Test #2: Solution

Version: MEI-19

Dummy unit: $\Omega \equiv 1$ $A \equiv 1$ $V \equiv 1$ $W \equiv 1$ $kW \equiv 1 \cdot 10^3 \cdot W$ $Hz \equiv 1$

$Nm \equiv 1$ $rpm \equiv 1$

Q1: DC Machine - speed of the motor under full load condition

Under full load: $P_{Rated} := 10.0kW$ $E_{aRated} := 100V$ $V_t := 100V$ $R_a := 0.1\Omega$

$$I_{aFL} := \frac{P_{Rated}}{E_{aRated}} \qquad I_{aFL} = 100\ A$$

$$E_{aFL} := V_t - I_{aFL} \cdot R_a \qquad E_{aFL} = 90\ V$$

Under no load: Given: $E_a = 99V$ at No load $\omega_{NL} := 1000rpm$

Therefore, $E_{aNL} := 99V$

(a) R_f and rotaional loss

3 marks

For $E_a := 99V$ $I_f := 0.95A$

$$R_f := \frac{V_t}{I_f} \qquad R_f = 105.263\ \Omega$$

$$I_a := \frac{V_t - E_a}{R_a} \qquad I_a = 10$$

Rotational loss: $P_{loss} := E_a \cdot I_a$ $P_{loss} = 990\ W$

(b) With armature reaction, $\phi_{FL} = 0.935\phi_{NL}$ (6.5% reduction in flux due to armature reaction)

$$\frac{E_{aFL}}{E_{aNL}} = \frac{0.935 \cdot \omega_{FL}}{\omega_{NL}}$$

Therefore, $\omega_{FL} := \frac{E_{aFL}}{E_{aNL}} \cdot \frac{\omega_{NL}}{0.935}$ $\omega_{FL} = 972.29\ rpm$

3 marks

(c) With armature reaction:

For differential compound motor,

$$I_{f\_eff} = I_f - I_{f\_AR} - \frac{N_{sr}}{N_f} \cdot I_t$$

$I_{f\_AR} := 0.15A$

$I_f := 0.95A$ $\quad I_{f\_eff} := I_f - I_{f\_AR}$ $\quad I_{f\_eff} = 0.8\,A$

$\frac{\omega_C}{\omega_D} = \frac{E_C}{E_D}$ $\quad \omega_C := 1000rpm$ $\quad \omega_D := 1100rpm$ $\quad E_D := E_{aFL}$ $\quad E_D = 90\,V$

$E_C := \frac{\omega_C}{\omega_D} \cdot E_D$ $\quad E_C = 81.818\,V$

From the graph, when $E_C = 81.8V$ $\quad I_f = 0.58A$ **(Point C)**

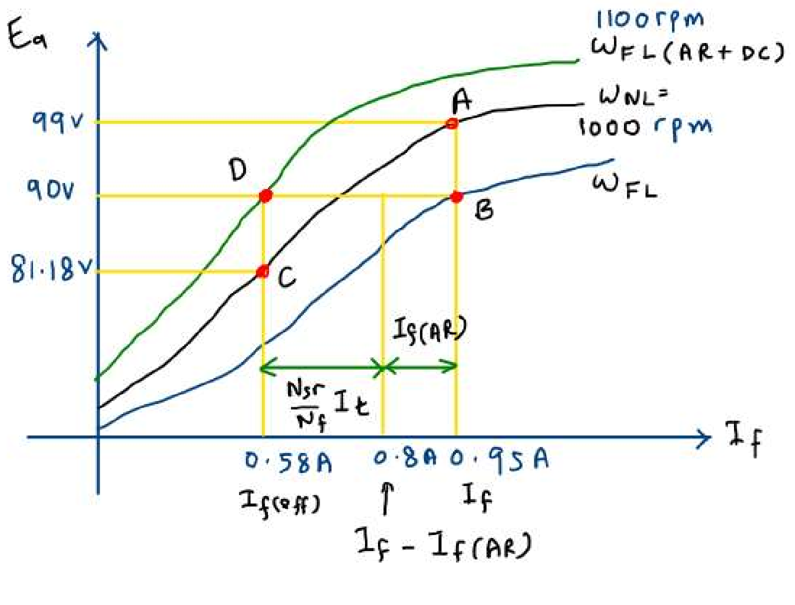
$I_{f\_eff} = I_f - I_{f\_AR} - \frac{N_{sr}}{N_f} \cdot I_t$ $\quad I_{f\_eff} := 0.58A$ $\quad I_{f\_AR} := 0.15$ $\quad N_f := 800$

$I_t := I_{aFL} + I_f$ $\quad I_{aFL} = 100$ $\quad I_f = 0.95$

$N_{sr} := \left(I_f - I_{f\_AR} + I_{f\_eff}\right) \cdot \frac{N_f}{I_t}$

$N_{sr} = 10.936$

3 marks

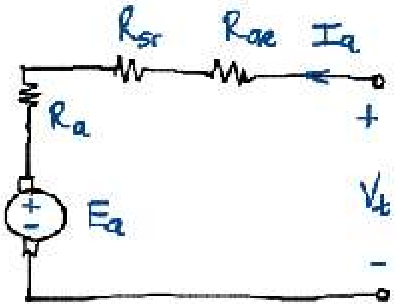


2 marks

Q2: DC Series Motor - Speed control of DC series motor by varying R_a.

