TEST#1 SOLUTION

Using Mathcad

FOR PUBLIC RELEASE

Units: $mm \equiv 10^{-3}$ $cm \equiv 10^{-2}$ $A \equiv 1$ $J \equiv 1$ $Nm \equiv 1$ $T \equiv 1$ $N \equiv 1$

$kW \equiv 10^{3}$ $V \equiv 1$ $turns \equiv 1$ $rpm \equiv 1$ $weber \equiv 1$ $\Omega \equiv 1$

Q1) $i = 2\lambda^2 + 3\lambda \cdot (x-4)^2 + \sin(x)$

$$W_f = \int i\, d\lambda = \int \left[2 \cdot \lambda^2 + 3 \cdot \lambda \cdot (x-4)^2 + \sin(x)\right] d\lambda$$

$$W_f = 2\frac{\lambda^3}{3} + 3 \cdot \frac{\lambda^2}{2} \cdot (x-4)^2 + \lambda\sin(x)$$

$$f_m = \frac{-\delta}{\delta x} W_f(\lambda, x) \qquad f_m = -\left[0 + \frac{3}{2} \cdot \lambda^2 \cdot 2 \cdot \left[(x-4)^1 \cdot (1) + \lambda\cos(x)\right]\right]$$

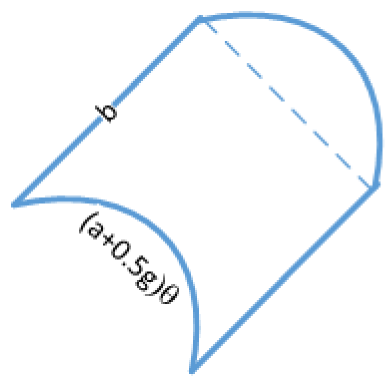
$\lambda := 2$ $x := 1$

$$f_m := -\left[0 + \frac{3}{2} \cdot \lambda^2 \cdot 2 \cdot (x-4)^1 \cdot (1) + \lambda \cdot \cos(x)\right]$$

$f_m = 34.919\,N$

Q2) $a := 15cm$ $b := 20cm$ $c := 0.05cm$ $N := 1000$ $B := 1T$

$\mu_o := 4\pi \cdot 10^{-7}$ $g := 2mm$



(a) Effective area normal to the flux,

$A_{eff} = (a + 0.5g)\theta \cdot b$

Total volume, $V_g = 2(a + 0.5g)\,\theta \cdot b \cdot g$

(b) To find torque,

$$W_{fp} = V_g \cdot \int B\, dH \qquad B = \mu_o \cdot H$$

$$W_{fp} = V_g \cdot \mu_o \cdot \frac{H^2}{2} = 2(a + 0.5g)\,\theta \cdot b \cdot g \cdot \frac{H^2}{2} \qquad \text{and} \qquad H = \frac{N \cdot i}{2 \cdot g}$$

$$W_{fP} = (a + 0.5g)\theta \cdot b \cdot g \cdot \mu_o \cdot \left(\frac{N \cdot i}{2 \cdot g}\right)^2$$

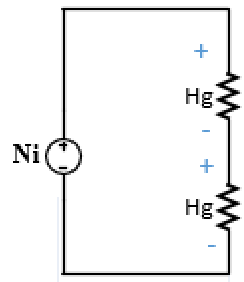
$$T = \frac{\delta}{\delta\theta} W_f(\lambda,\theta) = (a + 0.5g)\cdot b\cdot g\cdot \mu_o \left(\frac{N\cdot i}{2\cdot g}\right)^2$$

OR

$$T = \frac{1}{2}\cdot i^2\cdot \frac{d}{d\theta}L(\theta)$$ and $$L(\theta) = \frac{N^2}{R}$$ and $$R = \frac{2g}{\mu_o\cdot A_{eff}}$$

$$T = \frac{1}{2}\cdot i^2\cdot \frac{d}{d\theta}\left[\frac{N^2\cdot \mu_o\cdot (a + 0.5g)\cdot \theta}{2\cdot g}\right]$$

$$T = (a + 0.5\cdot g)\cdot b\cdot g\cdot \mu_o\cdot \left(\frac{N\cdot i}{2\cdot g}\right)^2$$



(c) The torque, T

$$T = (a + 0.5\cdot g)\cdot b\cdot g\cdot \mu_o\cdot \left(\frac{N\cdot i}{2\cdot g}\right)^2$$

$$T := (a + 0.5\cdot g)\cdot b\cdot g\cdot \mu_o\cdot \left(\frac{N\cdot i}{2\cdot g}\right)^2 \qquad T = 48.065\,\mathrm{Nm}$$

....

$$N\cdot i = 2\cdot H\cdot g = 2\cdot \frac{B}{\mu_o}\cdot g$$

$$i := 2\cdot \frac{B}{\mu_o}\cdot \frac{g}{N}$$

$$i = 3.183\,\mathrm{A}$$

....

