TEST#1 SOLUTION SKEE4633/SEE3433

Using Mathcad

April-2019

Units: $mm \equiv 10^{-3}$ $cm \equiv 10^{-2}$ $A \equiv 1$ $Joules \equiv 1$ $Nm \equiv 1$ $T \equiv 1$ $N \equiv 1$

$kW \equiv 10^{3}$ $V \equiv 1$ $turns \equiv 1$ $rpm \equiv 1$ $weber \equiv 1$ $\Omega \equiv 1$

$mWb \equiv 10^{-3}$ $kV \equiv 10^{3}$ $kN \equiv 10^{3}$

Q1) $i = 6 \cdot x^2 \cdot \lambda^2 + 8 \cdot (2 \cdot x - 7)^2 \cdot \sqrt{\lambda}$

$$W_f = \int i \, d\lambda = \int \left[6 \cdot x^2 \cdot \lambda^2 + 8 \cdot (2 \cdot x - 7)^2 \cdot \sqrt{\lambda}\right] d\lambda$$

$$W_f = 2 \cdot x^2 \cdot \lambda^3 + \frac{16}{3} \cdot \lambda^{\frac{3}{2}} \cdot (2 \cdot x - 7)^2$$

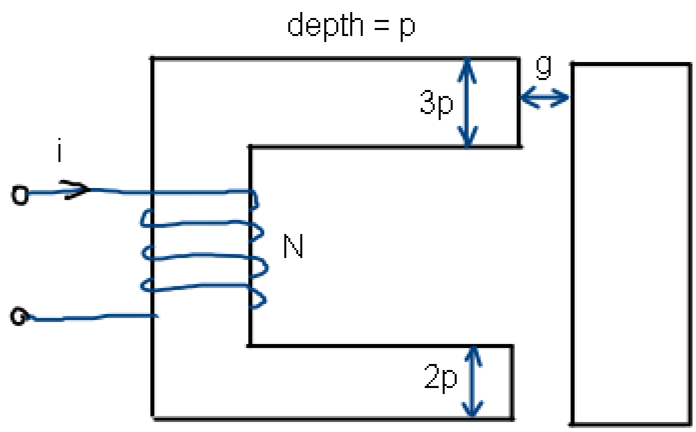
$$f_m = \frac{-\delta}{\delta x} W_f(\lambda, x) \qquad f_m = -\left[4 \cdot x \cdot \lambda^3 + \frac{64}{3} \cdot \lambda^{\frac{3}{2}} \cdot (2 \cdot x - 7)\right]$$

$\lambda := 2$ $x := 2$

$$f_m := -\left[4 \cdot x \cdot \lambda^3 + \frac{64}{3} \cdot \lambda^{\frac{3}{2}} \cdot (2 \cdot x - 7)\right] \qquad f_m = 117.019\ N$$

6 marks

Q2)



$p := 40\text{cm}$ $N := 1000$

$i := 5$ $\mu_o := 4\pi \cdot 10^{-7}$ $g := 4\text{cm}$

(a) The area normal to the flux,

3 marks

$A_1 = 3p \cdot p = 3p^2$ $A_2 = 2p \cdot p = 2p^2$

$R_1 = \dfrac{g}{\mu_o \cdot A_1}$ $R_2 = \dfrac{g}{\mu_o \cdot A_2}$ $R_T = R_1 + R_2 = \dfrac{g}{\mu_o} \cdot \left(\dfrac{1}{A_1} + \dfrac{1}{A_2}\right)$

$$R_T = \frac{g}{\mu_o}\left(\frac{1}{p \cdot q} + \frac{1}{p \cdot r}\right) = \frac{g}{\mu_o}\left(\frac{1}{3p^2} + \frac{1}{2p^2}\right) = \frac{g}{\mu_o \cdot p^2}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5g}{6 \cdot \mu_o \cdot p^2}$$

$$L = \frac{N^2}{R_T} = \frac{6\mu_o \cdot p^2 N^2}{5g}$$

(b)

