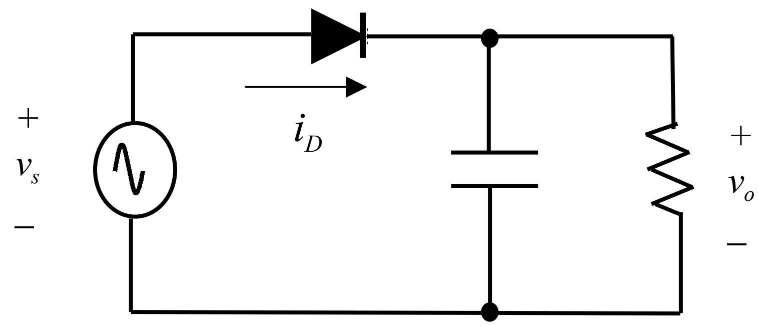
$$pf = \frac{P}{S}$$

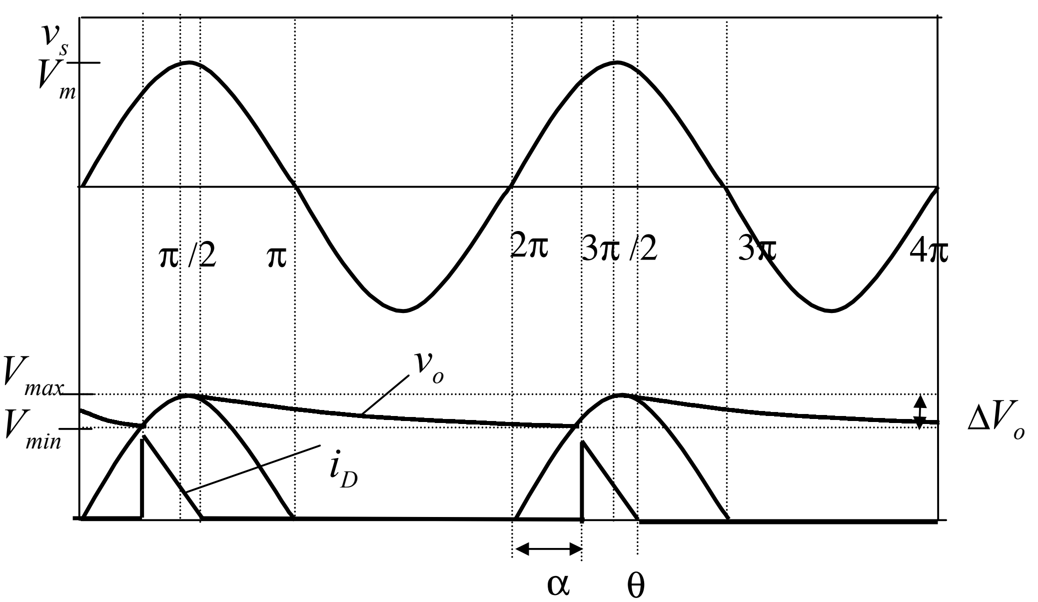
where P is the real power supplied by the source, which equal to the power absorbed by the load. S is the apparent power supplied by the source, i.e

$$S = (V_{s,RMS}).(I_{RMS})$$

$$\Rightarrow pf = \frac{P}{(V_{s,RMS}).(I_{RMS})}$$

Half wave rectifier, R-C Load





$$v_o = \begin{cases} V_m \sin(\omega t) & \text{when diode is ON} \\ V_\theta \, e^{-(\omega t - \theta)/\omega RC} & \text{when diode is OFF} \end{cases}$$

$$v_\theta = V_m \sin\theta$$

Operation

- Let C initially uncharged. Circuit is energised at $\omega t=0$
- Diode becomes forward biased as the source become positive
- When diode is ON the output is the same as source voltage. C charges until V_m
- After $\omega t=\pi/2$, C discharges into load (R).
- The source becomes less than the output voltage
- Diode reverse biased; isolating the load from source.
- The output voltage decays exponentially.

Estimation of θ

The slope of the functions are:

$$\frac{d\left(V_m \sin \omega t\right)}{d(\omega t)} = V_m \cos \omega t$$

and

$$\frac{d\left(V_m \sin\theta \cdot e^{-(\omega t - \theta)/\omega RC}\right)}{d(\omega t)}$$

$$= V_m \sin\theta \cdot \left(-\frac{1}{\omega RC}\right) \cdot e^{-(\omega t - \theta)/\omega RC}$$

At $\omega t = \theta$, the slopes are equal,

$$V_m \cos\theta = V_m \sin\theta \cdot \left(-\frac{1}{\omega RC}\right) \cdot e^{-(\theta - \theta)/\omega RC}$$

$$\Rightarrow \frac{V_m \cos\theta}{V_m \sin\theta \cdot} = -\frac{1}{\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

$$\theta = \tan^{-1}\left(-\omega RC\right) = -\tan^{-1}\left(\omega RC\right) + \pi$$

Estimation of α

For practical circuits, ωRC is large, then :

$$\theta = -\tan(\infty) + \pi = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

and $V_m \sin\theta = V_m$

At $\omega t = 2\pi + \alpha$,

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta) e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\sin(\alpha - (\sin\theta) e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$$

This equation must be solved numerically for α

Ripple Voltage

Max output voltage is $V_{\max}$.

Min output voltage occurs at $\omega t = 2\pi + \alpha$

Reffering to diagram, the ripple is :
$\Delta V_o = V_{\max} - V_{\min}$

$= V_m - V_m \sin(2\pi + \alpha) = V_m - V_m \sin\alpha$

If $V_\theta = V_m$ and $\theta = \pi/2$, and C is large such that DC output voltage is constant, then $\alpha \approx \pi/2$.

The output voltage evaluated at $\omega t = 2\pi + \alpha$ is :

$$v_o(2\pi + \alpha) = V_m e^{-\left(\frac{2\pi + \pi/2 - \pi/2}{\omega RC}\right)} = V_m e^{-\left(\frac{2\pi}{\omega RC}\right)}$$

The ripple voltage is approximated as :

$$\Delta V_o \approx V_m - V_m e^{-\left(\frac{2\pi}{\omega RC}\right)} = V_m\left(1 - e^{-\left(\frac{2\pi}{\omega RC}\right)}\right)$$

Voltage ripple-cont'd

Approximation of exponent term yields:

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

Substituting,

$$\Delta V_o \approx V_m \left(\frac{2\pi}{\omega RC} \right) = \frac{V_m}{fRC}$$

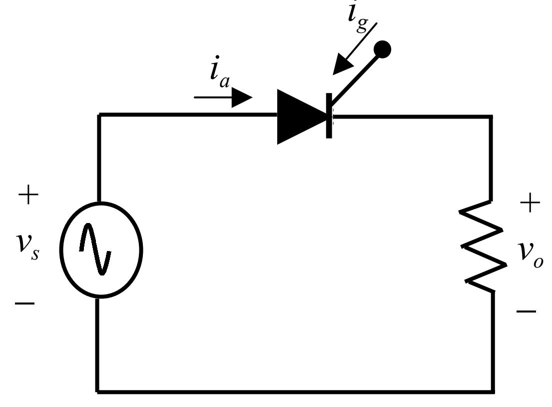
- The output voltage ripple is reduced by increasing C.

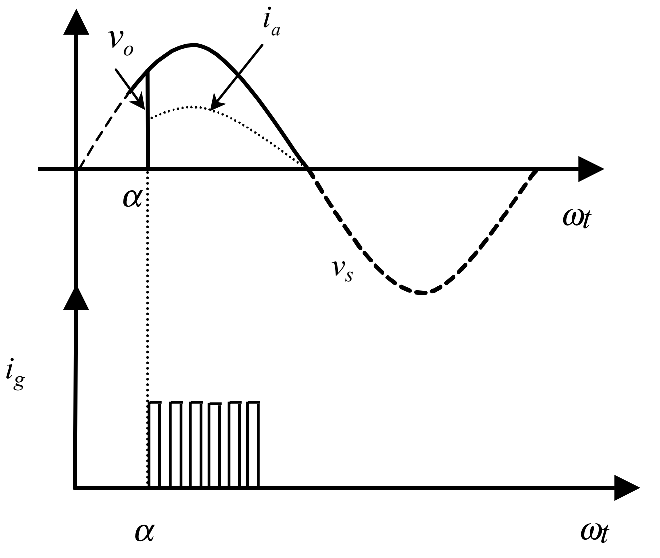
- As C is increased, the conduction interval for diode decreases.

