Test #2: Solution Version: 25Dec PRELIMINARY RESULTS

Dummy unit: $\Omega \equiv 1$ $A \equiv 1$ $V \equiv 1$ $W \equiv 1$ $kW \equiv 1 \cdot 10^3 \cdot W$ $Hz \equiv 1$

$Nm \equiv 1$ $rpm \equiv 1$

Q1: DC Machine - speed of the motor under full load condition

Under full load: $P_{Rated} := 10.0kW$ $E_{aRated} := 100V$ $V_t := 100V$ $R_a := 0.1\Omega$

$I_{aFL} := \frac{P_{Rated}}{E_{aRated}}$ $I_{aFL} = 100\,A$

$E_{aFL} := V_t - I_{aFL} \cdot R_a$ $E_{aFL} = 90\,V$

From the graph, $E_a = 90V$ $I_f = 0.7A$

Under no load: $I_{aNL} := 10A$ $\omega_{rpmNL} := 1000rpm$

$E_{aNL} := V_t - I_{aNL} \cdot R_a$ $E_{aNL} = 99\,V$

From the graph, $E_a = 99V$ $I_f = 0.95A$

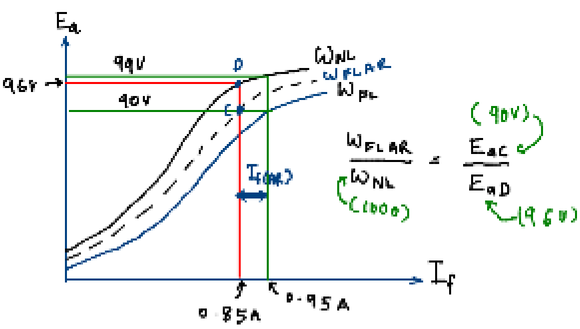
(a) With armature reaction, $\phi_{FL} = 0.94\phi_{NL}$ (6% reduction in flux due to armature reaction)

$\frac{E_{aFL}}{E_{aNL}} = \frac{0.94 \cdot \omega_{rpmFL}}{\omega_{rpmNL}}$ Therefore, $\omega_{rpmFL} := \frac{E_{aFL}}{E_{aNL}} \cdot \frac{\omega_{rpmNL}}{0.94}$ $\omega_{rpmFL} = 967.118\,rpm$

(b) With armature reaction: $I_{f\_AR} := 0.1A$

$I_f := 0.95A$ $I_{f\_eff} := I_f - I_{f\_AR}$ $I_{f\_eff} = 0.85\,A$

From the graph, when $I_f = 0.85A$ $E_a = 96V$ **(Point D)**



At the **point C,** $E_a = 90V$

Now, we know that:

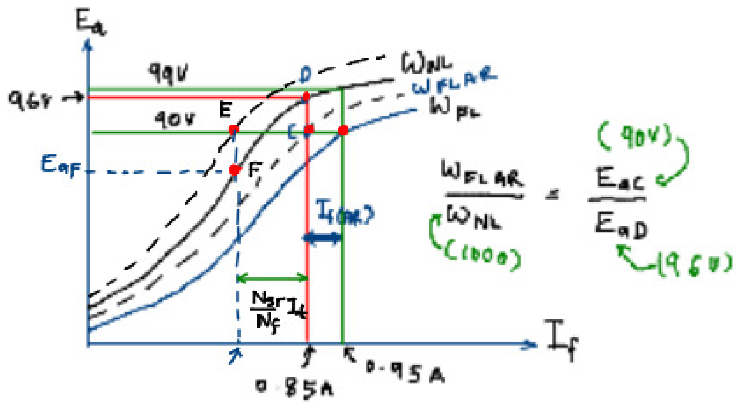
$E_{aD} := 96V$ $E_{aC} := 90V$ $\omega_D := 1000rpm$

Points C and D are related using:

$\frac{\omega_D}{\omega_C} = \frac{E_{aD}}{E_{aC}}$ and $\omega_C := \frac{E_{aC}}{E_{aD}} \cdot \omega_D$ $\omega_C = 937.5\,rpm$

(c) With armature reaction: $I_{f\_AR} := 0.1A$

$I_f := 0.95A$ $N_f := 750$ $I_a := 100A$



At the **point E**: $E_{aE} := 90V$ $\omega_E := 1050$

At the **point F**: E_{aF} unknown $\omega_F := 1000$

Therefore, $\dfrac{E_{aF}}{E_{aE}} = \dfrac{\omega_F}{\omega_E}$

then $E_{aF} := \dfrac{\omega_F}{\omega_E} \cdot E_{aE}$ $E_{aF} = 85.714\,V$

From the graph $E_a = 85.7V$ $I_f = 0.7A$ i.e. effective field current.