3 marks

$R_T := \dfrac{5g}{6 \cdot \mu_o \cdot p^2}$ $R_T = 1.658 \times 10^5$

$L := \dfrac{6\mu_o \cdot p^2 N^2}{5g}$ $L = 6.032$

$\phi := \dfrac{N \cdot i}{R_T}$ $\phi = 30.159\,\text{mWb}$ $\lambda := N \cdot \phi$ $\lambda = 30.159$

$A_1 := 3p^2 \qquad B_1 := \frac{\phi}{A_1} \qquad V_{ag1} := A_1 \cdot g$

$A_2 := 2p^2 \qquad B_2 := \frac{\phi}{A_2} \qquad V_{ag2} := A_2 \cdot g$

$$W_f := \frac{B_1^{\,2}}{2 \cdot \mu_o} \cdot V_{ag1} + \frac{B_2^{\,2}}{2 \cdot \mu_o} \cdot V_{ag2} \qquad W_f = 75.398\,\text{Joules}$$

OR

$$W_f := \frac{1}{2} \cdot L \cdot i^2 \qquad W_f = 75.398\,\text{Joules}$$

(c) To find expression for the force based on inductance **3 marks**

$$f_m = \frac{1}{2} \cdot i^2 \cdot \frac{d}{dx} L(x) \qquad \text{and} \qquad L(x) = \frac{N^2}{R_T}$$

but $dx = -dg$ and $L(g) = \dfrac{6\mu_o \cdot p^2 N^2}{5g}$

therefore, $f_m = -\frac{1}{2} \cdot i^2 \cdot \frac{d}{dg} L(g)$

$$f_m = -\frac{1}{2} \cdot i^2 \cdot \frac{d}{dg} \frac{6\mu_o \cdot p^2 N^2}{5g} = -\frac{1}{2} \cdot \frac{i^2 \cdot N^2 \cdot 6 \cdot \mu_o \cdot p^2}{5} \frac{d}{dg} \frac{1}{g} \qquad \frac{d}{dg}\frac{1}{g} = \frac{-1}{g^2}$$

$$f_m = \frac{1}{2} \cdot i^2 \cdot \frac{N^2 \cdot 6 \cdot \mu_o \cdot p^2}{5g^2}$$

$$f_m := \frac{1}{2} \cdot i^2 \cdot \frac{N^2 \cdot 6 \cdot \mu_o \cdot p^2}{5g^2} \qquad f_m = 1.885 \times 10^3\,\text{Nm}$$ **3 marks**

Q3) $p := 8 \qquad \omega_{rpm} := 2000\text{rpm} \qquad \phi := 40\text{mWb}$

$N := 64 \cdot 20 \qquad N = 1.28 \times 10^3\,\text{turns}$

a) To find the the induced voltage for lap-wound generator **2 marks**

The number of parallel path, $a := p$ for lap-wound

$E_a = K_a \cdot \phi \cdot \omega_m$ $\omega_m = \frac{E_a}{K_a \cdot \phi}$ $K_a := \frac{N \cdot p}{\pi \cdot a}$ $K_a = 407.437$

$E_a := K_a \cdot \phi \cdot \frac{\omega_{rpm} \cdot 2 \cdot \pi}{60}$ $E_a = 3.413 \times 10^3\ V$ $E_t := \frac{E_a}{\frac{N}{a}}$ $E_t = 21.333\ V$

