Question 1

The averaged circuit model of an ideal boost converter is shown in Figure Q1. The boost converter has L = 75 μH, C = 330 μF, input voltage, v_d = 18 V, output voltage, v_o = 30 V, load resistance, R = 30 Ω, and switching frequency f_s = 100 kHz. The circuit operates in continuous conduction mode.

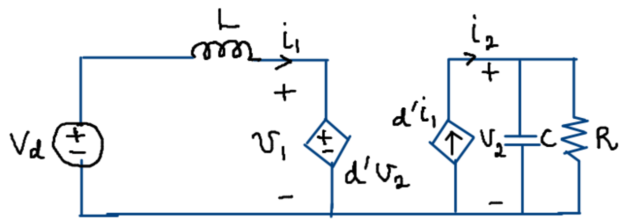


Figure Q1

(i) By doing perturbation and linearization on the averaged circuit model, obtain AC small-signal model and DC steady-state model. [5 marks]

(ii) Analyze DC steady-state model to determine all bias point constants. [5 marks]

Question 2

Analyze AC small signal model to obtain control to output transfer function, $\frac{\hat{v}_o}{\hat{d}}(s)$ for the AC small signal circuit in Figure Q2. A and B are circuit constants. [4 marks]

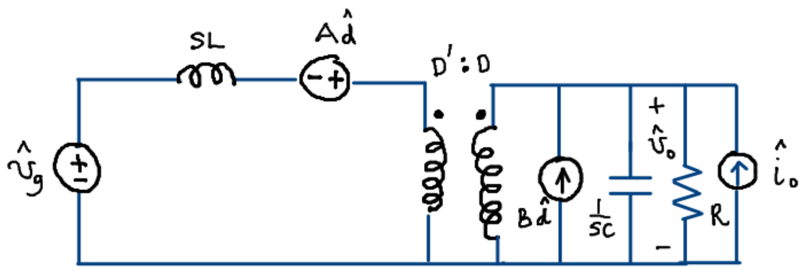


Figure Q2

Question 3

A buck converter has L = 35 μH, C = 440 μF, equivalent series resistance of capacitor, R_{ESR} = 25 mΩ, the input voltage, V_d = 18 V, the output voltage, v_o = 12 V, load resistance, R = 6 Ω, and the switching frequency f_s = 100 kHz. The PWM has the sawtooth peak V_m = 2 V and the error amplifier has V_{ref} = 2 V. The circuit operates in continuous conduction mode.

It is desired to design a type-3 error amplifier, as shown in the **Appendix B**, based on pole-zero placement, that will give the crossover frequency of 10 kHz. The poles and zeros of the error amplifier are placed as follows: (i) the two zeros are placed at resonant frequency, f_o; (ii) the first and second poles are placed at f_{zSER} and $0.75f_s$, respectively.

(i) Determine the magnitude in dB and phase in degrees of the loop gain at the crossover frequency, including error amplifier (with ω_i =1). [6 marks]

(ii) What is the gain of the error amplifier required at the crossover frequency? [2 marks]

(iii) Determine all resistance and capacitance values of the type-3 error amplifier (see Appendix A). [4 marks

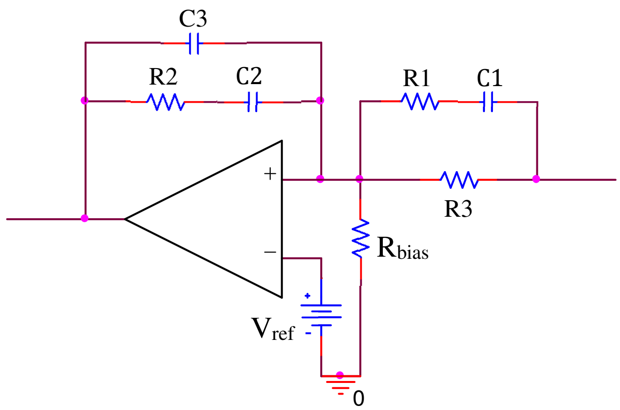
Potentially Useful Formula

Transfer Function	With s = jω	Gain	Phase
$1+\frac{s}{\omega_z}$	$1+j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$1-\frac{s}{\omega_z}$	$1-j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$-\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$\frac{1}{1+\frac{s}{\omega_p}}$	$\frac{1}{1+j\frac{\omega}{\omega_p}}$	$\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$
$\frac{1}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$\frac{1}{1-\left(\frac{\omega}{\omega_o}\right)^2+j\frac{\omega}{Q\omega_o}}$	$\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\left(\frac{\omega}{Q\omega_o}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega/Q\omega_o}{1-\left(\frac{\omega}{\omega_o}\right)^2}\right)$

Converter	Control-to-output transfer function, $\frac{\hat{v}_o}{\hat{d}}$	Parameters definition
Buck	$g_{do}\frac{\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=V_d$; $\omega_o=\frac{1}{\sqrt{LC}}$ $\omega_{zESR}=\frac{1}{rC}$; $Q=\frac{R}{\omega_o L}$
Boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{L_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$
Buck-boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{DL_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$

Appendix B

Pole-zero placement for type 3 error amplifier



1. Calculate the gain, Aco (in dB), and the phase, Pco (in degree), of the loop gain at the cross over frequency, fco. For the error amplifier, ω_i = 1.

2. Calculate the error amplifier gain, ω_i, required:

$$\omega_i = 10^{\frac{-Aco}{20}}$$

3. Calculate capacitances and resistances consecutively:

R_3 = 10 kΩ

$$C_3 = \frac{f_{z2}}{(\omega_i R_3 f_{p2})}$$

$$C_2 = C_3\left(\frac{f_{p2}}{f_{z2}} - 1\right)$$

$$R_2 = \frac{1}{2\pi f_{z2} C_2}$$

$$R_1 = \frac{R_3}{\left(\frac{f_{p1}}{f_{z1}} - 1\right)}$$

$$C_1 = \frac{1}{2\pi f_{p1} R_1}$$

$$R_{bias} = \frac{V_{ref}}{V_o - V_{ref}} R_3$$