Q3) $p := 8$ $\quad P := 25\mathrm{kW}$ $\quad V_{rated} := 120\mathrm{V}$ $\quad \omega_{rpm} := 2400\mathrm{rpm}$

$N := 64\cdot 16$ $\quad N = 1.024\times 10^3\ \mathrm{turns}$

The number of parallel path, $a := p$ for lap winding

a) To find flux per pole, ϕ

$E_a = K_a\cdot \phi\cdot \omega$ $\quad K_a := \frac{N\cdot p}{\pi\cdot a}$ $\quad K_a = 325.949$ $\quad E_a := 120\mathrm{V}$

$$\phi := \frac{E_a}{K_a\cdot \omega_{rpm}\cdot \frac{2\pi}{60}} \qquad \phi = 1.465\times 10^{-3}\ \mathrm{weber}$$

b) To find the current for each parallel path

Rated armature current, $V_{rated} = 120\ \mathrm{V}$ $\quad P = 25\,\mathrm{kW}$ $\quad I_a := \frac{P}{V_{rated}}$ $\quad I_a = 208.333\ \mathrm{A}$

Therefore, the current for each parallel path, $I := \frac{I_a}{a}$ $a = 8$

$I = 26.042\,A$

c) To find the induced torque

$T := K_a \cdot \phi \cdot I_a$ $T = 99.472\,Nm$ or $T := \frac{P}{\omega_{rpm} \cdot 2\frac{\pi}{60}}$ $T = 99.472\,Nm$

d) To find the armature resistance

Number of turns connected in series, $N_{series} := \frac{N}{a}$ $N = 1.024 \times 10^3$

(Number of turns for each parallel path) $a = 8$

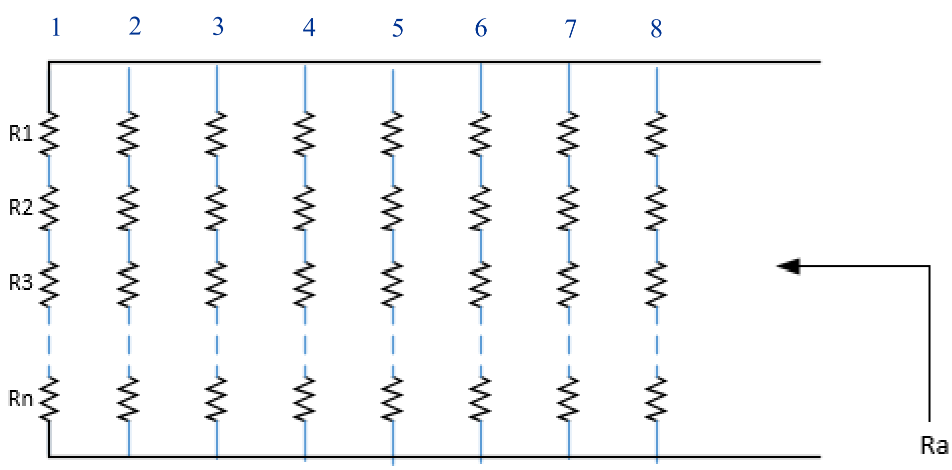
$N_{series} = 128$

$R_{turn} := 0.011 \cdot \Omega$

Therefore, the resistance for each parallel path, $R_{path} := N_{series} \cdot R_{turn}$