b) To find the the developed torque for lap-wound generator

2 marks

The number of parallel path, $a := p$ for lap-wound

$I_c := 50A$ conductor current

$I_a := a \cdot I_c$ $I_a = 400\ A$

$T_m := K_a \cdot \phi \cdot I_a$ $T_m = 6.519 \times 10^3\ Nm$

$T_t := \frac{T}{N}$ $T_t = 7.813 \times 10^{-4}$

c) The number of turns connected in series, N/a

Rated voltage accross each turn: 25 V
Rated current winding: 2.5 A

$V_{turn} := 25V$
$I_{turn} := 2.5A$

To find the kW rating of the generator for lap-wound generator

The number of parallel path, $a := p$ for lap-wound $a = 8$

2 marks

$E_a := V_{turn} \cdot \frac{N}{a}$ $E_a = 4 \times 10^3\ V$

$I_a := a \cdot I_{turn}$ $I_a = 20\ A$

$P := E_a \cdot I_a$ $P = 80\ kW$

Q4) A. Separately Excited DC generator

$R_a := 0.1\Omega$ $E_a := 108V$ $I_f := 1.15A$ $P_{max} := 10.8kW$

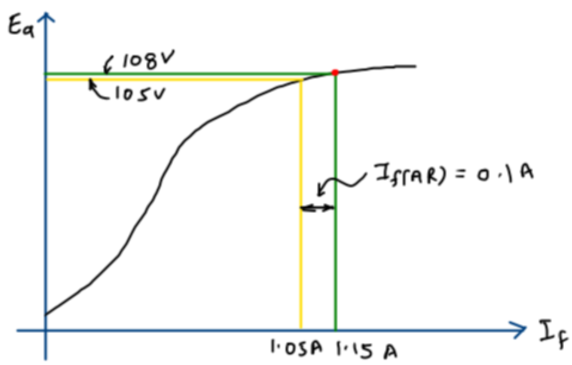
$I_a := \frac{10.8kW}{108}$ $I_a = 100\ A$ $V_f := 108$

(i) Terminal voltage at full-load with armature reaction

2 marks

$I_{f\_AR} := 0.1A$

$I_{f\_eff} := I_f - I_{f\_AR}$ $I_{f\_eff} = 1.05$



From the graph, $I_f = 1.05A$ $E_a := 105V$

$V_t := E_a - I_a \cdot R_a$ $V_t = 95\ V$

(ii) I_f required to make the terminal voltage $V_t = 108$ V

$V_t := 108V$ $I_{f\_AR} := 0.1A$

$E_a := V_t + I_a \cdot R_a$ $E_a = 118\ V$

From the graph, $E_a = 110V$ $I_f = 1.225A$ this is I_f effective $I_{f\_eff} := 1.225A$

Therefore, $I_f := I_{f\_eff} + I_{f\_AR}$ $I_f = 1.325\ A$

B. Shunt DC generator

(iii) The maximum armature current without armature reaction

From the graph, $I_a \cdot R_a = 30V$

2 marks

Therefore, $I_a := \frac{30}{R_a}$ $I_a = 300\ A$

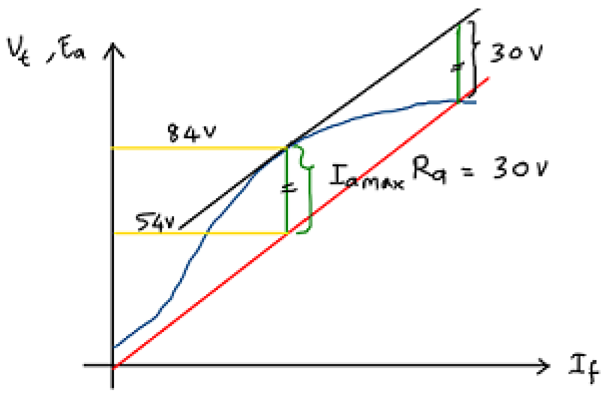
$V_t = 54V$ $E_a = 84V$

(iv) Terminal voltage at full-load with armature reaction

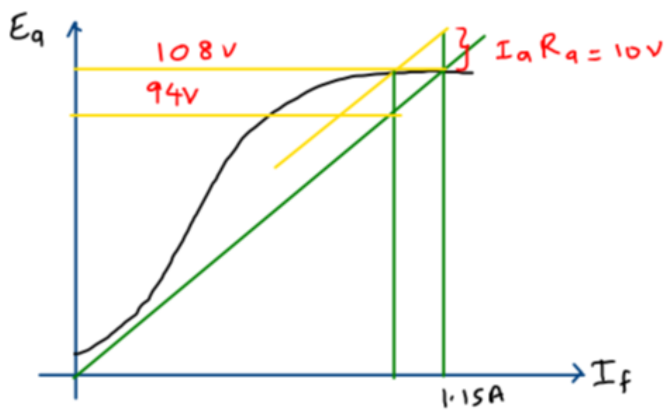
2 marks

$I_a := 100A$ $I_a \cdot R_a = 10\ V$ $I_{f\_AR} = 0.1A$

From the graph, $V_t = 88V$



B(iii)



B(iv)