$V_t := 360V$ $\quad R_a := 0.6\Omega$ $\quad R_{sr} := 0.4\Omega$

An Equivalent circuit of the series DC motor



(a) To determine key quantities at the operating point 1.

Operation at point 1: $\omega_{m1} := 600 \cdot rpm$ $\quad I_{a1} := 33A$ $\quad R_{ae1} := 0$

Therefore,

$$E_{a1} := V_t - I_{a1} \cdot \left(R_a + R_{sr} + R_{ae1}\right) \qquad E_{a1} = 327\,V$$

$$P_1 := E_{a1} \cdot I_{a1} \qquad P_1 = 10.791\,kW$$

$$T_1 := \frac{E_{a1} \cdot I_{a1}}{\frac{\omega_{m1}}{60} \cdot 2\pi} \qquad T_1 = 171.744\,Nm$$

2.5 marks

(b) To determine key quantities at the operating point 2.

Operation at point 2: $\omega_{m2} := 400 \cdot rpm$

Since the load torque is proportional to square root of the motor speed,

$$T_2 := \left(\frac{\omega_{m2}}{\omega_{m1}}\right)^{2.1} \cdot T_1 \qquad T_2 = 73.298$$

To find K_{sr}: $\quad T_1 = K_{sr} \cdot I_a^{\,2}$ $\quad K_{sr} := \frac{T_1}{I_{a1}^{\,2}}$ $\quad K_{sr} = 0.158$

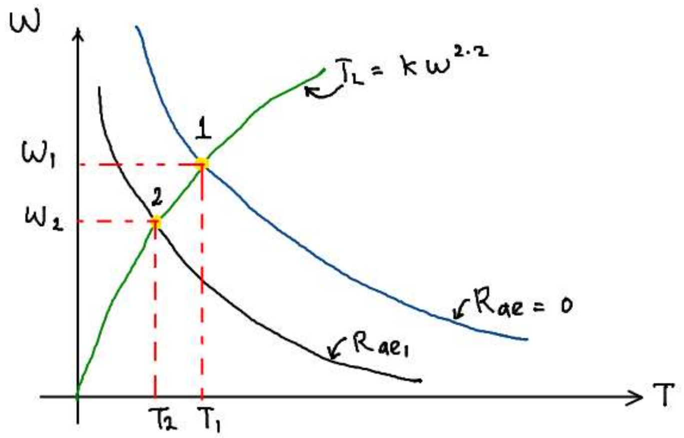
$$I_{a2} := \sqrt{\frac{T_2}{K_{sr}}} \qquad I_{a2} = 21.558\,A$$

$$E_{a2} := K_{sr} \cdot I_{a2} \cdot \omega_{m2} \cdot \frac{2\pi}{60} \qquad E_{a2} = 142.417\,V$$

$R_{ae2} := \dfrac{V_t - E_{a2}}{I_{a2}} - (R_a + R_{sr})$ $R_{ae2} = 9.093\,\Omega$

$P_2 := E_{a2} \cdot I_{a2}$ $P_2 = 3.07\,kW$ 3 marks

(c) Speed-torque characteristics



1.5 marks

Q3: Induction Machine - power flow

$V_L := 400V$ $I_L := 95A$ $P_{out} := 50kW$ $N_r := 580rpm$ $p := 10$ $f := 50Hz$

Δ-connected $P_{rotat} := 1kW$ $\eta := 0.90$

$N_s := \dfrac{120 \cdot f}{p}$ $N_s = 600\,rpm$

$s := \dfrac{N_s - N_r}{N_s}$ $s = 0.033$

i. $P_m := P_{out} + P_{rotat}$ $P_m = 51\,kW$ 1 mark

ii. $P_{ag} := \dfrac{P_m}{(1-s)}$ $P_{ag} = 52.759\,kW$ 1 mark

iii. $P_{rotor} := s \cdot P_{ag}$ $P_{rotor} = 1.759\,kW$ 1 mark

iv. $P_{in} := \dfrac{P_{out}}{\eta}$ $P_{in} = 55.556\,kW$ 1 mark

v. $P_{stator} := P_{in} - P_{ag}$ $P_{stator} = 2.797\,kW$ 1 mark

vi. $P_{Losses} := P_{rotat} + P_{rotor} + P_{stator}$ $P_{Losses} = 5.556\,kW$ 1 mark