- Therefore, reduction in output voltage ripple results in larger peak diode current.

- EXAMPLE:
  The half wave rectifier has 120V RMS source at 60Hz. R=500 Ohm and C=100uF. Determine (a) the expression for output voltage, (b) voltage ripple.

Controlled half-wave





Average voltage :

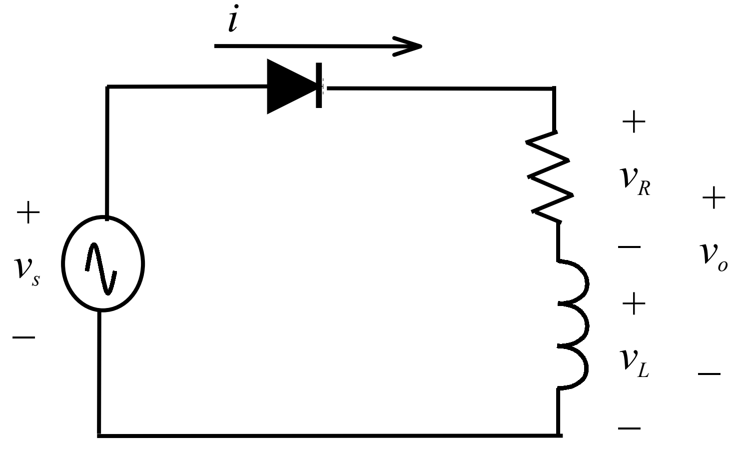
$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t = \frac{V_m}{2\pi}[1+\cos\alpha]$$

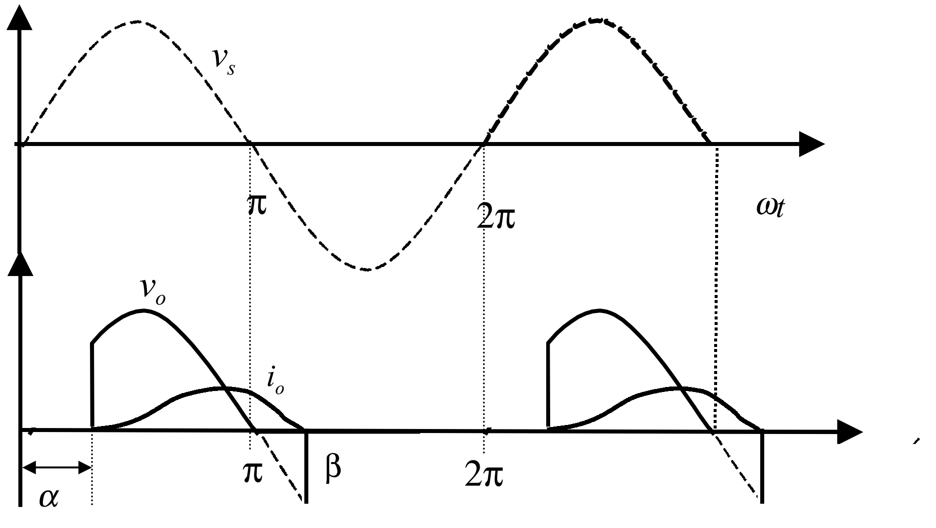
RMS volatge

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2 d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_{\alpha}^{\pi} [1-\cos(2\omega\, t] d\omega t} = \frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}$$

Controlled h/w, R-L load





$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \left(\frac{V_m}{Z}\right)\cdot \sin(\omega t - \theta) + Ae^{\frac{-\omega t}{\omega\tau}}$$

Initial condition : $i(\alpha) = 0$,

$$i(\alpha) = 0 = \left(\frac{V_m}{Z}\right)\cdot \sin(\alpha - \theta) + Ae^{\frac{-\alpha}{\omega\tau}}$$

$$A = -\left[\left(\frac{V_m}{Z}\right)\cdot \sin(\alpha - \theta)\right]e^{\frac{-\alpha}{\omega\tau}}$$

Extinction angle

Substituting for A and simplifying,

$$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\omega t - \theta) - \sin(\alpha - \theta) e^{\frac{-(\alpha - \omega t)}{\omega\tau}} \right] \\ \text{for } \alpha \le \omega t \le \beta \\ 0 \quad \text{otherwise} \end{cases}$$

Extinction angle, β is defined when $i = 0$,

$$i(\beta) = 0 = \left(\frac{V_m}{Z}\right)\left[\sin(\beta - \theta) - \sin(\beta - \theta) e^{\frac{(\alpha - \beta}{\omega\tau}} \right]$$

which can only be solved numerically.

Angle $(\beta - \theta)$ is called the conduction angel.
i.e the diode conducts for γ degrees.

RMS voltage and current

Average voltage:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{2\pi}[\cos\alpha - \cos\beta]$$

Average current

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d\omega$$

RMS current

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d\omega}$$

The power absorbed by the load is:

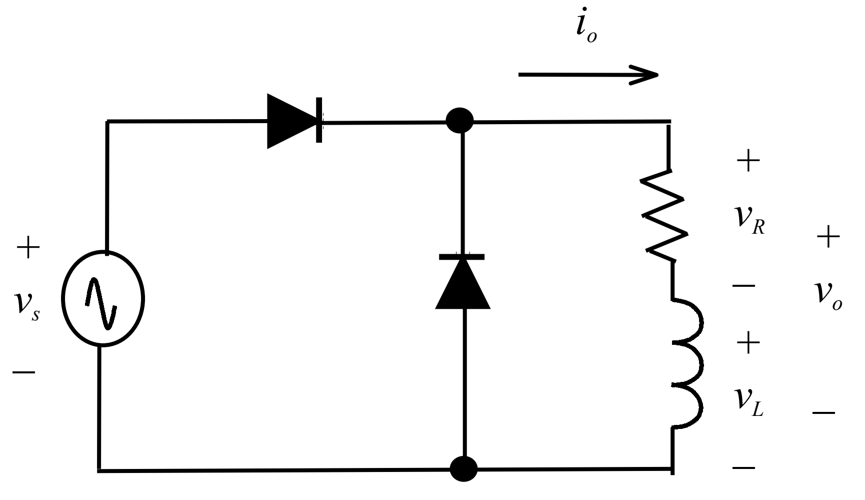
$$P_o = I_{RMS}^{\ 2} \cdot R$$

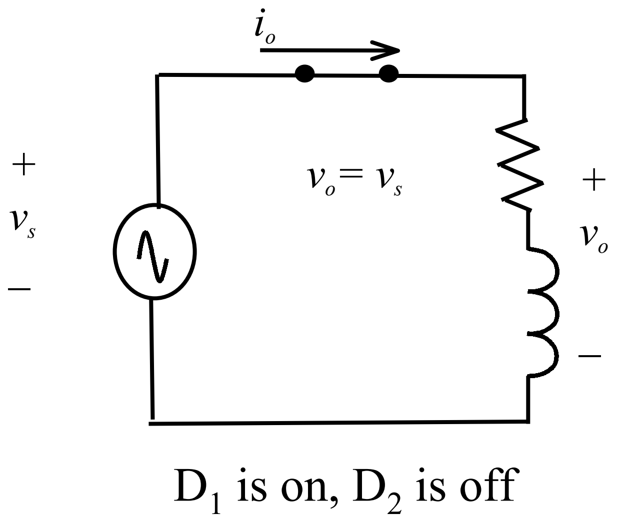
- EXAMPLES
- 1. Design a circuit to produce an average voltage of 40V across a 100 ohm load from a 120V RMS, 60Hz supply. Determine the power factor absorbed by the resistance.
- 2. A half wave rectifier has a source of 120V RMS at 60Hz. R=20 ohm, L=0.04H, and the delay angle is 45 degrees. Determine: (a) the expression for *i(ωt)*, (b) average current, (c) the power absorbed by the load.

