Question 3
AC small-signal Buck Converter

Define unit:

$\mu \equiv 10^{-6}$ $m \equiv 10^{-3}$ $V := 1$ $k\Omega \equiv 10^{3}$ $\Omega \equiv 1$ $k \equiv 10^{3}$ $H_Z \equiv 1$ $nF \equiv 10^{-9}$ $pF \equiv 10^{-12}$

$dB \equiv 1$ $rad \equiv 1$ $kHz \equiv 10^{3}$

Buck Parameters:

$L := 35\mu$ $C := 440\mu$ $R_{ESR} := 25m$ $V_d := 18V$ $V_o := 12V$ $R := 6\Omega$

$f_s := 100k$ $V_m := 2V$ $V_{ref} := 2V$

Compensator Specifications:

$f_{co} := 10k$ $\omega_{co} := 2 \cdot \pi f_{co}$

Pole-zero Locations:

$$\omega_{zESR} := \frac{1}{R_{ESR} \cdot C} \qquad \omega_{zESR} = 9.091 \times 10^{4} \qquad f_{zESR} := \frac{\omega_{zESR}}{2\pi} \qquad f_{zESR} = 14.469\,kHz$$

$$\omega_o := \frac{1}{\sqrt{L \cdot C}} \qquad \omega_o = 8.058 \times 10^{3} \qquad f_o := \frac{\omega_o}{2\pi} \qquad f_o = 1.283\,kHz$$

$$Q := \frac{R}{\omega_o \cdot L} \qquad Q = 21.274 \qquad g_{od} := V_d$$

Bode plot setting:

$N := 1000$ $r := 0 .. N - 1$ $f_{start} := 100$ $f_{stop} := 1000k$ $f_r := f_{start} \cdot \left(\frac{f_{stop}}{f_{start}}\right)^{\frac{r}{N-1}}$

$$G_{oc}(s) := g_{od} \cdot \frac{\left(1 + \frac{s}{\omega_{zESR}}\right)}{1 + \frac{s}{Q \cdot \omega_o} + \frac{s^2}{\omega_o{}^2}} \cdot \frac{1}{V_m}$$

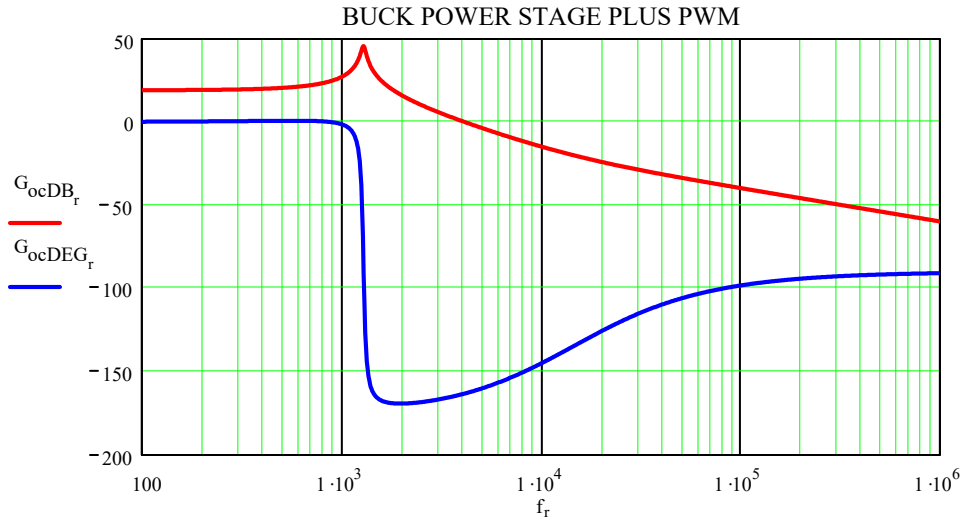
$20\log\left(\left|G_{oc}(j \cdot 2 \cdot \pi \cdot 16.666k)\right|\right) = -21.747$ Aco

$\arg\left(G_{oc}(j \cdot 2\pi \cdot 16.666k)\right) = -130.754\,deg$ Pco

Define quantities for Bode plotting:

$$G_{ocDB_r} := 20 \cdot \log\left(\left|G_{oc}(j \cdot 2 \cdot \pi \cdot f_r)\right|\right)$$

$$G_{ocDEG_r} := \arg\left(G_{oc}(j \cdot 2 \cdot \pi \cdot f_r)\right) \cdot \frac{180}{\pi}$$



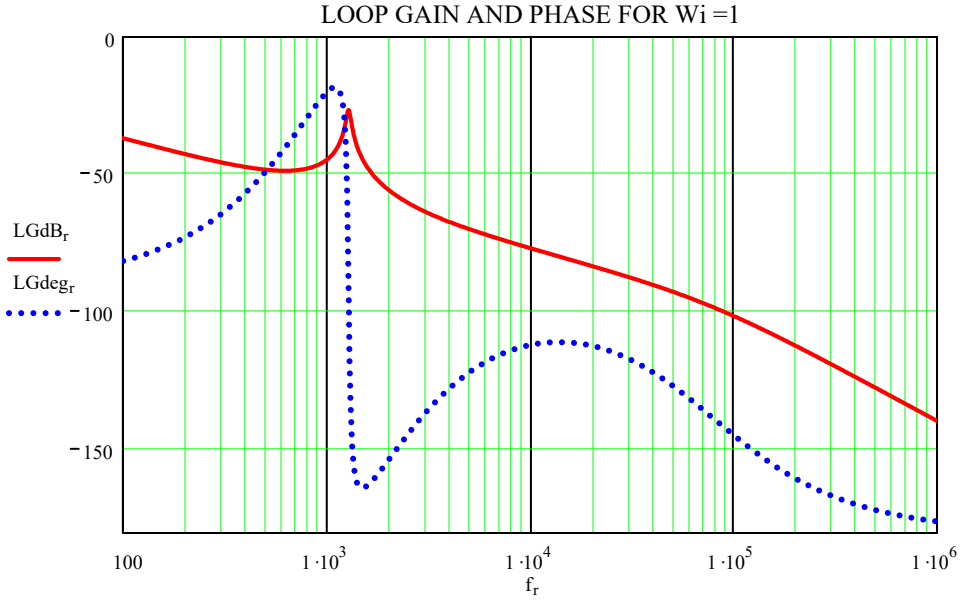
Pole-zero Placement for compensator design:

