Test #2: Solution

Version: Jan, 2020

Dummy unit: $\Omega \equiv 1$ $A \equiv 1$ $V \equiv 1$ $W \equiv 1$ $kW \equiv 1 \cdot 10^3 \cdot W$ $Hz \equiv 1$

$Nm \equiv 1$ $rpm \equiv 1$

Q1: DC Machine - speed of the motor under full load condition

Under full load: $P_{Rated} := 10.0kW$ $E_{aRated} := 100V$ $V_t := 100V$ $R_a := 0.1\Omega$

$I_{aFL} := \frac{P_{Rated}}{E_{aRated}}$ $I_{aFL} = 100\ A$

$E_{aFL} := V_t - I_{aFL} \cdot R_a$ $E_{aFL} = 90\ V$

Under no load: Given: $E_a = 99V$ at No load $\omega_{NL} := 1000rpm$

Therefore, $E_{aNL} := 99V$

R_f and rotaional loss

3 marks

For $E_a := 98V$ $I_f := 0.95A$

$R_f := \frac{V_t}{I_f}$ $R_f = 105.263\ \Omega$

$I_a := \frac{V_t - E_a}{R_a}$ $I_a = 20$

Rotational loss: $P_{loss} := E_a \cdot I_a$ $P_{loss} = 1.96 \times 10^3\ W$

(a) With armature reaction, 5.8% reduction in flux due to armature reaction.

3 marks

$$\Phi_{FL} = \left(1 - \frac{5.8}{100}\right) \cdot \Phi_{NL} \qquad \Phi_{FL} = 0.942 \cdot \Phi_{NL}$$

$$\frac{E_{aFL}}{E_{aNL}} = \frac{0.942 \cdot \omega_{FL}}{\omega_{NL}}$$

Therefore, $\omega_{FL} := \frac{E_{aFL}}{E_{aNL}} \cdot \frac{\omega_{NL}}{0.942}$ $\omega_{FL} = 965.065\ rpm$

(b) With armature reaction:

3 marks

For cumulative compound motor,

$$I_{f\_eff} = I_f - I_{f\_AR} + \frac{N_{sr}}{N_f} \cdot I_t$$

$I_{f\_AR} := 0.15A$

$I_f := 0.95A$

$\frac{\omega_C}{\omega_D} = \frac{E_C}{E_D}$ $\omega_C := 1000rpm$ $\omega_D := 818rpm$ $E_D := E_{aFL}$ $E_D = 90\,V$

$E_C := \frac{\omega_C}{\omega_D} \cdot E_D$ $E_C = 110.024\,V$

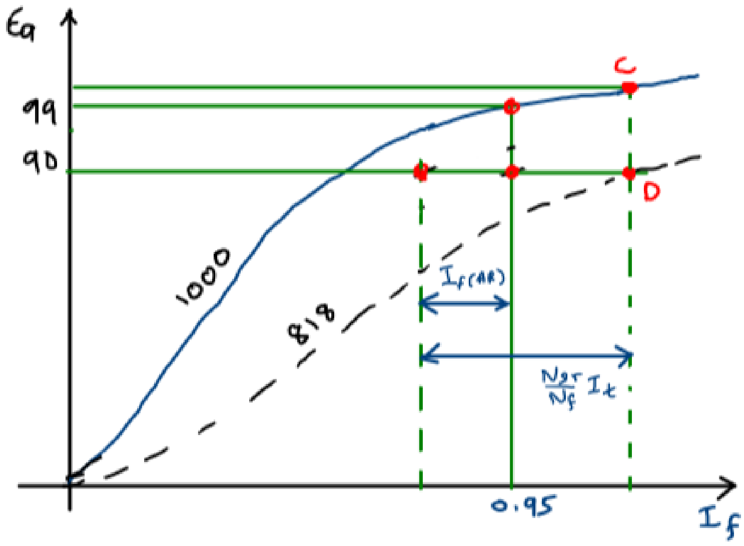
From the graph, when $E_C = 110V$ $I_f = 1.4A$ **(Point C)**

$I_{f\_eff} = I_f - I_{f\_AR} + \frac{N_{sr}}{N_f} \cdot I_t$ $I_{f\_eff} := 1.4A$ $I_{f\_AR} := 0.15$ $N_f := 800$