Freewheeling diode (FWD)

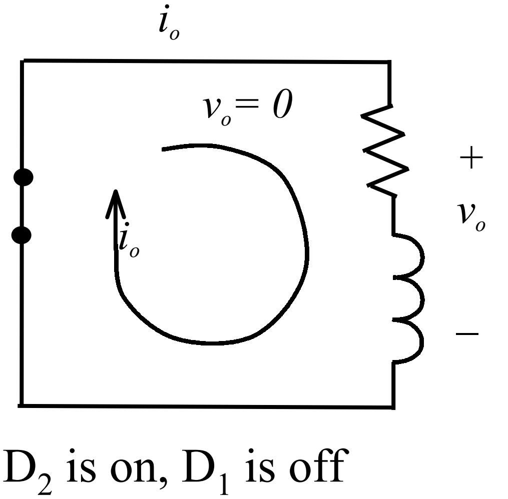
- Note that for single-phase, half wave rectifier with R-L load, the load (output) current is NOT continuos.

- A FWD (sometimes known as commutation diode) can be placed as shown below to make it continuos





D$_1$ is on, D$_2$ is off



D$_2$ is on, D$_1$ is off

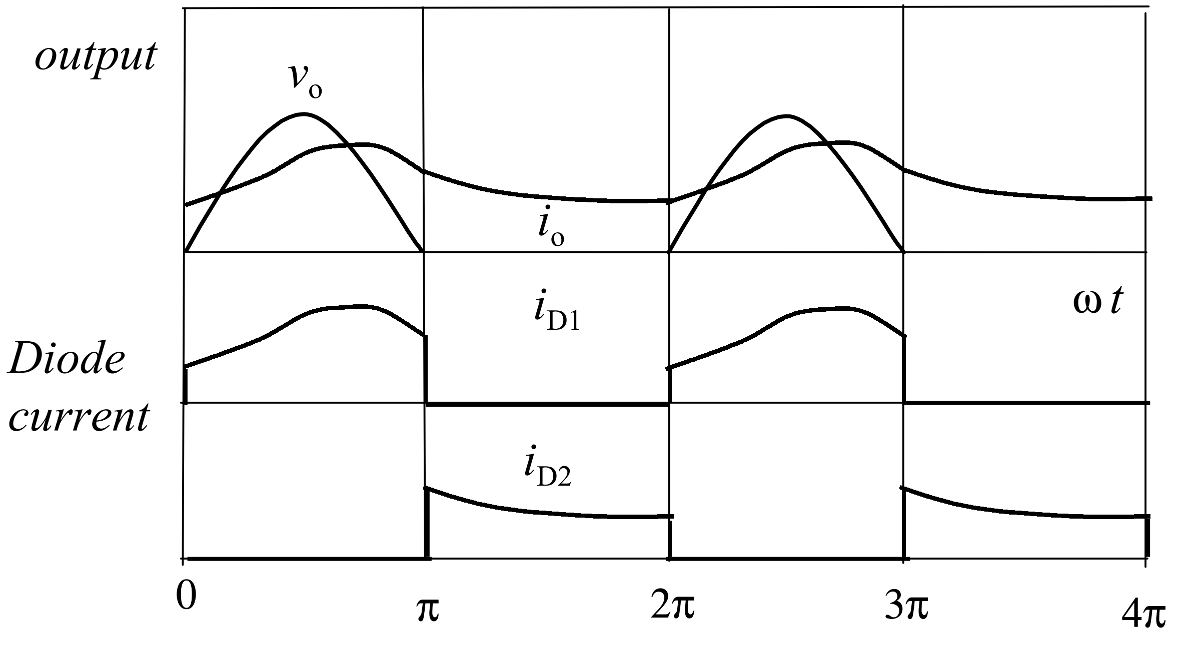
Operation of FWD

- Note that both D_1 and D_2 cannot be turned on at the same time.
- For a positive cycle voltage source,
  - D_1 is on, D_2 is off
  - The equivalent circuit is shown in Figure (b)
  - The voltage across the R-L load is the same as the source voltage.

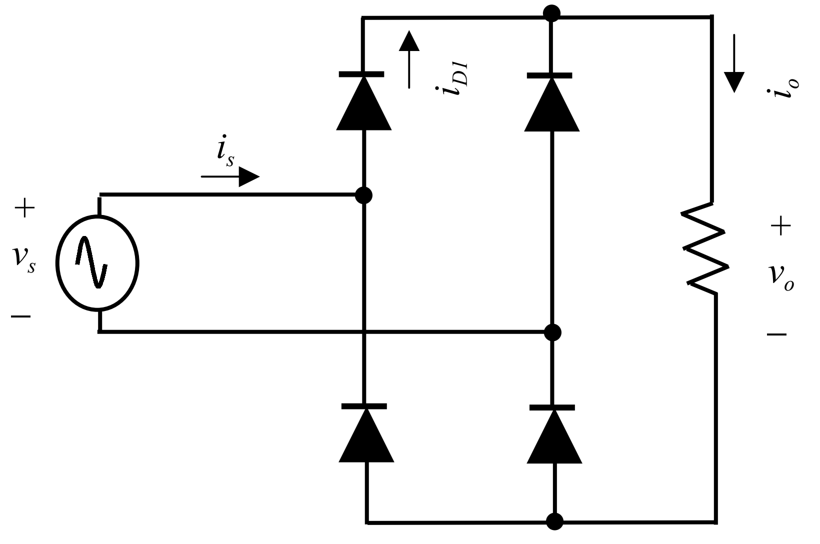
- For a negative cycle voltage source,
  - D_1 is off, D_2 is on
  - The equivalent circuit is shown in Figure (c)
  - The voltage across the R-L load is zero.
  - However, the inductor contains energy from positive cycle. The load current still circulates through the R-L path.
  - But in contrast with the normal half wave rectifier, the circuit in Figure (c) does not consist of supply voltage in its loop.
  - Hence the "negative part" of v_o as shown in the normal half-wave disappear.

FWD- Continuous load current

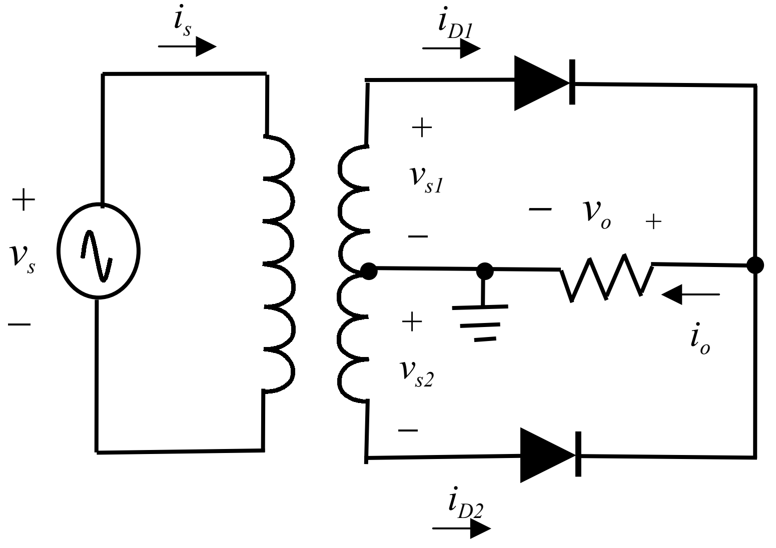
- The inclusion of FWD results in continuos load current, as shown below.
- Note also the output voltage has no negative part.



Full wave rectifier with R load



Bridge circuit



Center-tapped circuit

Notes on full-wave

- Center-tapped rectifier requires center-tap transformer. Bridge does not.

- Center tap requires only two diodes, compared to four for bridge. Hence, per half-cycle only one diode volt-drop is experienced. Conduction losses is half of bridge.

- However, the diodes ratings for center-tapped is twice than bridge.

