Solution For Test #2

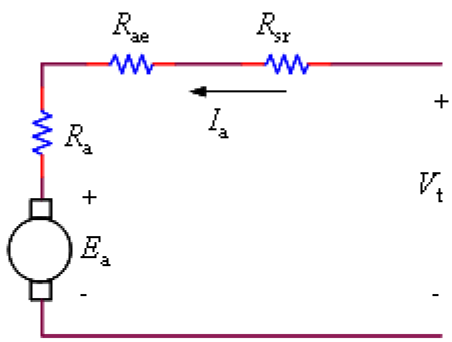
Question 1: DC Series Motor

Controlling speed of DC series motor by Resistance

Dummy unit: $\Omega \equiv 1$ rpm $\equiv 1$ $V \equiv 1$ $A \equiv 1$ $Hz \equiv 1$ $kW \equiv 10^3$ $Nm \equiv 1$ $W \equiv 1$ $k \equiv 10^3$

$V_t := 220 \cdot V$ $R_a := 0.55\Omega$ $R_{sr} := 0.25\Omega$ **(constants)**

(a)



Under steady state condition, L is short-circuited. So it does not appear in the circuit.

3 marks

(b) At the point 1: $I_{a1} := 30 \cdot A$ $R_{ae1} := 0$ $N_{m1} := 500\text{rpm}$

$E_{a1} := V_t - I_{a1} \cdot (R_a + R_{sr} + R_{ae1})$ $E_{a1} = 196\,V$

5 marks

$P_1 := E_{a1} \cdot I_{a1}$ $P_1 = 5.88\,kW$

$\omega_{m1} := \frac{N_{m1}}{60} \cdot 2\pi$ $T_1 := \frac{E_{a1} \cdot I_{a1}}{\omega_{m1}}$ $T_1 = 112.3\,Nm$

(c) At the point 2: $N_{m2} := 300\text{rpm}$

5 marks

Relationship between T and ω that is given by load torque characteristics.

Load torque characteristic is given as: $T = k \cdot \omega_m^2$

It can also be stated as: $\frac{T_2}{T_1} = \left(\frac{\omega_{m2}}{\omega_{m1}}\right)^2$ or $\frac{T_2}{T_1} = \left(\frac{N_{m2}}{N_{m1}}\right)^2$

$T_2 := \left(\frac{N_{m2}}{N_{m1}}\right)^2 \cdot T_1$ $T_2 = 40.428\,Nm$

Relationship between T and I_a is given by motor torque equation.

$T = K_{sr} \cdot I_a^2$

At the point 1: $T_1 = 112.3\,Nm$ $I_{a1} = 30\,A$ $K_{sr} := \frac{T_1}{I_{a1}^2}$ $K_{sr} = 0.125$

Motor's coefficient K_{sr} is the same for any operating point.

Relationship between T and ω that is given by motor torque characteristics.

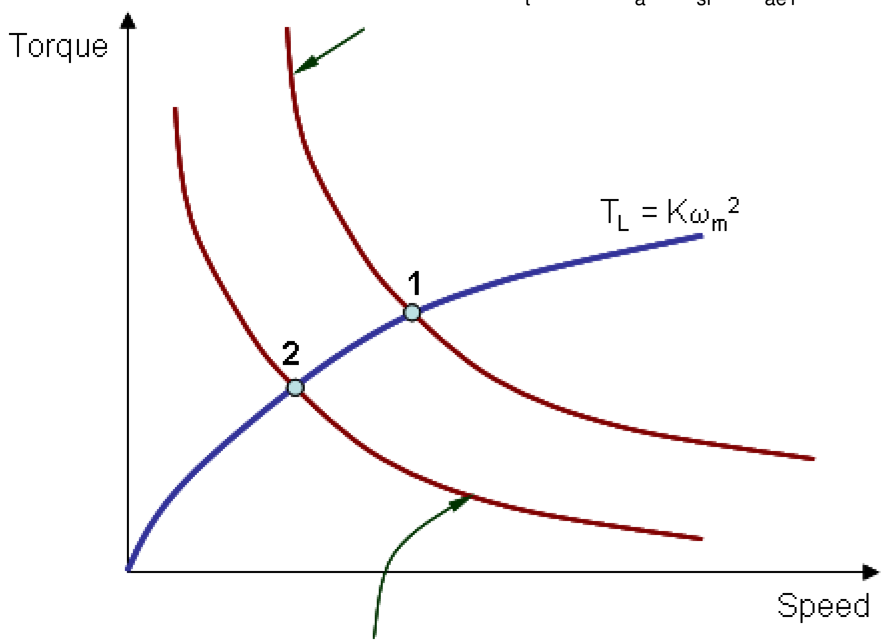
$$\omega_m = \frac{V_t}{\sqrt{K_{sr}} \cdot \sqrt{T}} - \frac{R_a + R_{sr} + R_{ae}}{K_{sr}}$$