$I_t := I_{aFL} + I_f$ $I_{aFL} = 100$ $I_f = 0.95$

$N_{sr} := \left(I_{f\_eff} - I_f + I_{f\_AR}\right) \cdot \frac{N_f}{I_t}$

$N_{sr} = 4.755$



(c) From (a) $\omega_F := 965$ $E_F := 90V$ 3 marks

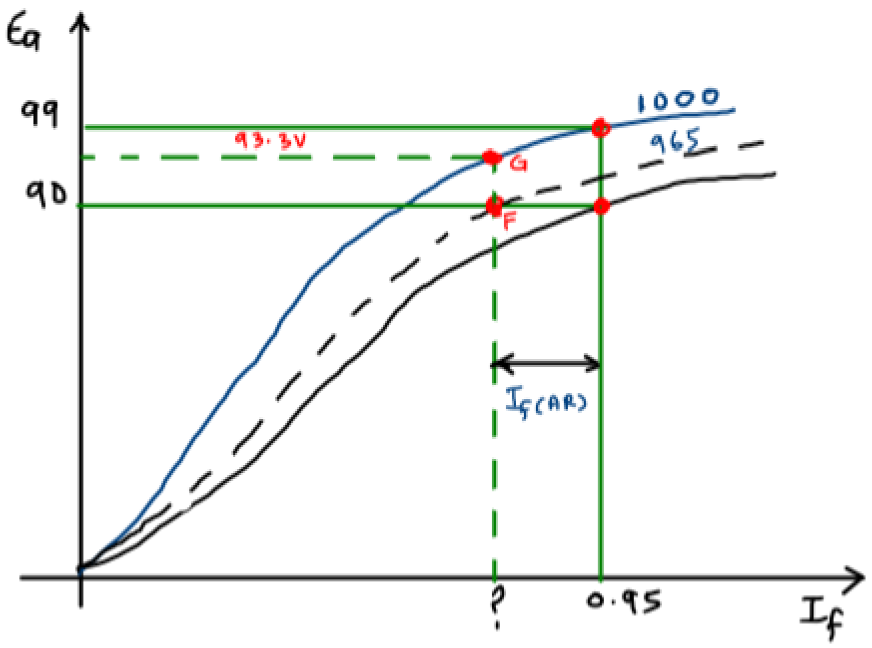
$\omega_G := 1000$ $E_G =$ unknown

$E_G := \frac{\omega_G}{\omega_F} \cdot E_F$ $E_G = 93.264\,V$

From the graph, for $E_G = 93.264$ $I_{f\_eff} := 0.775$

and $I_f = 0.95$

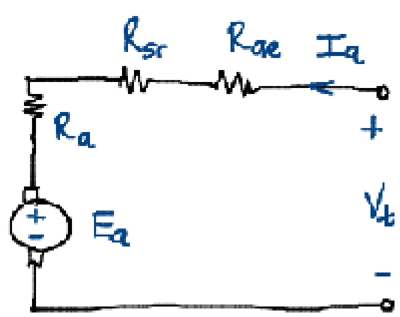
$I_{f\_AR} := I_f - I_{f\_eff}$ $\qquad I_{f\_AR} = 0.175\,A$



Q2: DC Series Motor - Speed control of DC series motor by varying R_a.

$V_t := 330V \qquad R_a := 0.6\Omega \qquad R_{sr} := 0.4\Omega$

An Equivalent circuit of the series DC motor



(a) To determine key quantities at the operating point 1.

Operation at point 1: $\omega_{m1} := 500 \cdot rpm \qquad I_{a1} := 28A \qquad R_{ae1} := 0$

Therefore,

$$E_{a1} := V_t - I_{a1} \cdot (R_a + R_{sr} + R_{ae1}) \qquad E_{a1} = 302\,\text{V}$$

$$P_1 := E_{a1} \cdot I_{a1} \qquad P_1 = 8.456\,\text{kW}$$

$$T_1 := \frac{E_{a1} \cdot I_{a1}}{\frac{\omega_{m1}}{60} \cdot 2\pi} \qquad T_1 = 161.498\,\text{Nm}$$

3 marks

(b) To determine key quantities at the operating point 2.

Operation at point 2: $\omega_{m2} := 400 \cdot \text{rpm}$

Since the load torque is proportional to square root of the motor speed,

$$T_2 := \left(\frac{\omega_{m2}}{\omega_{m1}}\right) \cdot T_1 \qquad T_2 = 129.198$$

To find K_{sr}: $T_1 = K_{sr} \cdot I_a^{\,2}$ $\qquad K_{sr} := \frac{T_1}{I_{a1}^{\,2}} \qquad K_{sr} = 0.206$

$$I_{a2} := \sqrt{\frac{T_2}{K_{sr}}} \qquad I_{a2} = 25.044\,\text{A}$$

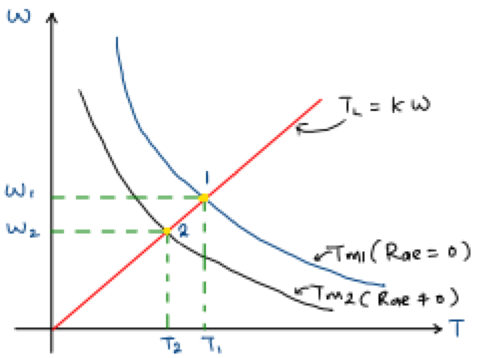
$$E_{a2} := K_{sr} \cdot I_{a2} \cdot \omega_{m2} \cdot \frac{2\pi}{60} \qquad E_{a2} = 216.094\,\text{V}$$

$$R_{ae2} := \frac{V_t - E_{a2}}{I_{a2}} - (R_a + R_{sr}) \qquad R_{ae2} = 3.548\,\Omega$$