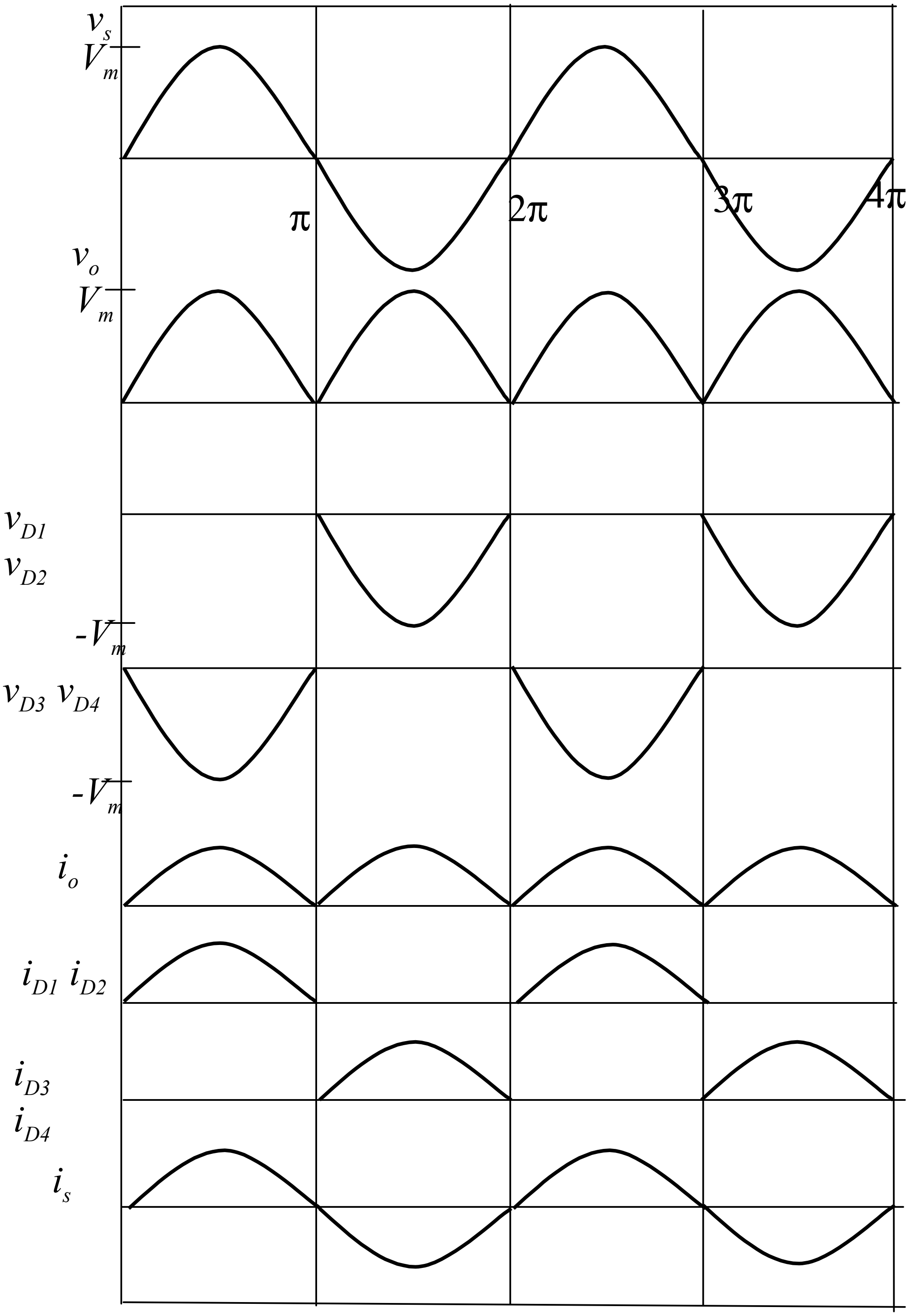
For both circuits,

$$v_o = \begin{cases} V_m \sin\omega t & 0 \le \omega t \le \pi \\ -V_m \sin\omega t & \pi \le \omega t \le 2\pi \end{cases}$$

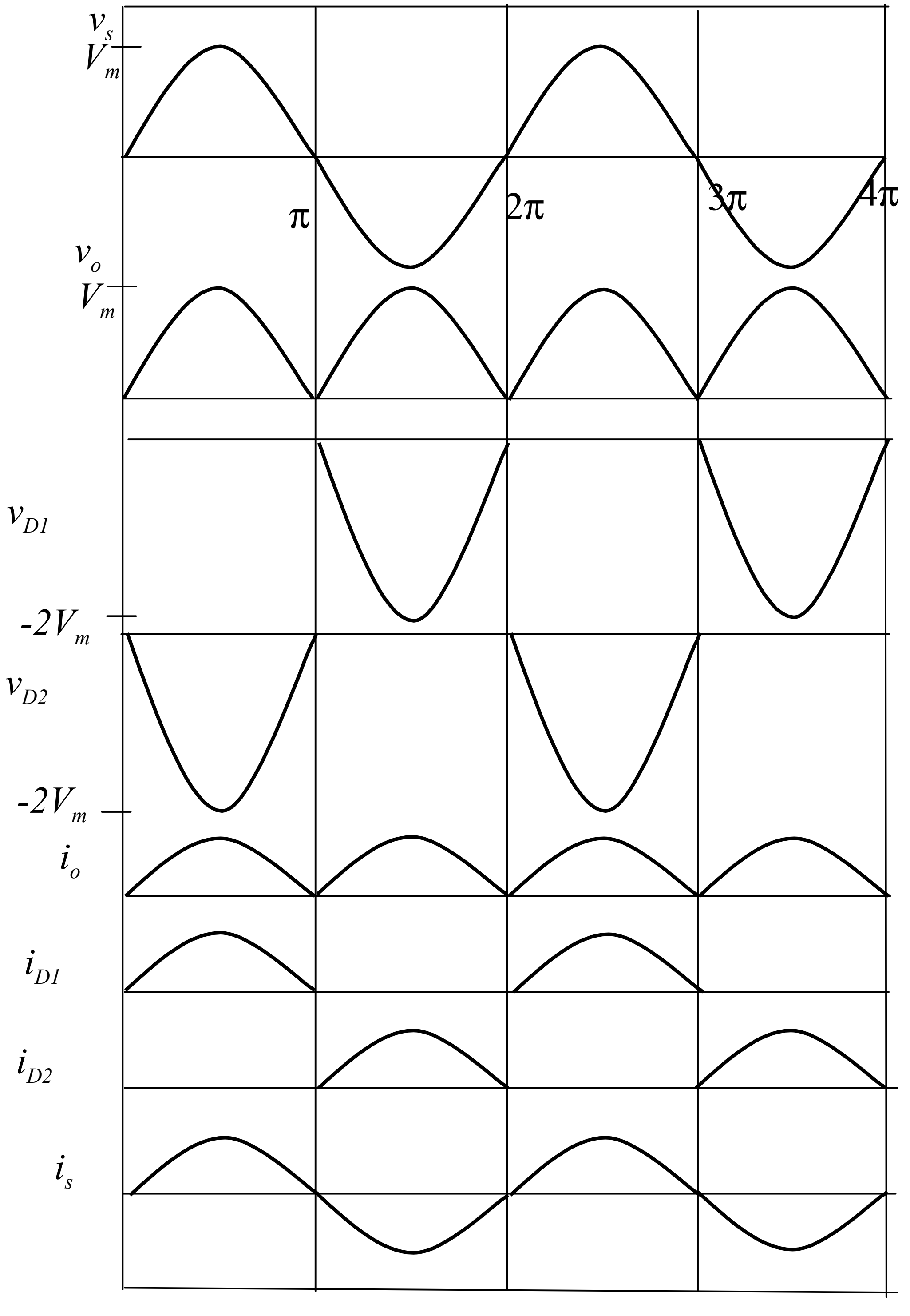
DC voltage:

$$V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{2V_m}{\pi} = 0.637 V_m$$

