



Using Mathcad

April-2019

Units: mm $\equiv 10^{-3}$ cm $\equiv 10^{-2}$ A $\equiv 1$ J $\equiv 1$ Nm $\equiv 1$ T $\equiv 1$ N $\equiv 1$
 kW $\equiv 10^3$ V $\equiv 1$ turns $\equiv 1$ rpm $\equiv 1$ weber $\equiv 1$ $\Omega \equiv 1$
 mWb $\equiv 10^{-3}$ kV $\equiv 10^3$ kN $\equiv 10^3$

Q1)
$$i = 6 \cdot x^2 \cdot \lambda^2 + 8 \cdot \lambda \cdot (2 \cdot x - 7)^2$$

$$W_f = \int i \, d\lambda = \int [6 \cdot x^2 \cdot \lambda^2 + 8 \cdot \lambda \cdot (2 \cdot x - 7)^2] \, d\lambda$$

$$W_f = 2 \cdot x^2 \cdot \lambda^3 + 4 \cdot \lambda^2 \cdot (2 \cdot x - 7)^2$$

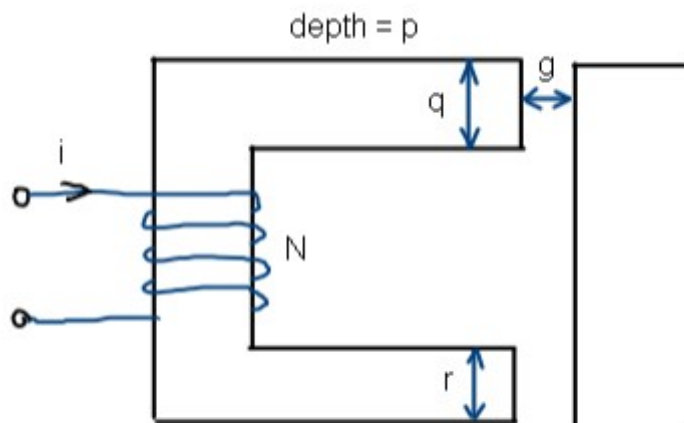
$$f_m = \frac{-\delta}{\delta x} W_f(\lambda, x) \quad f_m = -[4 \cdot x \cdot \lambda^3 + 16 \cdot \lambda^2 \cdot (2 \cdot x - 7)]$$

$$\lambda := 2 \quad x := 2$$

$$f_m := -[4 \cdot x \cdot \lambda^3 + 16 \cdot \lambda^2 \cdot (2 \cdot x - 7)] \quad f_m = 128 \text{ N}$$

6 marks

Q2)



$$p := 15\text{cm} \quad q := 40\text{cm} \quad r := 30\text{cm} \quad N := 1000$$

$$i := 5 \quad \mu_0 := 4\pi \cdot 10^{-7} \quad g := 4\text{cm}$$

(a) The area normal to the flux,

3 marks

$$A_1 = p \cdot q$$

$$A_2 = p \cdot r$$

$$R_1 = \frac{g}{\mu_0 \cdot A_1}$$

$$R_2 = \frac{g}{\mu_0 \cdot A_2}$$

$$R_T = R_1 + R_2 = \frac{g}{\mu_0} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2} \right)$$

$$R_T = \frac{g}{\mu_0} \left(\frac{1}{p \cdot q} + \frac{1}{p \cdot r} \right) = \frac{g}{\mu_0 \cdot p} \left(\frac{1}{q} + \frac{1}{r} \right) = \frac{g}{\mu_0 \cdot p} \cdot \frac{r+q}{q \cdot r}$$

$$L = \frac{N^2}{R_T} = \frac{N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{g \cdot (r+q)}$$

(b) $R_T := \frac{g}{\mu_0 \cdot p} \cdot \frac{r+q}{q \cdot r}$

$$R_T = 1.238 \times 10^6$$

3 marks

$$\phi := \frac{N \cdot i}{R_T}$$

$$\phi = 4.039 \text{ mWb}$$

$$A_1 := p \cdot q$$

$$B_1 := \frac{\phi}{A_1}$$

$$V_{ag1} := A_1 \cdot g$$

$$A_2 := p \cdot r$$

$$B_2 := \frac{\phi}{A_2}$$

$$V_{ag2} := A_2 \cdot g$$

$$W_f := \frac{B_1}{2 \cdot \mu_0} \cdot V_{ag1} + \frac{B_2}{2 \cdot \mu_0} \cdot V_{ag2}$$

$$W_f = 128.571$$

(c) To find expression for the force based on inductance

3 marks

$$f_m = \frac{1}{2} \cdot i^2 \cdot \frac{d}{dx} L(x)$$

and $L(x) = \frac{N^2}{R_T}$

but $dx = -dg$ and $L(g) = \frac{N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{g \cdot (r+q)}$

therefore, $f_m = -\frac{1}{2} \cdot i^2 \cdot \frac{d}{dg} L(g)$

$$f_m = -\frac{1}{2} \cdot i^2 \cdot \frac{d}{dg} \frac{N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{g \cdot (r+q)} = -\frac{1}{2} \cdot \frac{i^2 \cdot N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{(r+q)} \frac{d}{dg} \frac{1}{g}$$