$I_{f\_eff} = I_f - I_{f\_AR} - \dfrac{N_{sr}}{N_f} \cdot I_t$ $I_{f\_eff} := 0.7A$ $I_{f\_AR} = 0.1\,A$ $N_f = 750$

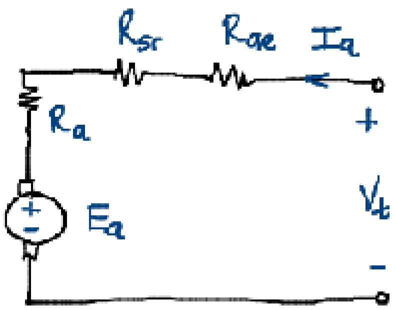
$I_t := I_a + I_f$ $I_t = 100.95$

$N_{sr} := \left(I_f - I_{f\_AR} - I_{f\_eff}\right) \cdot \dfrac{N_f}{I_t}$ $N_{sr} = 1.114$

Q2: DC Series Motor - Speed control of DC series motor by varying R_a.

$V_t := 300V$ $R_a := 0.6\Omega$ $R_{sr} := 0.3\Omega$

(a) An Equivalent circuit of the series DC motor



(b) To determine key quantities at the operating point 1.

Operation at point 1: $\omega_{m1} := 600 \cdot rpm$ $I_{a1} := 30A$ $R_{ae1} := 0$

Therefore,

$$E_{a1} := V_t - I_{a1} \cdot (R_a + R_{sr} + R_{ae1}) \qquad E_{a1} = 273\,V$$

$$P_1 := E_{a1} \cdot I_{a1} \qquad P_1 = 8.19\,kW$$

$$T_1 := \frac{E_{a1} \cdot I_{a1}}{\frac{\omega_{m1}}{60} \cdot 2\pi} \qquad T_1 = 130.348\,Nm$$

(c) To determine key quantities at the operating point 2.

Operation at point 2: $\omega_{m2} := 300 \cdot rpm$

Since the load torque is proportional to square root of the motor speed,

$$T_2 := \left(\frac{\omega_{m2}}{\omega_{m1}}\right)^{\frac{1}{2}} \cdot T_1 \qquad T_2 = 92.17\,Nm$$

To find K_{sr}: $T_1 = K_{sr} \cdot I_a{}^2$ $\qquad K_{sr} := \frac{T_1}{I_{a1}{}^2}$ $\qquad K_{sr} = 0.145$

$$I_{a2} := \sqrt{\frac{T_2}{K_{sr}}} \qquad I_{a2} = 25.227\,A$$

$$E_{a2} := K_{sr} \cdot I_{a2} \cdot \omega_{m2} \cdot \frac{2\pi}{60} \qquad E_{a2} = 114.782\,V$$

$$R_{ae2} := \frac{V_t - E_{a2}}{I_{a2}} - (R_a + R_{sr}) \qquad R_{ae2} = 6.442\,\Omega$$

$$P_2 := E_{a2} \cdot I_{a2} \qquad P_2 = 2.896\,kW$$

Q3: Induction Machine - power flow

$V_L := 415V$ $\qquad I_L := 98A$ $\qquad P_{out} := 50kW$ $\qquad N_r := 727rpm$ $\qquad p := 8$ $\qquad f := 50Hz$

Δ-connected $\qquad \xi := 0.87$ $\qquad P_{rotor} := 1.68kW$

$$N_s := \frac{120 \cdot f}{p} \qquad N_s = 750\,rpm$$

$$s := \frac{N_s - N_r}{N_s} \qquad s = 0.031$$

(a) To find input power, Pin $\qquad \xi = \frac{P_{out}}{P_{in}}$

$$P_{in} := \frac{P_{out}}{\xi} \qquad P_{in} = 57.471\,\text{kW}$$

(b) To find total losses, Ploss

$$P_{loss} := P_{in} - P_{out} \qquad P_{loss} = 7.471\,\text{kW}$$

(c) To find airgap power, Pag $\qquad P_{rotor} = s \cdot P_{ag}$

$$P_{ag} := \frac{P_{rotor}}{s} \qquad P_{ag} = 54.783\,\text{kW}$$

(d) Slip speed $\qquad N_{sl} := N_s - N_r \qquad N_{sl} = 23\,\text{rpm}$

(e) To find power factor, pf = cos(θ)

$$P_{in} = \sqrt{3} \cdot V_L \cdot I_L \cdot pf \qquad pf := \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L} \qquad pf = 0.816$$

$$V_L = 415 \qquad I_L = 98$$

(f) Rotational loss $\qquad P_m := (1 - s) \cdot P_{ag} \qquad P_m = 53.103\,\text{kW}$