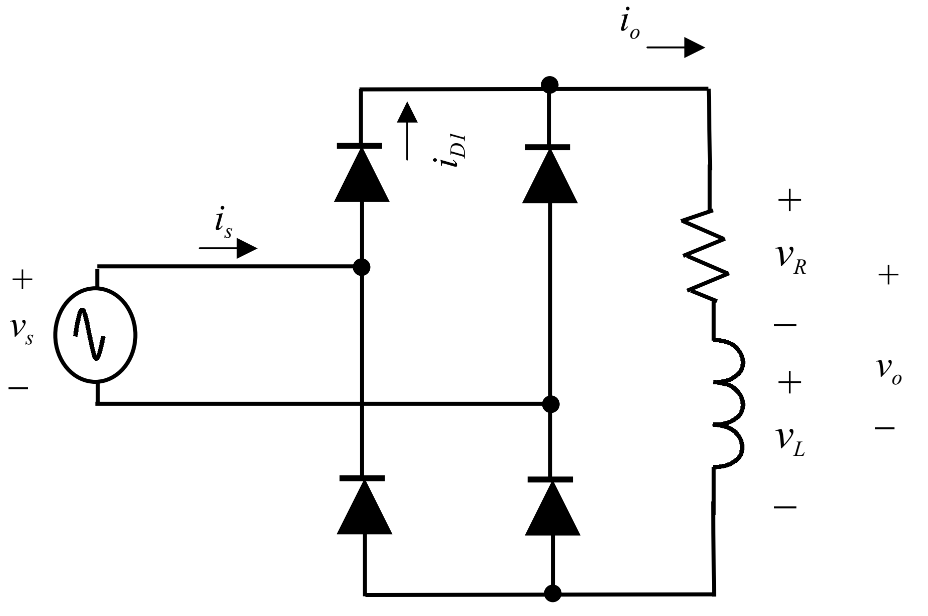
Bridge waveforms

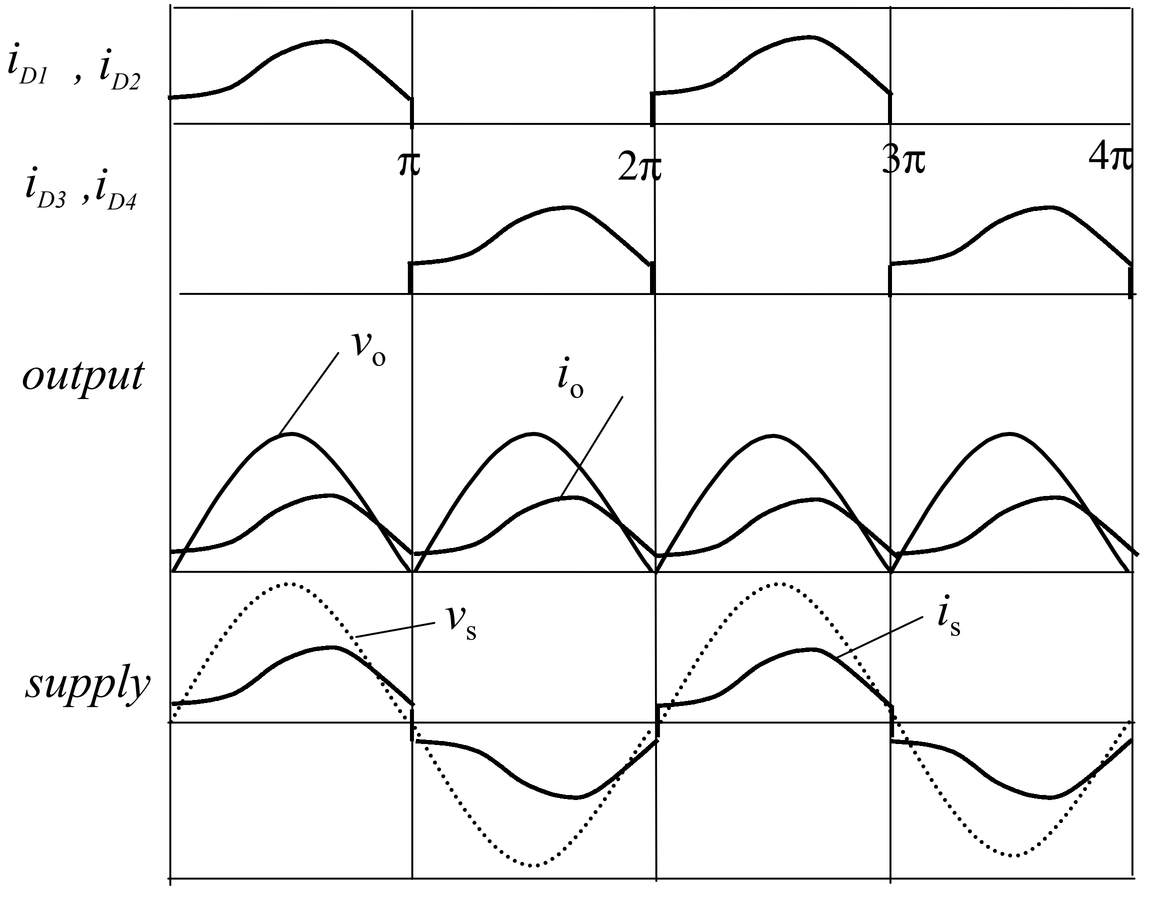


Center-tapped waveforms



Full wave bridge, R-L load





R-L load analysis: approximation with large L

Using Fourier Series, output voltage is described as :

$$v_o(\omega t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega t + \pi)$$

where

$$V_o = \frac{2V_m}{\pi}$$

$$V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

The DC and harmonic currents are :

$$I_o = \frac{V_o}{R} \qquad I_n = \frac{V_n}{Z_n} = \frac{V_n}{\left|R + jn\omega L\right|}$$

Note that as n increases, voltage amplitude for the nth harmonic decreases.

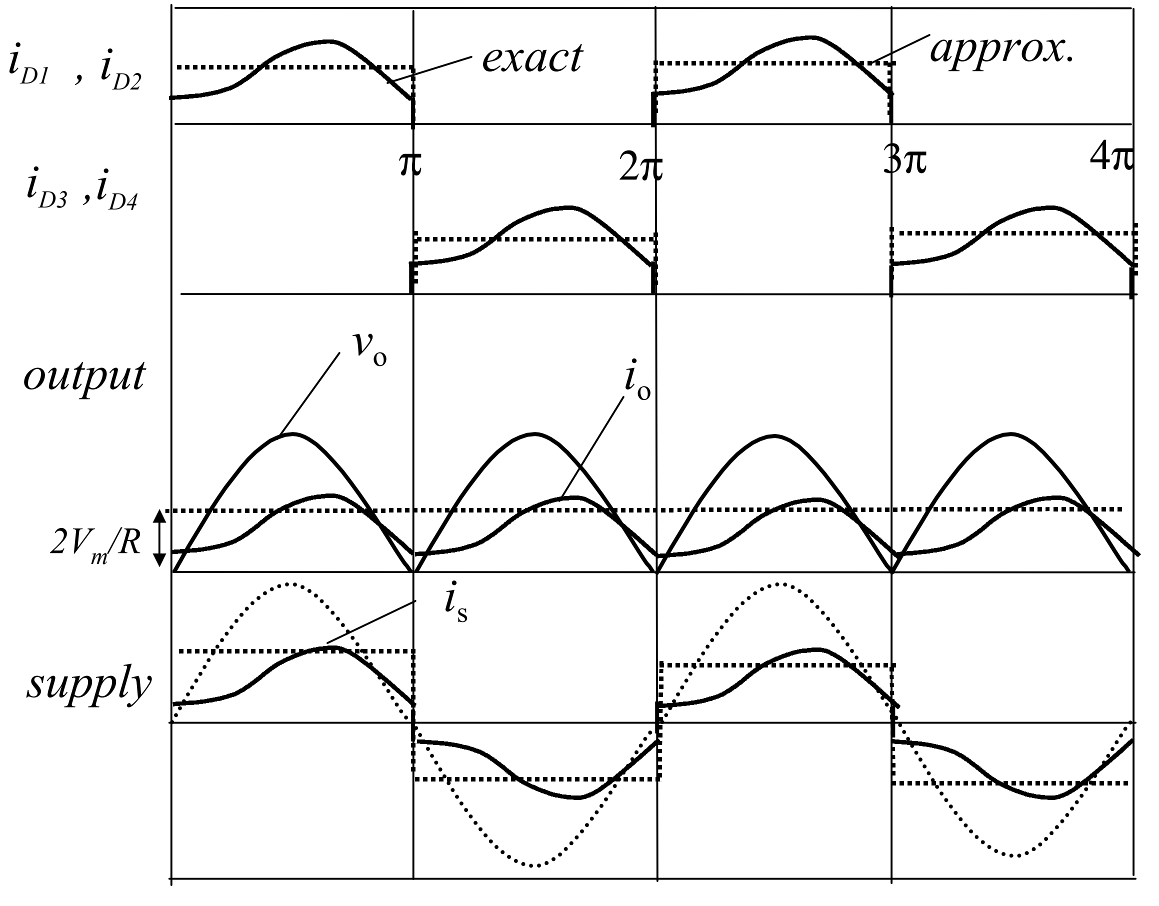
This makes I_n decreases rapidly for increasing n. Only a few terms sufficient to approximate output.