At the point 2: $\omega_{m2} := \frac{N_{m2}}{60} \cdot 2\pi$ $\quad T_2 = 40.428\,\text{Nm}$

$$R_{ae2} := \left(\frac{V_t}{\sqrt{K_{sr}} \cdot \sqrt{T_2}} - \omega_{m2} \right) \cdot K_{sr} - \left(R_a + R_{sr} \right) \qquad R_{ae2} = 7.502\,\Omega$$

$P_2 := T_2 \cdot \omega_{m2}$ $\quad P_2 = 1.27\,\text{kW}$

$V_t \qquad R_a + R_{sr} + R_{ae1}$



$$\omega_{m2} = \frac{V_t}{\sqrt{K_{sr}} \cdot \sqrt{T_2}} - \frac{R_a + R_{sr} + R_{ae2}}{K_{sr}}$$

(d) By introducing extra R_a to the rotor circuit causes extra losses to the system. The loss due to extra R_a is significant because of high current flows in the rotor circuit. As such, controlling speed by changing R_a is performed at the expense of lower efficiency.

1.5 marks

(e) A DC series motor must always be started under load because the speed at no load is dangerously high. Example: lift (hoist system). The lift carriage itself essentially acts as a load, when the lift without load (no passengger).

1.5 marks

Question 2: Induction Machine

Dummy unit: $\Omega \equiv 1$ $\quad$ $rpm \equiv 1$ $\quad$ $V \equiv 1$ $\quad$ $A \equiv 1$ $\quad$ $Hz \equiv 1$ $\quad$ $kW \equiv 10^3$ $\quad$ $Nm \equiv 1$ $\quad$ $W \equiv 1$ $\quad$ $k \equiv 10^3$

Y-connected Induction Machine:

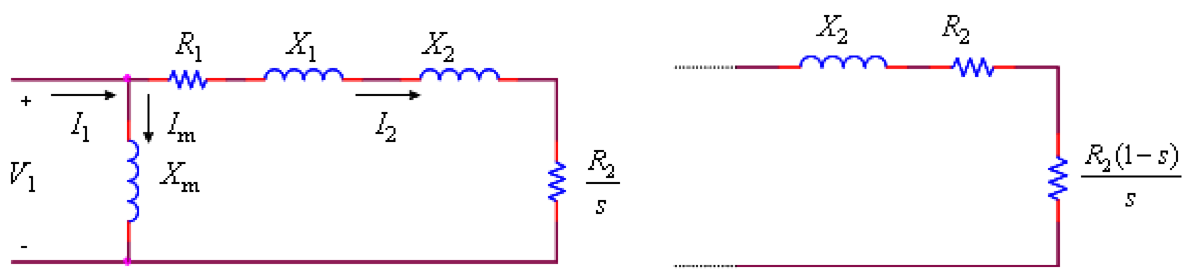
$V_{3\phi} := 415V$ $\quad$ $N_r := 1400rpm$ $\quad$ $f := 50Hz$

$R_1 := 0.25\Omega$ $\quad$ $R_2 := 0.20\Omega$ $\quad$ $p := 4$

$X_1 := 0.55$ $\quad$ $X_2 := 0.45$ $\quad$ $X_m := 50\Omega$

$V_1 := \frac{V_{3\phi}}{\sqrt{3}}$ $\quad$ $V_1 = 239.6\,V$ $\quad$ $N_s := \frac{120 \cdot f}{p}$ $\quad$ $N_s = 1.5 \times 10^3\,rpm$

We use approximate equivalent circuit for all analysis and calculations.