$$P_{rotational} := P_m - P_{out} \qquad P_{rotational} = 3.103 \times 10^3$$

(g) Output torque $\qquad P_m = T \cdot \omega_r$

$$T := \frac{P_m}{N_r \cdot \frac{2\pi}{60}} \qquad T = 697.514\,\text{Nm}$$

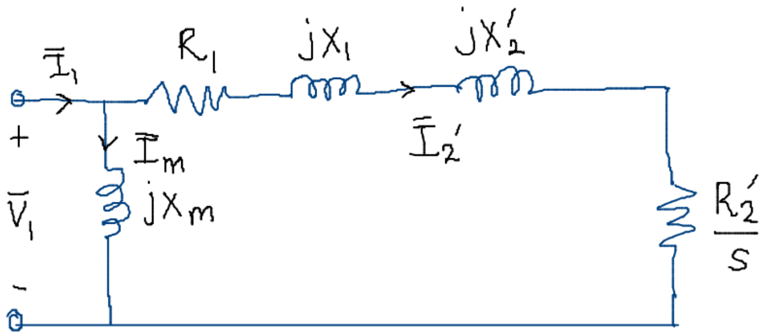
or

$$T := \frac{P_{ag}}{N_s \cdot \frac{2\pi}{60}} \qquad T = 697.514\,\text{Nm}$$

Q4: Induction Machine - Equivalent Circuit Analysis

$R_1 := 1.4\Omega$ $R_2 := 0.6\Omega$ $X_1 := 2\Omega$ $X_2 := 1\Omega$ $X_m := 50\Omega$

We use an approximate equivalent circuit. The calculation uses per phase quantities.



Connection Type: Delta

$V_L := 415V$ $V_1 := V_L$ $V_1 = 415\,V$

(a) Under full load, i.e. when $s := 0.035$; **to find the air gap power and output power**

	mag	phase
$I_2 := \dfrac{V_1}{R_1 + \dfrac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 22.093\,A$	$\arg(I_2) = -9.19\,deg$
$I_m := \dfrac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8.3\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 24.811\,A$	$\arg(I_1) = -28.473\,deg$

$P_{ag} := (\lvert I_2 \rvert)^2 \cdot \dfrac{R_2}{s}$ $P_{ag} = 8.368\,kW$ $P_{ag3ph} := 3 \cdot P_{ag}$ $P_{ag3ph} = 25.103\,kW$

$P_m := (\lvert I_2 \rvert)^2 \cdot \dfrac{R_2}{s} \cdot (1 - s)$ $P_m = 8.075\,kW$ $P_{m3ph} := 3 \cdot P_m$ $P_{m3ph} = 24.224\,kW$

$P_{rotational} := 0.275kW$ $P_{out} := P_{m3ph} + P_{rotational}$ $P_{out} = 24.499\,kW$

(b) Under starting condition, i.e. when $s := 1$; **to find line current and torque**

	mag	phase
$I_2 := \dfrac{V_1}{R_1 + \dfrac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 115.1\,A$	$\arg(I_2) = -56.31\,deg$
$I_m := \dfrac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8.3\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 122.093\,A$	$\arg(I_1) = -58.471\,deg$

The line current, since the connection is delta, $I_L := \sqrt{3}\,|I_1|$ $I_L = 211.472\,A$

$P_{ag} := (|I_2|)^2 \cdot \frac{R_2}{s}$ $P_{ag} = 7.949\,kW$ $P_{ag3ph} := 3 \cdot P_{ag}$ $P_{ag3ph} = 23.847\,kW$

$f := 50Hz$ $p := 6$ $\omega_s := 120 \cdot \frac{f}{p} \cdot \frac{2\pi}{60}$ $\omega_s = 104.72$

$T := \frac{P_{ag}}{\omega_s}$ $T = 75.906\,Nm$ $T_{3ph} := 3 \cdot T$ $T_{3ph} = 227.718\,Nm$

(c) Under maximum torque condition, i.e. when $s = s_{Tmax}$ **; to find power factor and rotor current**

$$s_{Tmax} := \frac{R_2}{\left[R_1^{\,2} + (X_1 + X_2)^2\right]^{\frac{1}{2}}}$$ $s_{Tmax} = 0.181$

$s := s_{Tmax}$

	mag	**phase**
$I_2 := \dfrac{V_1}{R_1 + \dfrac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\|I_2\| = 74.309\,A$	$arg(I_2) = -32.492\,deg$
$I_m := \dfrac{V_1}{j \cdot X_m}$	$\|I_m\| = 8.3\,A$	$arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\|I_1\| = 79.078\,A$	$arg(I_1) = -37.571\,deg$

The power factor, $pf = \cos(\theta)$

$pf := \cos(arg(I_1))$ $pf = 0.793$ lagging

The rotor current, $|I_2| = 74.309\,A$