R-L load analysis

If ωL is large enough, it is possible to drop all the harmonic terms, i.e. :

$$i(\omega t) \approx I_o = \frac{V_o}{R} = \frac{2V_m}{R}, \quad \text{for } \omega L >> R,$$

The approximation with large L is shown below.



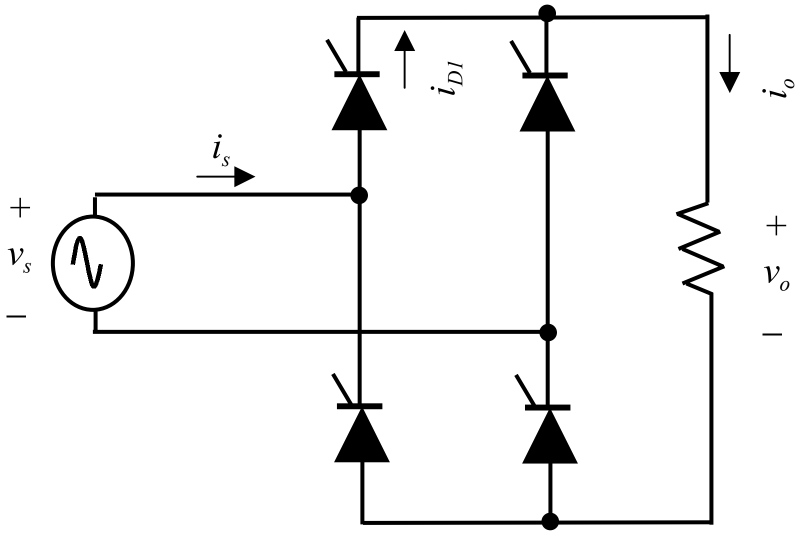
Examples

$$I_{RMS} = \sqrt{I_o^2 + \sum \left(I_{n,RMS}^2 \right)} = I_o$$

$$\text{Power delivered to the load}: P_o = I_{RMS}^2 R$$

- EXAMPLE: Given a bridge rectifier has an AC source V_m=100V at 50Hz, and R-L load with R=10ohm, L=10mH
  - a) determine the average current in the load
  - b) determine the first two higher order harmonics of the load current
  - c) determine the power absorbed by the load

Controlled full wave, R load



$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

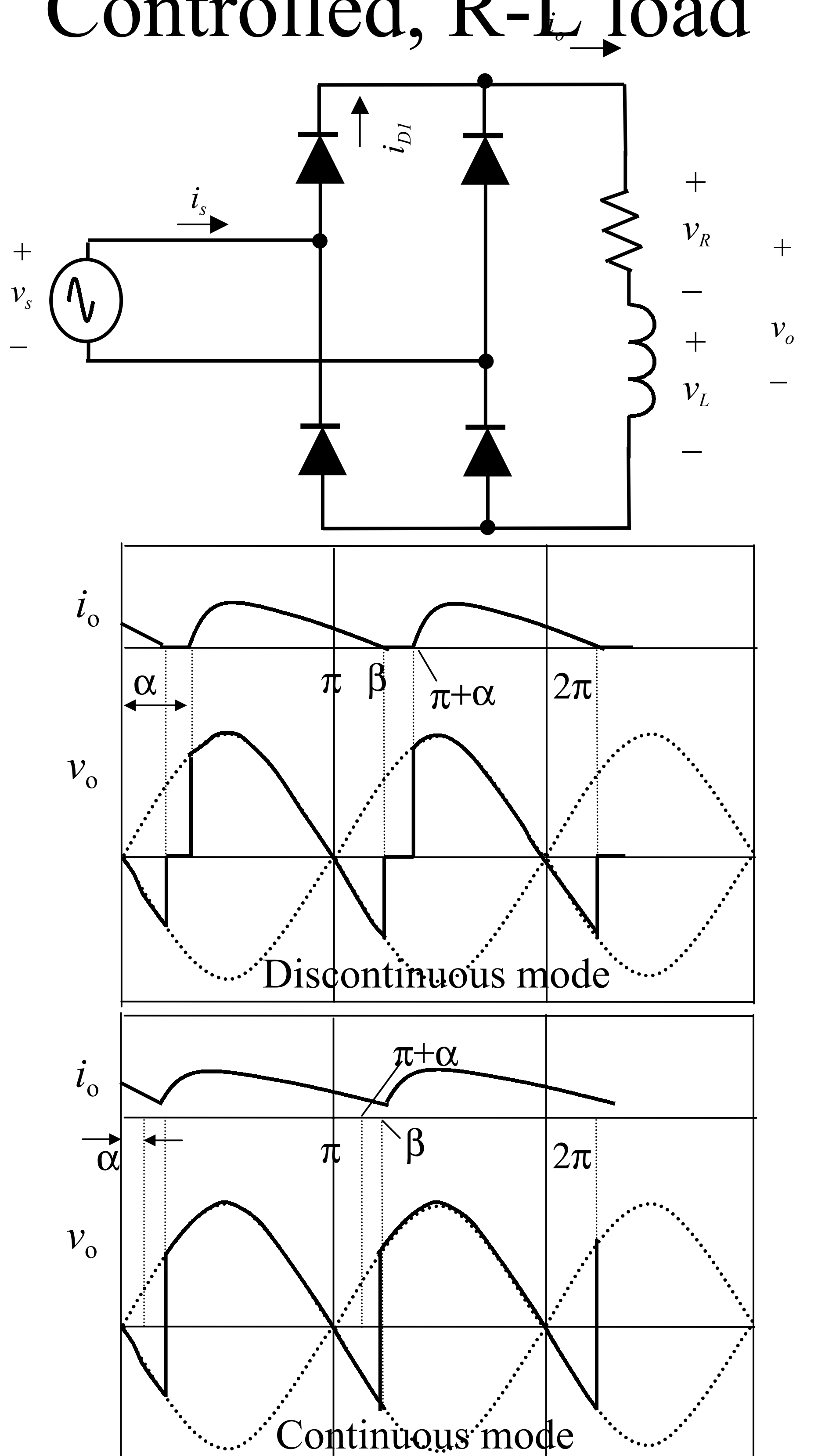
$$V_{RMS} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left[V_m \sin(\omega t)\right]^2 d\omega t}$$

$$= V_m \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The power absorbed by the R load is :

$$P_o = \frac{{V_{RMS}}^2}{R}$$

Controlled, R-L load



Discontinuous mode

Analysis similar to controlled half wave with R - L load :

$$i(\omega t) = \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\omega t - \theta) - \sin(\alpha - \theta) e^{-(\omega t - \alpha)/\omega\tau}\right]$$

for $\alpha \le \omega t \le \beta$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$; $\tau = \frac{L}{R}$

For discontinous mode, need to ensure :

$$\beta < (\alpha + \pi)$$

Note that β is the extinction angle and must be solved numerically with condition :

$$i_o(\beta) = 0$$

The boundary between continous and discontinous current mode is when β in the output current expression is $(\pi + \alpha)$.