$R_{path} = 1.408\,\Omega$

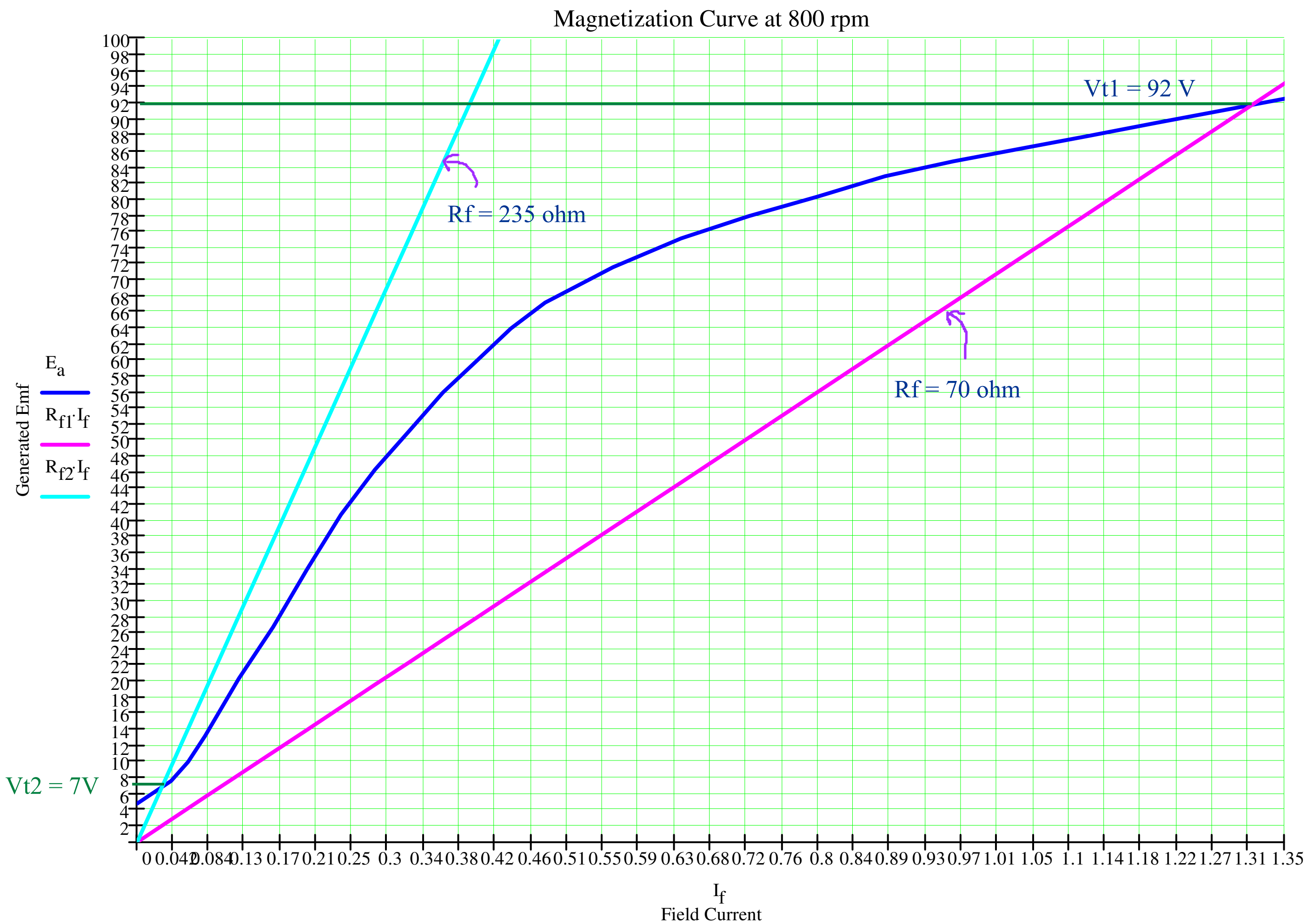
The armature resistance, $R_a := \frac{R_{path}}{a}$ $R_a = 0.176\,\Omega$



n = 128 (128 resistances in series)