$f_{z1} := f_o \qquad f_{z2} := f_o \qquad f_{p1} := f_{zESR} \qquad f_{p2} := 0.75 f_s \qquad \omega_i := 1$

$$C(s) := \frac{\omega_i \cdot \left(1 + \frac{s}{2 \cdot \pi \cdot f_{z1}}\right) \cdot \left(1 + \frac{s}{2 \cdot \pi \cdot f_{z2}}\right)}{s \cdot \left(1 + \frac{s}{2 \cdot \pi \cdot f_{p1}}\right) \cdot \left(1 + \frac{s}{2 \cdot \pi \cdot f_{p2}}\right)} \qquad LG(s) := G_{oc}(s) \cdot C(s)$$

Define quantities for Bode plot of Loop Gain LG:

$$LGdeg_r := \arg\left(LG\left(j \cdot 2 \cdot \pi \cdot f_r\right)\right) \cdot \frac{180}{\pi} \qquad LGdB_r := 20 \cdot \log\left(\left|LG\left(j \cdot 2 \cdot \pi \cdot f_r\right)\right|\right)$$



Evaluate LG at fco for gain and phase:

$$LGdB := 20 \cdot \log\left(\left|LG\left(j \cdot 2 \cdot \pi \cdot f_{co}\right)\right|\right) \qquad LGdB = -76.67 \qquad 10^{\frac{-76.67}{20}} = 1.467 \times 10^{-4}$$

$$LGph := \arg\left(LG\left(j \cdot 2 \cdot \pi \cdot f_{co}\right)\right) \cdot \frac{180}{\pi} \qquad LGph = -111.86$$

Phase Margin:

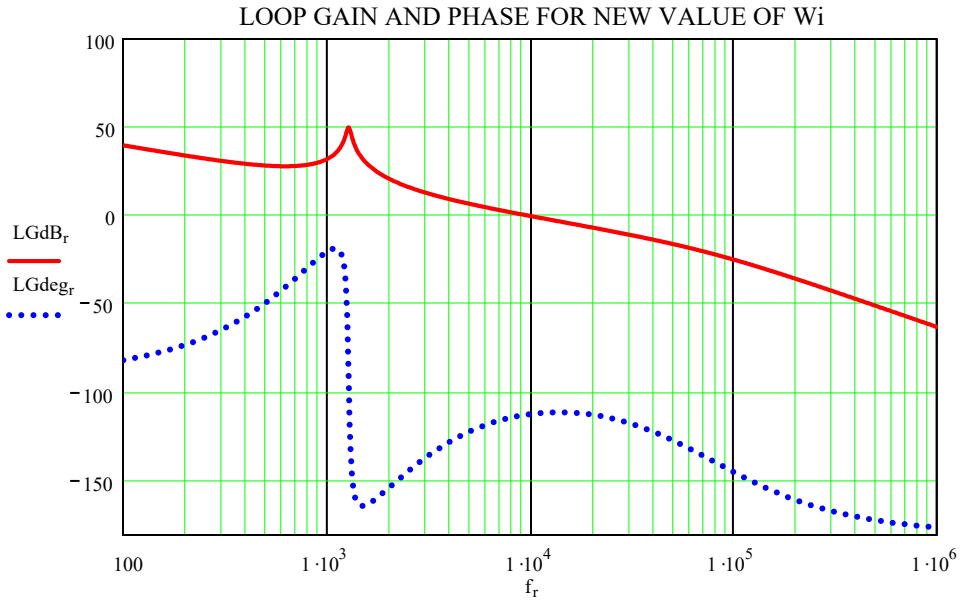
$$PM := 180 + LGph \qquad PM = 68.14$$

The gain, ω_i, required:

$$\omega_i := 10^{\frac{-LGdB}{20}} \qquad \omega_i = 6.815 \times 10^{3}$$

Bode plot for new value for ω_i:

$$LGdB_r := 20 \cdot \log\left(\left|LG\left(j \cdot 2 \cdot \pi \cdot f_r\right)\right|\right) - LGdB$$



To find component values of error amplifier:

Let $R_3 := 10k\Omega$

$$C_3 := \frac{f_{z2}}{\omega_i \cdot R_3 \cdot f_{p2}} \qquad C_3 = 250.908\ pF$$

$$C_2 := C_3 \cdot \left(\frac{f_{p2}}{f_{z2}} - 1\right) \qquad C_2 = 14.422\ nF$$

$$R_2 := \frac{1}{2 \cdot \pi \cdot f_{z2} \cdot C_2} \qquad R_2 = 8.605\ k\Omega$$

$$R_1 := \frac{R_3}{\left(\frac{f_{p1}}{f_{z1}} - 1\right)} \qquad R_1 = 0.973\ k\Omega$$

$$C_1 := \frac{1}{2 \cdot \pi \cdot f_{p1} \cdot R_1} \qquad C_1 = 11.31\ nF$$

$$R_{bias} := \frac{V_{ref}}{V_o - V_{ref}} \cdot R_3 \qquad R_{bias} = 2\ k\Omega$$