For continous operation current at $\omega t = (\pi + \alpha)$ must be greater than zero.

Continuous mode

$$i(\pi + \alpha) \geq 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta) e^{-(\pi + \alpha - \alpha)/\omega\tau} \geq 0$$

Using Trigonometry identity :

$$\sin(\pi + \alpha - \theta) = \sin(\theta - \alpha),$$

$$\sin(\theta - \alpha)\left[1 - e^{-(\pi/\omega\tau)}\right] \geq 0,$$

Solving for α

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

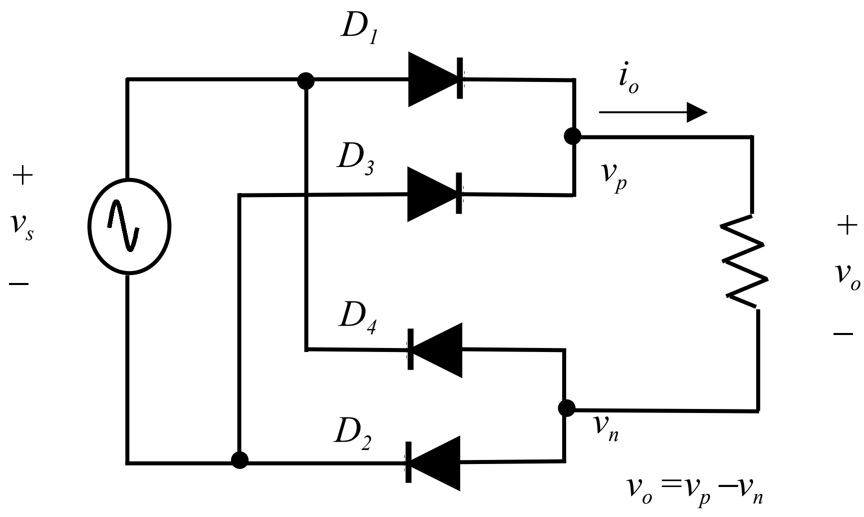
Thus for continuous current mode,

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Average (DC) output voltage is given as :

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t) d\omega t = \frac{2V_m}{\pi} \cos\alpha$$

Single-phase diode groups



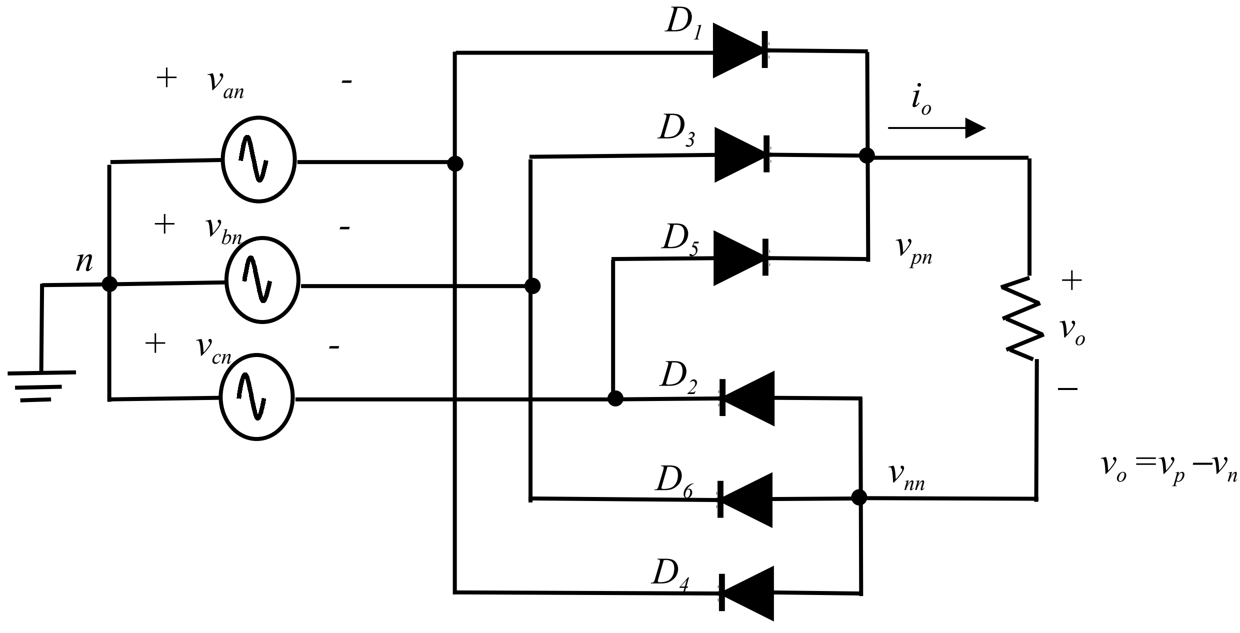
- In the top group (D_1, D_3), the cathodes (-) of the two diodes are at a common potential. Therefore, the diode with its anode (+) at the highest potential will conduct (carry) i_d.

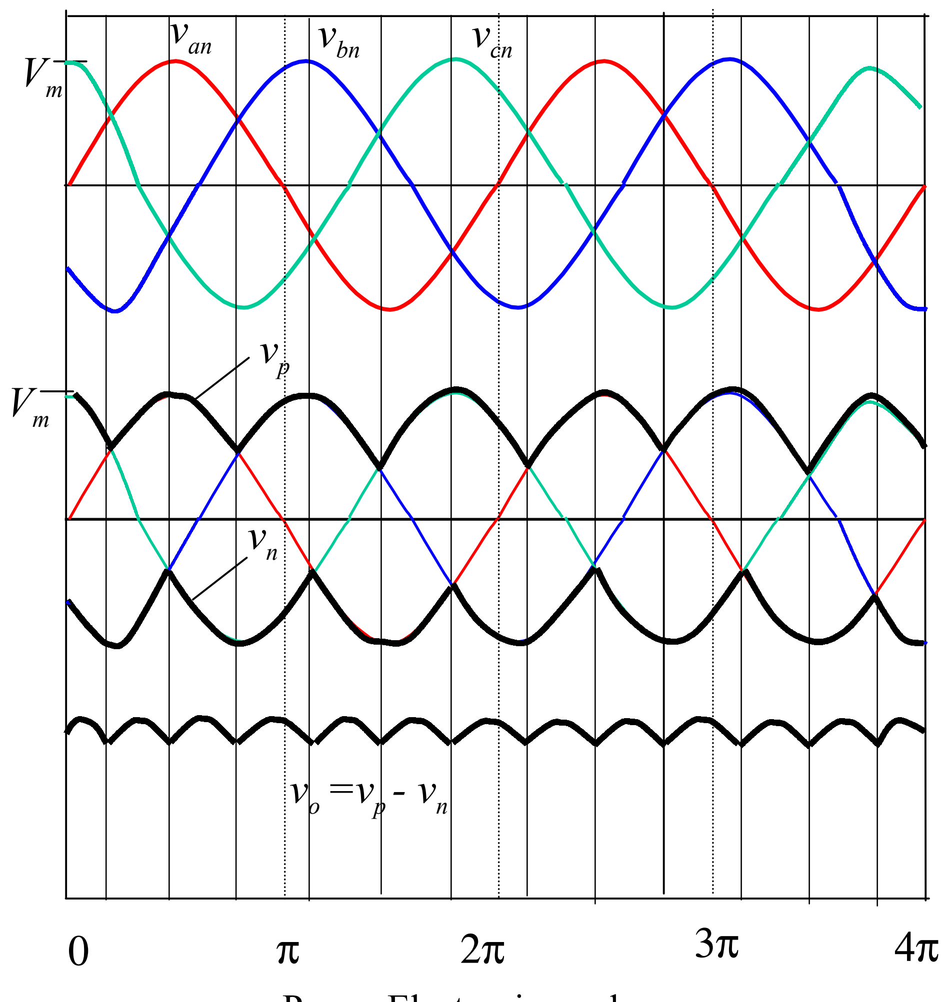
- For example, when v_s is (+), D_1 conducts i_d and D_3 reverses (by taking loop around v_s, D_1 and D_3). When v_s is (-), D_3 conducts, D_1 reverses.