$P_{losses} := P_{in} - P_{out}$ $\quad\quad$ $P_{losses} = 5.556\,kW$

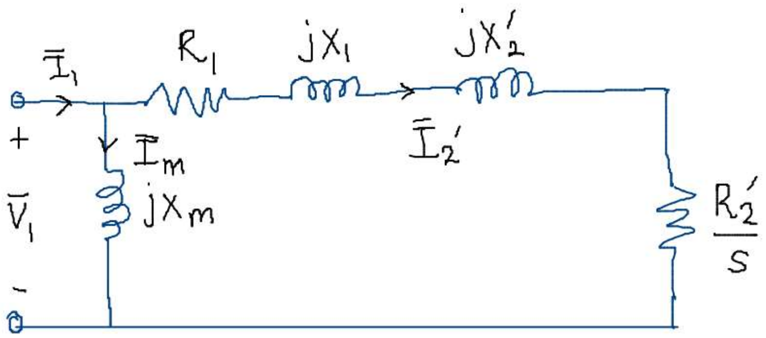
vii. $P_{in} = \sqrt{3} \cdot V_L \cdot I_L \cdot pf$ $\quad\quad$ $pf := \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L}$ $\quad\quad$ $pf = 0.844$ lagging $\quad\quad$ 1 mark

Q4: Induction Machine - Equivalent Circuit Analysis

$R_1 := 1.5\Omega$ $\quad$ $R_2 := 0.65\Omega$ $\quad$ $X_1 := 2\Omega$ $\quad$ $X_2 := 1.2\Omega$ $\quad$ $X_m := 50\Omega$

$V_L := 400V$ $\quad$ $N_r := 720rpm$ $\quad$ $p := 8$ $\quad$ $f := 50Hz$

We use an approximate equivalent circuit. The calculation uses per phase quantities.



Connection Type:
Delta

$V_1 := V_L$ $\quad$ $V_1 := 400V$ $\quad$ $N_s := \frac{120f}{p}$ $\quad$ $N_s = 750\,rpm$

(a) Under starting condition, i.e. when $s := 1$; **to find line current and power factor**

	mag	phase
$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 103.756\,A$	$arg(I_2) = -56.104\,deg$
$I_m := \frac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8\,A$	$arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 110.487\,A$	$arg(I_1) = -58.418\,deg$

The power factor, $pf = \cos(\theta)$

$pf := \cos(arg(I_1))$ $\quad\quad$ $pf = 0.524$ lagging $\quad\quad$ 3 marks

$P_{ag} := (\lvert I_2 \rvert)^2 \frac{R_2}{s}$ $\quad\quad$ $P_{ag} = 6.997\,kW$

and $\omega_s := N_s \cdot \frac{2\pi}{60}$

$T := \frac{P_{ag}}{\omega_s}$ $\quad\quad$ $T = 89.095\,Nm$ $\quad\quad$ per phase

(b) Under maximum torque condition, i.e. when $s = s_{Tmax}$ **; to find the maximum torque**

$$s_{Tmax} := \frac{R_2}{\sqrt{R_1^{\ 2} + (X_1 + X_2)^2}} \qquad s_{Tmax} = 0.184$$

$$s := s_{Tmax}$$

	mag	phase
$I_2 := \dfrac{V_1}{R_1 + \dfrac{R_2}{s} + j\cdot(X_1 + X_2)}$	$\lvert I_2 \rvert = 67.057\,A$	$\arg(I_2) = -32.443\,deg$
$I_m := \dfrac{V_1}{j\cdot X_m}$	$\lvert I_m \rvert = 8\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 71.667\,A$	$\arg(I_1) = -37.848\,deg$

$$P_{ag} := (\lvert I_2 \rvert)^2 \frac{R_2}{s} \qquad P_{ag} = 15.892\,kW$$

and $\omega_s := N_s \cdot \dfrac{2\pi}{60}$

$$T := \frac{P_{ag}}{\omega_s} \qquad T = 202.338\,Nm$$

per phase

3 marks

(c) Under full load, i.e. when $s := \dfrac{N_s - N_r}{N_s}$ **; to find efficiency at the full-load**

$$s = 0.04$$

	mag	phase
$I_2 := \dfrac{V_1}{R_1 + \dfrac{R_2}{s} + j\cdot(X_1 + X_2)}$	$\lvert I_2 \rvert = 22.178\,A$	$\arg(I_2) = -10.22\,deg$
$I_m := \dfrac{V_1}{j\cdot X_m}$	$\lvert I_m \rvert = 8\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 24.876\,A$	$\arg(I_1) = -28.671\,deg$

$$P_{ag} := (\lvert I_2 \rvert)^2 \cdot \frac{R_2}{s} \qquad P_{ag} = 7.993\,kW$$

$$P_m := (\lvert I_2 \rvert)^2 \cdot \frac{R_2}{s} \cdot (1 - s) \qquad P_m = 7.673\,kW$$

$P_{rotational} := 0.275kW$ $\quad P_{out} := P_m - \frac{P_{rotational}}{3}$ $\quad P_{out} = 7.581\,kW$

$P_{in} := |V_1| \cdot |I_1| \cdot \cos(\arg(I_1))$ $\quad P_{in} = 8.73\,kW$

Efficiency, $\eta := \frac{P_{out}}{P_{in}} \cdot 100$ $\quad \eta = 86.837$

3 marks