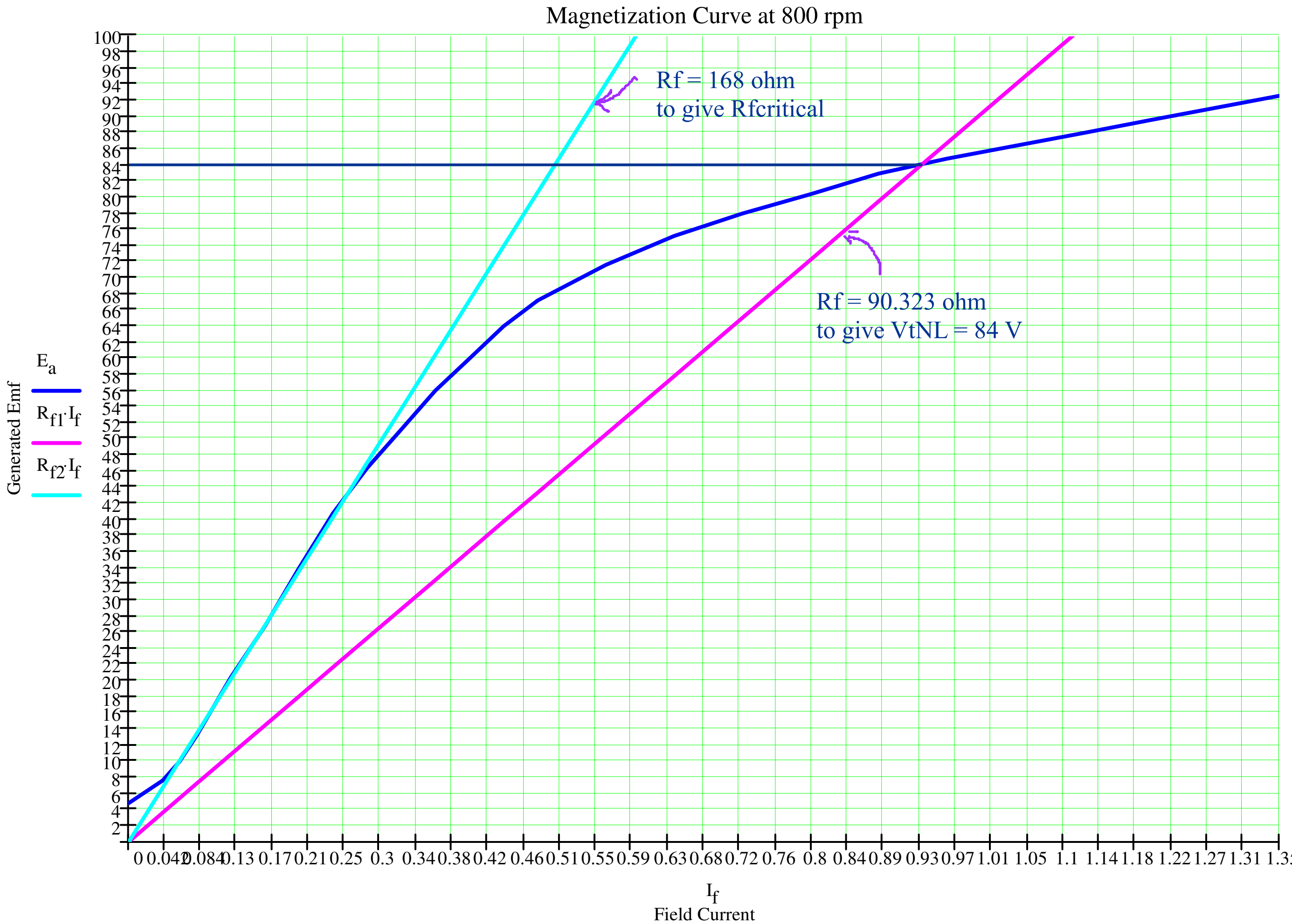
Q4a) $R_{fw} := 70\Omega$ $R_{fc1} := 0\Omega$ $R_{f1} := R_{fc1} + R_{fw}$ $R_{f1} = 70\,\Omega$ From the graph, $V_{t1} = 90V$ 92 V

$R_{fc2} := 165\Omega$ $R_{f2} := R_{fc2} + R_{fw}$ $R_{f2} = 235\,\Omega$ From the graph, $V_{t2} = 7V$



Q4b) From the graph, when $V_{t1} := 84V$ $I_{f1} := 0.93\Omega$ $R_{f1} := \frac{V_{t1}}{I_{f1}}$ $R_{f1} = 90.323\,\Omega$ $R_{fc} := R_{f1} - R_{fw}$ $R_{fc} = 20.323\,\Omega$

Q4c) From the graph, when $V_{t2} := 42V$ $I_{f2} := 0.25A$ $R_{f2} := \frac{V_{t2}}{I_{f2}}$ $R_{f2} = 168\,\Omega$ $R_{fc} := R_{f2} - R_{fw}$ $R_{fc} = 98\,\Omega$



Q4d) From the graph, when $V_{t1} := 80V$ $I_{f1} := 0.8\Omega$ $R_{f1} := \dfrac{V_{t1}}{I_{f1}}$ $R_{f1} = 100\,\Omega$ $R_{fc} := R_{f1} - R_{fw}$ $R_{fc} = 30\,\Omega$

$I_a := \dfrac{8 \cdot 10^3}{80}$ $I_a = 100\,A$ $R_a := 0.1\Omega$ $I_a \cdot R_a = 10\,V$