- In the bottom group, the anodes of the two diodes are at common potential. Therefore the diode with its cathode at the lowest potential conducts i_d.

- For example, when v_s (+), D_2 carry i_d. D_4 reverses. When v_s is (-), D_4 carry i_d. D_2 reverses.

Three-phase rectifiers





Three-phase waveforms

- Top group: diode with its anode at the highest potential will conduct. The other two will be reversed.

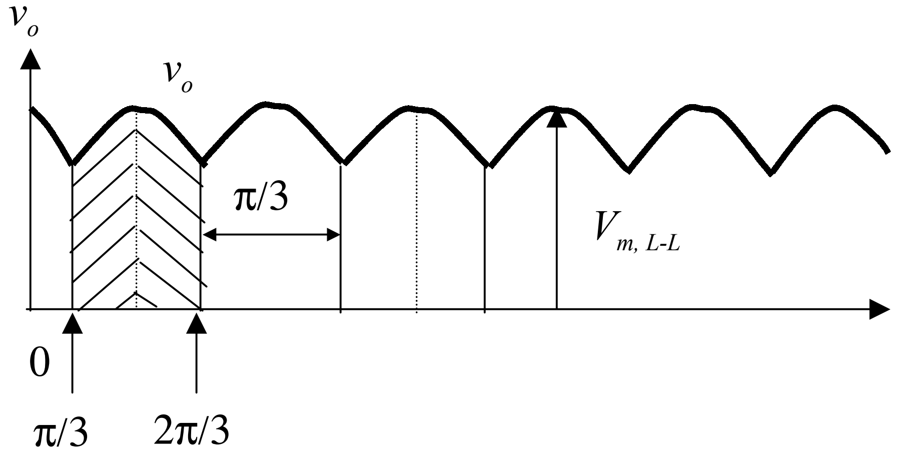
- Bottom group: diode with the its cathode at the lowest potential will conduct. The other two will be reversed.

- For example, if D_1 (of the top group) conducts, v_p is connected to $v_{an.}$. If D_6 (of the bottom group) conducts, v_n connects to v_{bn}. All other diodes are off.

- The resulting output waveform is given as: $v_o = v_p - v_n$

- For peak of the output voltage is equal to the peak of the line to line voltage v_{ab} .

Three-phase, average voltage



Considers only one of the six segments. Obtain its average over 60 degrees or $\pi/3$ radians.

Average voltage:

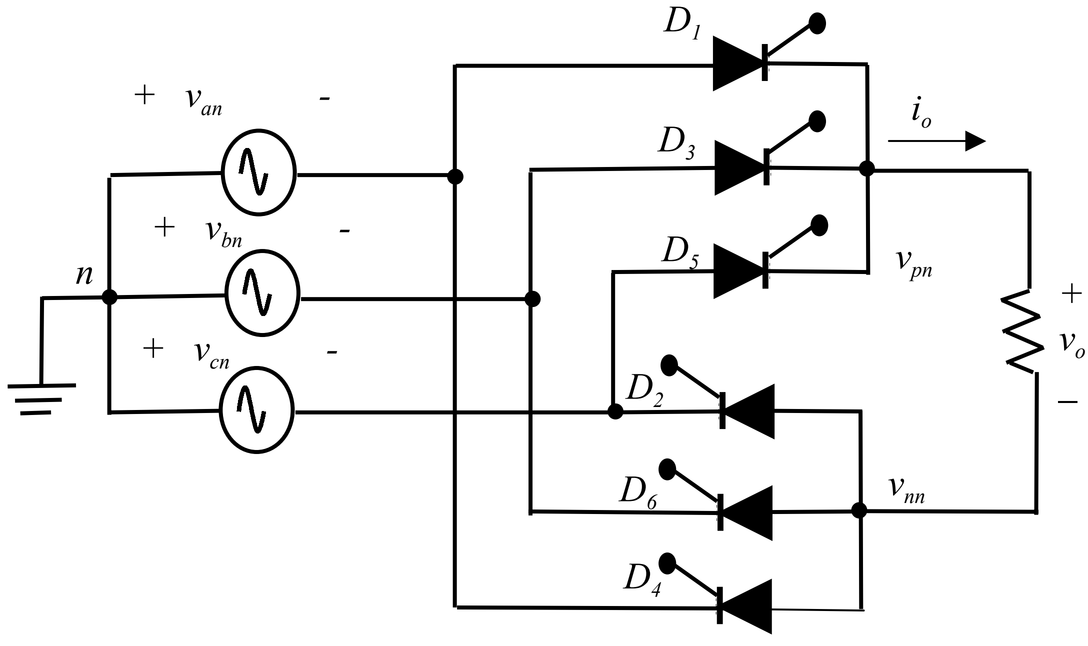
$$V_o = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{m,L-L}\sin(\omega t)d\omega t$$

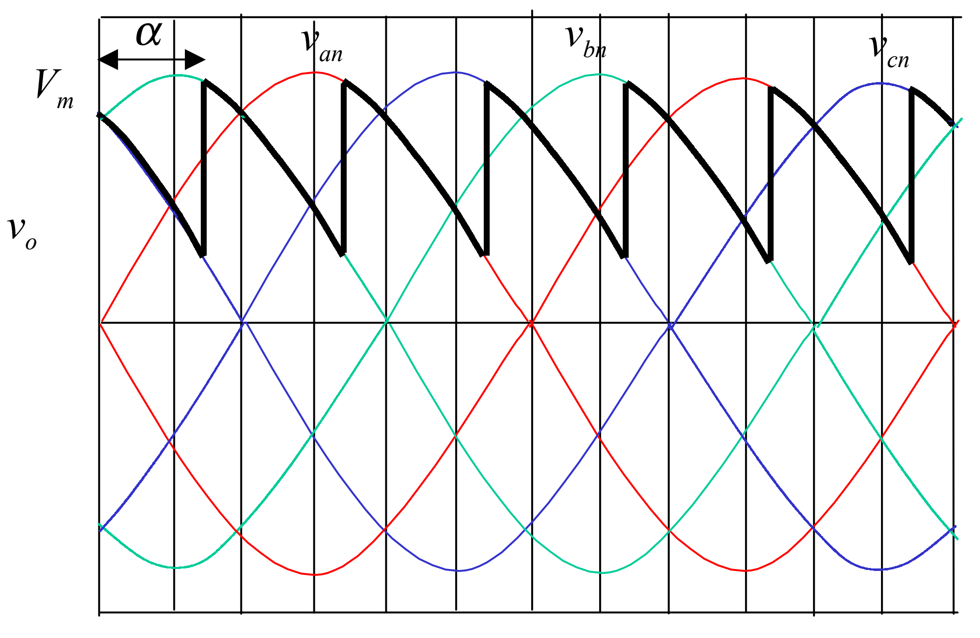
$$= \frac{3V_{m,L-L}}{\pi}\left[\cos(\omega t)\right]_{\pi/3}^{2\pi/3}$$

$$= \frac{3V_{m,L-L}}{\pi} = 0.955V_{m,L-L}$$

Note that the output DC voltage component of a three-phase rectifier is much higher than of a single-phase.

Controlled, three-phase





Output voltage of controlled three phase rectifier

From the previous Figure, let α be the delay angle of the SCR.

Average voltage can be computed as :

$$V_o = \frac{1}{\pi/3} \int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_{m,L-L} \sin(\omega t) d\omega t$$

$$= \left(\frac{3V_{m,L-L}}{\pi} \right) \cdot \cos\alpha$$

- EXAMPLE: A three-phase controlled rectifier has an input voltage of 415V RMS at 50Hz. The load R=10 ohm. Determine the delay angle required to produce current of 50A.