$$\frac{d}{dg} \frac{1}{g} = \frac{-1}{g^2}$$

$$f_m = \frac{1}{2} \cdot i^2 \cdot \frac{N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{g^2 \cdot (r + q)}$$

$$f_m := \frac{1}{2} \cdot i^2 \cdot \frac{N^2 \cdot \mu_0 \cdot p \cdot q \cdot r}{g^2 \cdot (r + q)}$$

$$f_m = 252.449 \text{ Nm}$$

3 marks

Q3) $p := 8$ $\omega_{\text{rpm}} := 2000 \text{ rpm}$ $\phi := 40 \text{ mWb}$
 $N := 64 \cdot 20$ $N = 1.28 \times 10^3 \text{ turns}$

a) To find the the induced voltage for lap-wound generator

2 marks

The number of parallel path, $a := p$ for lap-wound

$$E_a = K_a \cdot \phi \cdot \omega_m \quad \omega_m = \frac{E_a}{K_a \cdot \phi} \quad K_a := \frac{N \cdot p}{\pi \cdot a} \quad K_a = 407.437$$

$$E_a := K_a \cdot \phi \cdot \frac{\omega_{\text{rpm}} \cdot 2 \cdot \pi}{60} \quad E_a = 3.413 \times 10^3 \text{ V} \quad E_t := \frac{E_a}{\frac{N}{a}} \quad E_t = 21.333 \text{ V}$$

b) To find the the induced voltage for wave-wound generator

2 marks

The number of parallel path, $a := 2$ for wave-wound

$$E_a = K_a \cdot \phi \cdot \omega_m \quad K_a := \frac{N \cdot p}{\pi \cdot a} \quad K_a = 1.63 \times 10^3$$

$$E_a := K_a \cdot \phi \cdot \frac{\omega_{\text{rpm}} \cdot 2 \cdot \pi}{60} \quad E_a = 1.365 \times 10^4 \text{ V} \quad E_t := \frac{E_a}{\frac{N}{a}} \quad E_t = 21.333 \text{ V}$$

c) The number of turns connected in series, N/a

Rated voltage across each turn: 25 V
 Rated current winding: 2.5 A

$V_{\text{turn}} := 25\text{V}$
 $I_{\text{turn}} := 2.5\text{A}$

To find the kW rating of the generator for lap-wound generator

The number of parallel path, $a := p$ for lap-wound $a = 8$

1.5 marks

$$E_a := V_{\text{turn}} \cdot \frac{N}{a} \quad E_a = 4 \times 10^3 \text{ V}$$

$$I_a := a \cdot I_{\text{turn}} \quad I_a = 20 \text{ A}$$

$$P := E_a \cdot I_a \quad P = 80 \text{ kW}$$

d) To find the kW rating of the generator for wave-wound generator

The number of parallel path, $a := 2$

1.5 marks

$$E_a := V_{\text{turn}} \cdot \frac{N}{a} \quad E_a = 1.6 \times 10^4 \text{ V}$$

$$I_a := a \cdot I_{\text{turn}} \quad I_a = 5 \text{ A}$$

$$P := E_a \cdot I_a \quad P = 80 \text{ kW}$$

Q4) A. Separately Excited DC generator

$R_a := 0.1\Omega$ $E_a := 108\text{V}$ $I_f := 1.15\text{A}$ $P_{\text{max}} := 10.8\text{kW}$

$I_a := \frac{P_{\text{max}}}{E_a}$ $I_a = 100 \text{ A}$ $V_f := 108$

(i) Terminal voltage at full-load without armature reaction

$$V_t := E_a - I_a \cdot R_a \quad V_t = 98 \text{ V}$$

2 marks

(ii) Terminal voltage at full-load with armature reaction

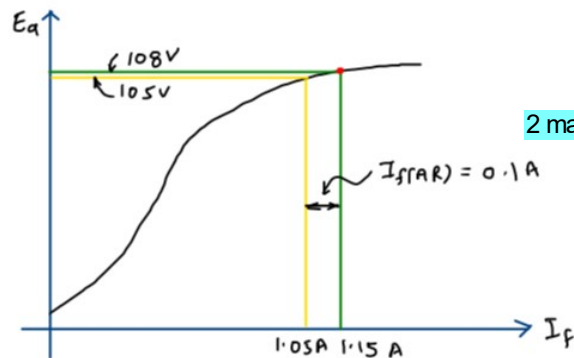
$$I_{f_AR} := 0.1\text{A}$$

$$I_{f_eff} := I_f - I_{f_AR} \quad I_{f_eff} = 1.05$$

From the graph, $I_f = 1.05\text{A}$ $E_a := 105\text{V}$

$$V_t := E_a - I_a \cdot R_a \quad V_t = 95 \text{ V}$$

2 marks



B. Shunt DC generator

(iii) Terminal voltage at full-load without armature reaction

2 marks

$$I_a \cdot R_a = 10 \text{ V} \quad I_{f_AR} = 0$$

From the graph, $V_t = 86 \text{ V}$ $V_t = 94 \text{ V}$

(iv) Terminal voltage at full-load with armature reaction

2 marks

$$I_a \cdot R_a = 10 \text{ V} \quad I_{f_AR} = 0.1 \text{ A}$$

From the graph, $V_t = 88 \text{ V}$

