Question 1

(a) What do you understand by averaged switch modeling? Explain briefly the procedure to obtain the averaged switch model. [5 marks]

(b) In the boost converter as shown in **Figure Q1**, the averaged model of the switch network is desired. The terminals voltage and current of the switch network are given as v_1, v_2, i_1 and i_2, respectively. The input voltage is constant, v_d. The capacitor, C, is sufficiently large so that the output voltage, v_o, can considerably be constant. The converter operates in continuous conduction mode (CCM) with inductor current repeatedly changes between i_{Lmin} and i_{Lmax} under the steady-state condition.

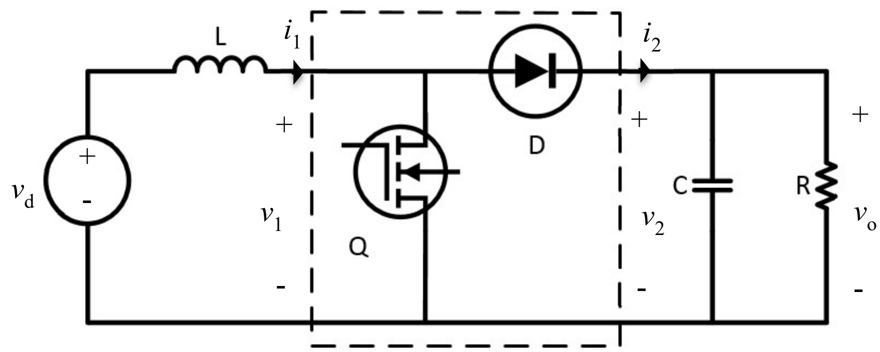


Figure Q1

(i) Sketch the terminal switch network waveforms: v_1, v_2, i_1 and i_2. Label all the key quantities. [5 marks]

(ii) Find the average voltage conversion ratio, $\langle v_2\rangle/\langle v_1\rangle$ and the average current conversion ratio, $\langle i_2\rangle/\langle i_1\rangle$, of the switch network. [5 marks]

(iii) Construct averaged circuit model of the complete boost converter by replacing the switch network with the averaged switch model (DC transformer model) based on the equations found in (ii). [5 marks]

(iv) Obtain the voltage conversion ratio, M_V, and the efficiency, η, of the converter in steady-state under ideal condition. [5 marks]

Question 2

(a) State two types of losses occur in non-isolated DC-DC converters. Explain briefly. [5 marks]

(b) In a buck converter, the MOSFET has the on resistance, R_Q, the diode forward voltage drop can be modeled by constant voltage V_D with resistor R_D, and the inductor has an internal resistance r_L.

(i) Sketch an equivalent circuit, which models the DC properties of this converter. The averaged switch model can be represented by dependent sources or a DC transformer. [5 marks]

(ii) Derive an expression for the efficiency of the converter. [6 marks]

(iii) Derive an expression for the conversion ratio, $M_V(D) = V_o/V_d$. [4 marks]

(iv) The converter is used to step-down the input voltage of V_d = 18 V to the output voltage, V_o = 12 V. If R_Q = 50 mΩ, V_D = 0.7 V, R_D = 60 mΩ and r_L = 0.1 Ω, determine the efficiency of the buck converter. Also, determine the required duty ratio, D. [5 marks]

Question 3

(a) What do you understand by AC small-signal model? Why do we need AC small signal model? [5 marks]

(b) The averaged circuit model of an ideal boost converter is shown in **Figure Q3.** The boost converter has the inductor, L = 75 μH, capacitor, C = 330 μF, the input voltage, v_d = 12 V, the output voltage, v_o = 18 V, the load resistance, R = 6 Ω, and the switching frequency, f_s = 100 kHz. The circuit operates in continuous conduction mode.

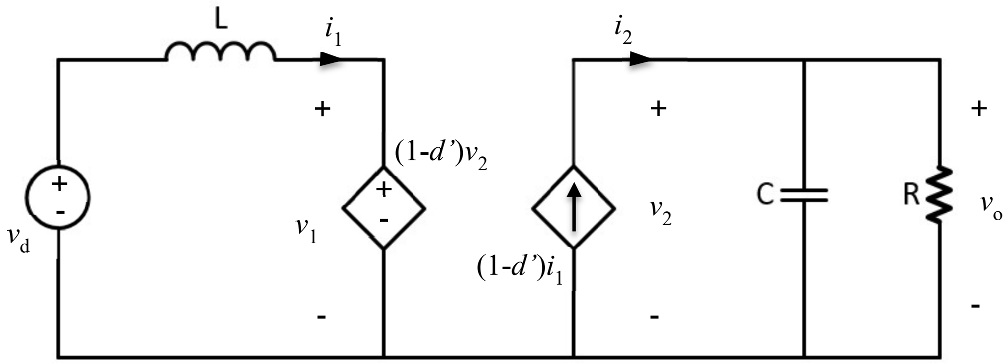


Figure Q3

(i) By doing perturbation and linearization on the averaged circuit model, obtain AC small-signal model and DC steady-state model. [9 marks]