$$P_2 := E_{a2} \cdot I_{a2} \qquad P_2 = 5.412\,\text{kW}$$

3 marks

(c) Speed-torque characteristics



2 marks

Q3: Induction Machine - power flow

$V_L := 415V$ $I_p := 90A$ $P_{ag} := 53kW$ $N_r := 700rpm$ $p := 8$ $f := 50Hz$

Δ-connected $P_{rotat} := 1kW$ $P_{stator} := 1.2kW$ $\eta := \frac{85}{100}$

i. $N_s := \frac{120 \cdot f}{p}$ $N_s = 750\,rpm$ 1 mark

$s := \frac{N_s - N_r}{N_s}$ $s = 0.067$

ii. $P_m := (1 - s) \cdot P_{ag}$ $P_m = 49.467\,kW$ 1 mark

iii. $P_m = P_o + P_{rotat}$ 1 mark

$P_o := P_m - P_{rotat}$ $P_o = 48.467\,kW$

iv. $P_{rotor} := s \cdot P_{ag}$ $P_{rotor} = 3.533\,kW$ 1 mark

v. $\eta = \frac{P_o}{P_{in}}$ 1 mark

$P_{in} := \frac{P_o}{\eta}$ $P_{in} = 57.02\,kW$

vi. $P_{in} = P_{ag} + P_{stator}$ 1 mark

$P_{stator} := P_{in} - P_{ag}$ $P_{stator} = 4.02\,kW$

vii. $P_{loss} := P_{in} - P_o$ $P_{loss} = 8.553\,kW$

viii. $I_L := \sqrt{3} \cdot I_p$ $I_L = 155.885\,A$ 1 mark

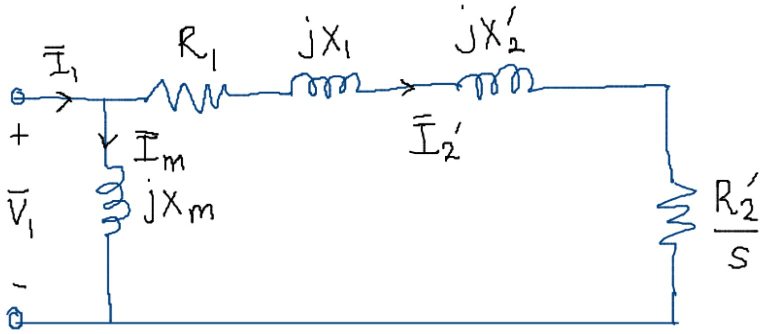
$P_{in} = \sqrt{3} \cdot V_L \cdot I_L \cdot pf$ $pf := \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L}$ $pf = 0.509$ lagging

Q4: Induction Machine - Equivalent Circuit Analysis

$R_1 := 0.5\Omega$ $R_2 := 1.2\Omega$ $X_1 := 2\Omega$ $X_2 := 1.0\Omega$ $X_m := 60\Omega$

$V_L := 460V$ $N_r := 970rpm$ $p := 6$ $f := 50Hz$ Δ connected

We use an approximate equivalent circuit. The calculation uses per phase quantities.



Connection Type: Delta

$V_1 := V_L$ $\quad V_1 = 460\,V$ $\quad N_s := \frac{120f}{p}$ $\quad N_s = 1 \times 10^3\,rpm$

i. $N_s := \frac{120 \cdot f}{p}$ $\quad N_s = 1 \times 10^3\,rpm$ $\quad s := \frac{N_s - N_r}{N_s}$ $\quad s = 0.03$ **2 marks**

slip speed, $N_{slip} := N_s - N_r$ $\quad N_{slip} = 30\,rpm$

Under full-load condition, i.e. when $s_{FL} := s$ $\quad s_{FL} = 0.03$

		mag	**phase**
ii.	$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 11.327\,A$	$\arg(I_2) = -4.236\,deg$
	$I_m := \frac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 7.667\,A$	$\arg(I_m) = -90\,deg$
	$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 14.139\,A$	$\arg(I_1) = -36.972\,deg$

2 marks

The power factor, $pf = \cos(\theta)$

$pf := \cos(\arg(I_1))$ $\quad pf = 0.799$ lagging

iii. **2 marks**

$P_{ag} := (\lvert I_2 \rvert)^2 \frac{R_2}{s}$ $\quad P_{ag} = 5.132\,kW$ per phase

$P_{rotor} := s \cdot P_{ag}$ $\quad P_{rotor} = 153.961\,W$ per phase

iv. $P_m := (1 - s) \cdot P_{ag}$ $\quad P_m = 4.978\,kW$ per phase **1 mark**

v. $P_m = P_o + P_{rotat}$ $P_{rotat} := \frac{900}{3}$ per phase 2 marks

$P_o := P_m - P_{rotat}$ $P_o = 4.678\,kW$ per phase

$P_{in} := |V_1| \cdot |I_1| \cdot pf$ $P_{in} = 5.196\,kW$ per phase

vi. Efficiency, $\eta := \frac{P_o}{P_{in}}$ $\eta = 90.029\,\%$ 1 mark