Question 1

The averaged circuit model of an ideal boost converter is shown in Figure Q1. The boost converter has L = 25 μH, C = 660 μF, r_{ESR} = 50 mΩ, input voltage, v_g = 20 V, output voltage, v_o = 30 V, load resistance, R = 15 Ω, and switching frequency f_s = 100 kHz. The circuit operates in continuous conduction mode.

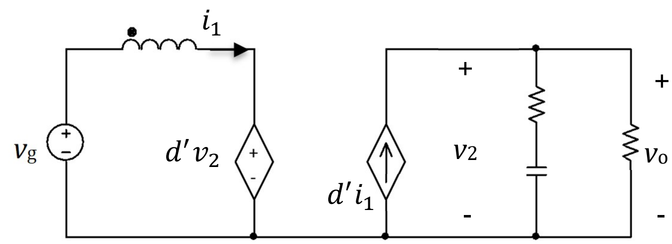

Figure Q1

(i) By doing perturbation and linearization on the averaged circuit model, obtain AC small-signal and DC steady-state model models. [6 marks]

(ii) Analyze DC steady-state model to determine all bias point constants. [3 marks]

(iii) Based on the transfer function for the boost converter in **Table II**, find g_{do}, f_o, Q, f_{zESR}, and f_{zRHP}. Sketch Bode plot for the gain and phase and label key quantities. [5 marks]

Question 2

The boost converter in Question 1 is required to design a controller to regulate the output voltage. The PWM has the sawtooth peak V_m = 4 V and the error amplifier has V_{ref} = 3 V.

It is desired to design a type-3 error amplifier based on pole-zero placement procedure as in **Appendix A**, that will give the crossover frequency, f_{co} = 4 kHz and the phase margin, PM = 50°. The poles and zeros of the error amplifier are placed as follows: (i) the two zeros (f_{z1} and f_{z2}) are placed at f_o, (ii) the first pole, f_{p1} is placed at f_{zRHP}, (iii) The second pole, f_{p2}, is placed as in the Appendix A.

(i) Determine the magnitude in dB, A_{codB}, and phase in degrees, P_{co}, of the boost converter (including PWM) at the crossover frequency. [[7 marks]

(ii) Determine the phase boost, Pboost, required, the location of the second pole of error amplifier, f_{p2} and the gain of the integrator, ω_i. [5 marks]

(iii) Determine all resistance and capacitance values of the type-3 error amplifier (see Appendix A). [4 marks]

Potentially Useful Formula

Table I

Transfer Function	With s = jω	Gain	Phase
$1+\frac{s}{\omega_z}$	$1+j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$1-\frac{s}{\omega_z}$	$1-j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$-\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$\frac{1}{1+\frac{s}{\omega_p}}$	$\frac{1}{1+j\frac{\omega}{\omega_p}}$	$\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$
$\frac{1}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$\frac{1}{1-\left(\frac{\omega}{\omega_o}\right)^2+j\frac{\omega}{Q\omega_o}}$	$\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\left(\frac{\omega}{Q\omega_o}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega/Q\omega_o}{1-\left(\frac{\omega}{\omega_o}\right)^2}\right)$

Table 2

Converter	Control-to-output transfer function, $\frac{\hat{v}_o}{\hat{d}}$	Parameters definition
Buck	$g_{do}\frac{\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=V_d$; $\omega_o=\frac{1}{\sqrt{LC}}$ $\omega_{zESR}=\frac{1}{r_{ESR}C}$; $Q=R\sqrt{\frac{C}{L}}$
Boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{r_{ESR}C}$; $\omega_{zRHP}=\frac{R}{L_{eq}}$ $Q=R\sqrt{\frac{C}{L_{eq}}}$; $L_{eq}=\frac{L}{(1-D)^2}$
Buck-boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{r_{ESR}C}$; $\omega_{zRHP}=\frac{R}{DL_{eq}}$ $Q=R\sqrt{\frac{C}{L_{eq}}}$; $L_{eq}=\frac{L}{(1-D)^2}$

Appendix A

Design of the type III compensator using pole-zero placement: Boost

Given desired crossover frequency, f_{co}, and phase margin, PM, and the power stage gain, A_{co}, and phase, P_{co}, at the crossover frequency:

(i) The phase boost required by the compensator at the crossover frequency is

$$P_{boost} = PM - 90° - P_{co} \tag{1}$$

(ii) Place the double zero at the LC network resonant frequency, f_o, to avoid oscillatory.

(iii) One pole, f_{p1}, is placed at f_{zRHP} to cancel the gain effect of RHP zero.

(iv) From (3), f_{p2} can be found as follows:

$$\tan^{-1}\left(\frac{f_{co}}{f_{p2}}\right) = \tan^{-1}\left(\frac{f_{co}}{f_{z1}}\right) + \tan^{-1}\left(\frac{f_{co}}{f_{z2}}\right) - \tan^{-1}\left(\frac{f_{co}}{f_{p1}}\right) - P_{boost} \equiv \emptyset_{fp2} \tag{2}$$

$$f_{p2} = \frac{f_{co}}{\tan(\emptyset_{fp2})} \tag{3}$$

(v) The gain of the integrator, ω_i, can be found as follows:

$$A_{co} = 10^{\frac{A_{codB}}{20}} \qquad G = \frac{1}{A_{co}} \quad \text{or} \quad G = 10^{\frac{-A_{codB}}{20}} \tag{4}$$

$$\omega_i = (G)(2\pi f_{co})\frac{\sqrt{\left(1+\left(\frac{f_{co}}{f_{p1}}\right)^2\right)\left(1+\left(\frac{f_{co}}{f_{p2}}\right)^2\right)}}{\sqrt{\left(1+\left(\frac{f_{co}}{f_{z1}}\right)^2\right)\left(1+\left(\frac{f_{co}}{f_{z2}}\right)^2\right)}}$$

$$f_{z1} = f_o, \qquad f_{z2} = f_o, \qquad f_{p1} = f_{zRHP}, \qquad f_{p2} = \frac{f_{co}}{\tan(\emptyset_{fp2})}$$

(vi) Let,

$$R_1 = 4.7\ \text{k}\Omega$$

$$C_2 = \frac{1}{\omega_i R_1 \frac{f_{p2}}{f_{z1}}}$$

$$C_1 = C_2\left(\frac{f_{p2}}{f_{z1}} - 1\right)$$

$$R_2 = \frac{1}{2\pi f_{z1} C_1}$$

$$R_3 = \frac{R_1}{\left(\frac{f_{p1}}{f_{z2}} - 1\right)}$$

$$C_3 = \frac{1}{2\pi f_{p1} R_3}$$

$$R_{bias} = \frac{V_{ref}}{V_o - V_{ref}} R_2$$