(ii) Analyze the DC steady-state model to determine all bias point constants. [5 marks]

(iii) Analyze the AC small signal model to obtain the control to output transfer function, $\frac{\hat{v}_o}{\hat{d}}(s)$. [6 marks]

Question 4

(a) With the help of a functional block diagram explain the benefits of negative feedback. [5 marks]

(b) A buck-boost converter has the inductor, L = 15 μH, the capcatitor, C = 440 μF, the equivalent series resistance of capacitor, R_{ESR} = 20 mΩ, the input voltage, V_d = 12 V, the output voltage, $|v_o|$ = 5 V, the load resistance, R = 2 Ω, and the switching frequency, f_s = 200 kHz. The PWM has the sawtooth peak, V_m = 3 V and the error amplifier has the voltage reference, V_{ref} = 2.5 V. The circuit operates in continuous conduction mode and the control to output transfer function for voltage mode control is given in **Appendix B**.

It is desired to design a voltage mode controller for the feedback system to regulate the output voltage. The controller is type-3 error amplifier, as shown in the **Appendix E**, and the calculation is based on k-factor approach. The required phase margin is at least 45° and the crossover frequency is at 1 kHz.

(i) Determine the magnitude in dB and phase in degrees of the loop gain at the crossover frequency, excluding error amplifier. [9 marks]

(ii) What are the gain and the phase boost of the error amplifier required at the crossover frequency? [5 marks]

(iii) Determine all resistance and capacitance values of the type-3 error amplifier (see **Appendix E**). [6 marks]

Question 5

(a) Explain the advantages of current mode control over voltage mode control.

[5 marks]

(b) A buck converter has the L = 25 μH, C = 660 μF, the equivalent series resistance of capacitor, R_{ESR} = 20 mΩ, the input voltage, V_d = 12 V, the output voltage, v_o = 5 V, the load resistance, R = 1.25 Ω, and the switching frequency f_s = 200 kHz. The error amplifier has V_{ref} = 2.5 V. The circuit operates in continuous conduction mode and the control to output transfer function is given in **Appendix C**.

It is desired to design a current mode controller for the outer loop of the feedback system. The controller is type-2 error amplifier, as shown in the **Appendix D**, and the calculation is based on k-factor approach. The required phase margin is at least 65° and the crossover frequency is at 6 kHz.

(i) Determine the required amplitude of the external ramp. [4 marks]

(ii) Determine the magnitude in dB and the phase in degrees of the loop gain at the crossover frequency. [8 marks]

(iii) What are the gain and the phase boost of the error amplifier required at the crossover frequency? [3 marks]

(iv) Determine all resistance and capacitance values of the type-2 error amplifier (see **Appendix D**). [5 marks]

Appendix A

Potentially Useful Formula

Transfer Function	With s = jω	Gain	Phase
$1+\frac{s}{\omega_z}$	$1+j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$1-\frac{s}{\omega_z}$	$1-j\frac{\omega}{\omega_z}$	$\sqrt{1+\left(\frac{\omega}{\omega_z}\right)^2}$	$-\tan^{-1}\left(\frac{\omega}{\omega_z}\right)$
$\frac{1}{1+\frac{s}{\omega_p}}$	$\frac{1}{1+j\frac{\omega}{\omega_p}}$	$\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$
$\frac{1}{1+\frac{s}{Q\omega_o}+\left(\frac{\omega}{\omega_o}\right)^2}$	$\frac{1}{1-\left(\frac{\omega}{\omega_o}\right)^2+j\frac{\omega}{Q\omega_o}}$	$\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\left(\frac{\omega}{Q\omega_o}\right)^2}}$	$-\tan^{-1}\left(\frac{\omega/Q\omega_o}{1-\left(\frac{\omega}{\omega_o}\right)^2}\right)$

Appendix B

Control to output transfer functions for voltage mode control

Converter	Control-to-output transfer function, $\frac{\hat{v}_o}{\hat{d}}$	Parameters definition
Buck	$g_{do}\frac{\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=V_d$; $\omega_o=\frac{1}{\sqrt{LC}}$ $\omega_{zESR}=\frac{1}{rC}$; $Q=\frac{R}{\omega_o L}$
Boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{L_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$
Buck-boost	$g_{do}\frac{\left(1-\frac{s}{\omega_{zRHP}}\right)\left(1+\frac{s}{\omega_{zESR}}\right)}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$	$g_{do}=\frac{V_d}{(1-D)^2}$; $\omega_o=\frac{1}{\sqrt{L_{eq}C}}$ $\omega_{zESR}=\frac{1}{rC}$; $\omega_{zRHP}=\frac{R}{DL_{eq}}$ $Q=\frac{R}{\omega_o L_{eq}}$; $L_{eq}=\frac{L}{(1-D)^2}$