(a) (i) The line current at the point A 5 marks

Point A is at start $\quad$ $s := 1$ $\quad$ $R_2 \cdot \frac{(1-s)}{s} = 0$

$V_1 := \frac{V_{3\phi}}{\sqrt{3}}$ $\quad$ $V_1 = 239.6\,V$

$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$ $\quad$ $I_2 = 89.663 - 199.252i$ $\quad$ $|I_2| = 218.497\,A$ $\quad$ $\arg(I_2) = -65.772\,deg$

$I_m := \frac{V_1}{j \cdot X_m}$ $\quad$ $I_m = -4.792i$ $\quad$ $|I_m| = 4.792$ $\quad$ $\arg(I_m) = -90\,deg$

$I_1 := I_m + I_2$ $\quad$ $I_1 = 89.663 - 204.044i$ $\quad$ $|I_1| = 222.875$ $\quad$ $\arg(I_1) = -66.278\,deg$

Starting current, $\quad$ $I_{start} := |I_1|$ $\quad$ $I_{start} = 222.875\,A$

(ii) power factor

$\theta := \arg(I_1)$ $\quad$ $\theta = -66.278\,deg$ $\quad$ $pf := \cos(\theta)$ $\quad$ $pf = 0.402$ lagging

(b) (i) Input power at the point C 5 marks

Point C is full-load. Full load slip $s := \frac{N_s - N_r}{N_s}$ $s = 0.067$

$$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$$ $I_2 = 67.347 - 20.722i$ $|I_2| = 70.463\,A$ $\arg(I_2) = -17.103\,deg$

$I_m := \frac{V_1}{j \cdot X_m}$ $I_m = -4.792i$ $|I_m| = 4.792$ $\arg(I_m) = -90\,deg$

$I_1 := I_m + I_2$ $I_1 = 67.347 - 25.514i$ $|I_1| = 72.018$ $\arg(I_1) = -20.749\,deg$

$\theta := \arg(I_1)$ $\theta = -20.749\,deg$ $pf := \cos(\theta)$ $pf = 0.935$ lagging

Input power: $P_{input} := 3\,|V_1| \cdot |I_1| \cdot pf$ $P_{input} = 48.409\,kW$

(ii) Motor Efficiency

$P_{rot} := 1500W$ rotational losses (windage plus friction)

Air gap power: $P_{ag} := \frac{3(|I_2|)^2 \cdot R_2}{s}$ $P_{ag} = 44.685\,kW$

Rotor Copper loss: $P_2 := s \cdot P_{ag}$ $P_2 = 2.979 \times 10^3$

Mechanical power: $P_m := (1 - s) \cdot P_{ag}$ $P_m = 41.706\,kW$

Output power: $P_{out} := P_m - P_{rot}$ $P_{out} = 40.206\,kW$

Motor efficiency, $Eff_{motor} := \frac{P_{out}}{P_{input}} \cdot 100$ $Eff_{motor} = 83.055$

(c) (i) Air gap power and developed torque at the point B 5 marks

(Point B is at which maximum torque developed)

$s_{Tmax} := \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$ $s_{Tmax} = 0.194$ $s := s_{Tmax}$

$$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$$ $I_2 = 116.223 - 90.744i$ $|I_2| = 147.453\,A$ $\arg(I_2) = -37.982\,deg$

Air gap power, $P_{ag} := \frac{3\left(\left|I_2\right|\right)^2 \cdot R_2}{s}$ $P_{ag} = 67.235\,kW$

$\omega_s := \frac{N_s}{60} \cdot 2\pi$ $T := \frac{P_{ag}}{\omega_s}$ $T_{max} := T$ $T_{max} = 428.029\,Nm$

(ii) Total losses of the motor

Stator copper loss: $P_1 := 3 \cdot \left(\left|I_2\right|\right)^2 \cdot R_1$ $P_1 = 16.307\,kW$

Rotor copper loss: $P_2 := 3 \cdot \left(\left|I_2\right|\right)^2 \cdot R_2$ $P_2 = 13.045\,kW$

Rotational losses: (windage and friction) $P_{rot} = 1.5\,kW$

$P_{total} := P_1 + P_2 + P_{rot}$ $P_{total} = 30.852\,kW$

(d) Speed of the motor (rotor), N_r, at the point D is synchronous speed, N_s. At this point slip s = 0, which means that no relative motion between rotating magnetic field of the stator and rotor conductors. As such, no emf would be induced and no current would flow in the rotor conductors. Therefore, no torque would be developed. Naturally, the rotor speed would not achieve the synchronous speed; the rotor speed, N_r, is always slightly less than synchronous speed.

1.5 marks

(e) X_m, in both transformer and induction machine, represents magnetizing inductance. This value reflects the amount of magnetizing current required to establish flux in the magnetic circuit. The major different between magnetic circuit in transformer and induction machine is that induction machine has air gap, while transformer does not have. Due to the air gap, higher value of current is required to establish flux in the induction machine. This corresponds to lower value of X_m in induction machine.

1.5 marks