Question 1

The averaged circuit model of an ideal buck converter is shown in Figure Q1. The buck converter has L = 75 μH, C = 330 μF, input voltage, v_d = 18 V, output voltage, v_o = 12 V, load resistance, R = 6 Ω, and switching frequency f_s = 100 kHz. The circuit operates in continuous conduction mode.

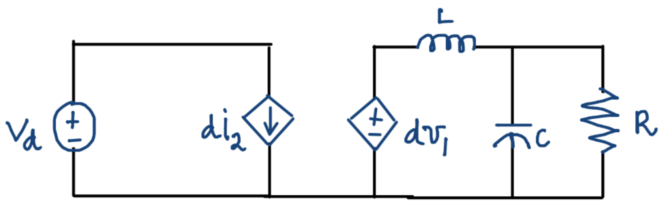


Figure Q1

(i) By doing perturbation and linearization on the averaged circuit model, obtain AC small-signal model and DC steady-state model. [5 marks]

(ii) Analyze DC steady-state model to determine all bias point constants. [5 marks]

Question 2

Analyze AC small signal model to obtain control to output transfer function, $\frac{\hat{v}_o}{\hat{d}}(s)$ for the AC small signal circuit in Figure Q2. A and B are circuit constants.

[3 marks]

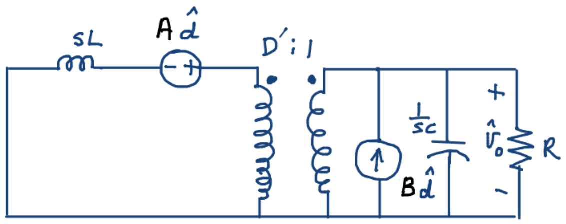


Figure Q2

Question 3

A boost converter has L = 22 μH, C = 440 μF, equivalent series resistance of capacitor, R_{ESR} = 15 mΩ, input voltage, V_d = 12 V, output voltage, v_o = 18 V, load resistance, R = 6 Ω, and switching frequency f_s = 100 kHz. The PWM has the sawtooth peak V_m = 2 V and the error amplifier has V_{ref} = 2 V. The circuit operates in continuous conduction mode.

It is desired to design a type-3 compensated error amplifier, as shown in the **Appendix B**, based on k-factor approach, that will give a phase margin of at least 45° at the crossover frequency of 4 kHz.

(i) Determine the magnitude in dB and phase in degrees of the loop gain at the crossover frequency, excluding error amplifier. [5 marks]

(ii) What are the gain and the phase boost of the error amplifier required at the crossover frequency? [2 marks]

(iii) Determine all resistance and capacitance values of the type-3 compensated error amplifier (see Appendix A). [4 marks

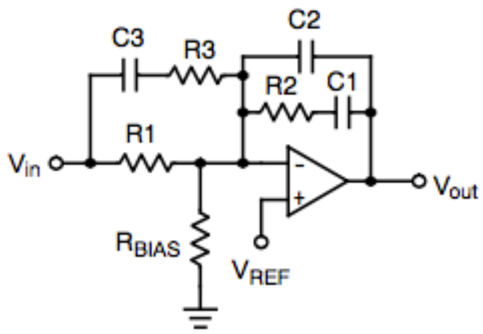
Potentially Useful Formula

Transfer Function	With s = jω	Gain	Phase
$1+\frac{s}{\omega_z}$	$1+j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$1-\frac{s}{\omega_z}$	$1-j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$-\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$\frac{1}{1+\frac{s}{\omega_p}}$	$\frac{1}{1+j\frac{\omega}{\omega_p}}$	$\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$
$\frac{1}{1+\frac{s}{Q\omega_o}+\left(\frac{\omega}{\omega_o}\right)^2}$	$\frac{1}{1-\left(\frac{\omega}{\omega_o}\right)^2+j\frac{\omega}{Q\omega_o}}$	$\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\left(\frac{\omega}{Q\omega_o}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega / Q\omega_o}{1-\left(\frac{\omega}{\omega_o}\right)^2}\right)$

Converter	Control-to-output transfer function, $\frac{\hat{v}_o}{\hat{d}}$	Parameters definition
Buck	$g_{do}\frac{\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=V_d$; $\omega_o=\frac{1}{\sqrt{LC}}$ $\omega_{zESR}=\frac{1}{rC}$; $Q=\frac{R}{\omega_o L}$
Boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{L_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$
Buck-boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{DL_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$

Appendix B

K-factor approach for type 3 error amplifier



1. Calculate the gain, Aco (in dB), and the phase, Pco (in degree), of the power stage including PWM at the cross over frequency, fco.

2. Calculate the amplifier phase boost, Pboost, and gain, G, required:

$$Pboost = PM - 90^\circ - Pco$$

$$G = 10^{\frac{-Aco}{20}}$$

3. Calculate K factor:

$$K = \tan^2\left(\frac{Pboost}{4} + 45^\circ\right)$$

4. Calculate capacitances and resistances consecutively:

$$R1 = 4.7\,\text{k}\Omega$$

$$R_{bias} = \frac{V_{ref}R_1}{V_o - V_{ref}}$$

$$C2 = \frac{1}{2\pi f_{co} G R1}$$

$$C1 = C2(K-1)$$

$$R2 = \frac{\sqrt{K}}{2\pi f_{co} C1}$$

$$R3 = \frac{R1}{(K-1)}$$

$$C3 = \frac{1}{2\pi f_{co}\sqrt{K}R3}$$