Appendix C

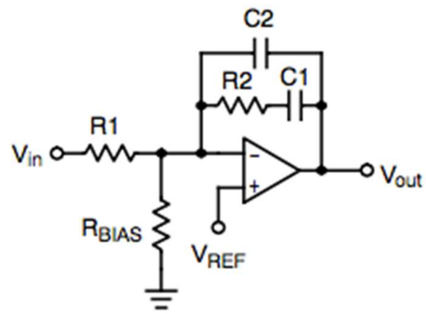
Control to output transfer functions, $\frac{\hat{v}_o}{\hat{i}_L}$, for current mode control

Converter Transfer Functions for Average Current Mode Controller

Converter	$\frac{\hat{i}_L}{\hat{d}}$	$\frac{\hat{v}_o}{\hat{i}_L}$
Buck	$\frac{V_g}{R}\frac{(1+sRC)}{1+s\left(\frac{L}{R}+rC\right)+s^2LC}$	$\frac{R(1+srC)}{1+sRC}$
Boost	$\frac{V_g}{R(1-D)^3}\frac{(2+sRC)}{1+s\frac{L}{R(1-D)^2}+s^2\frac{LC}{(1-D)^2}}$	$\frac{R(1-D)\left(1-\frac{sL}{R(1-D)^2}\right)(1+srC)}{2+sRC}$
Buck-boost	$\frac{V_g(1+D)}{R(1-D)^3}\frac{\left(1+\frac{1}{1+D}sRC\right)}{1+s\frac{L}{R(1-D)^2}+s^2\frac{LC}{(1-D)^2}}$	$\frac{R(1-D)\left(1-\frac{sLD}{R(1-D)^2}\right)(1+srC)}{(1+D)\left(1+\frac{1}{1+D}sRC\right)}$

Appendix D

K-factor approach for type 2 error amplifier



1. Calculate the gain, Aco, and the phase, Pco, of the power stage including PWM at the cross over frequency, fco.

2. Calculate the phase boost, Pboost, required:

$$Pboost = PM - 90^{\circ} - Pco$$

3. Calculate k factor:

$$K = \tan\left(\frac{Pboost}{2} + 45^{\circ}\right)$$

$$G = 10^{\frac{-Aco}{20}}$$

4. Calculate capacitances and resistances consecutively:

$$R1 = 4.7\,\mathrm{k}\Omega$$

$$R_{bias} = \frac{V_{ref} R_1}{V_o - V_{ref}}$$

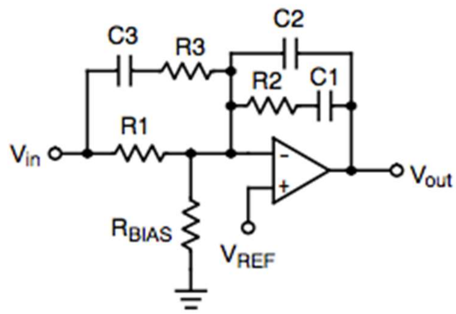
$$C2 = \frac{1}{2\pi f_{co} G K R1}$$

$$C1 = C2(K^2 - 1)$$

$$R2 = \frac{K}{2\pi f_{co} C1}$$

Appendix E

K-factor approach for type 3 error amplifier



1. Calculate the gain, Aco (in dB), and the phase, Pco (in degree), of the power stage including PWM at the cross over frequency, fco.

2. Calculate the amplifier phase boost, Pboost, and gain, G, required:

$$Pboost = PM - 90^{\circ} - Pco$$

$$G = 10^{\frac{-Aco}{20}}$$

3. Calculate K factor:

$$K = \tan^2\left(\frac{Pboost}{4} + 45^{\circ}\right)$$

4. Calculate capacitances and resistances consecutively:

$$R1 = 4.7\,\text{k}\Omega$$

$$R_{bias} = \frac{V_{ref} R_1}{V_o - V_{ref}}$$

$$C2 = \frac{1}{2\pi f_{co} G R1}$$

$$C1 = C2(K-1)$$

$$R2 = \frac{\sqrt{K}}{2\pi f_{co} C1}$$

$$R3 = \frac{R1}{(K-1)}$$

$$C3 = \frac{1}{2\pi f_{co}\sqrt{K} R3}$$