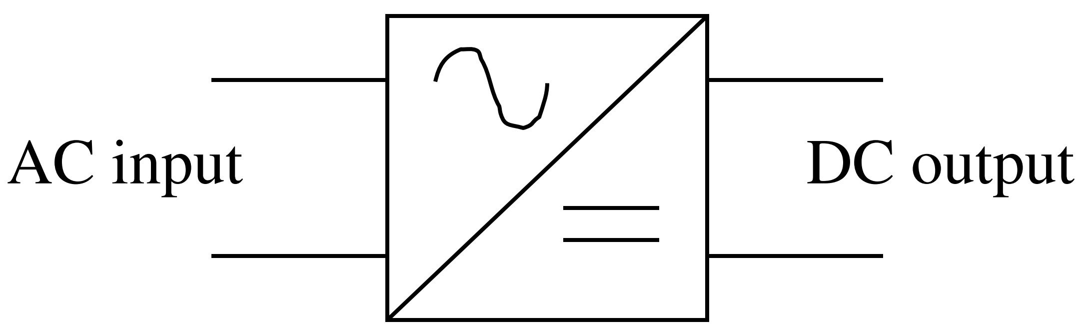
Chapter 2
AC to DC CONVERSION (RECTIFIER)

- Single-phase, half wave rectifier
  - Uncontrolled: R load, R-L load, R-C load
  - Controlled
  - Free wheeling diode
- Single-phase, full wave rectifier
  - Uncontrolled: R load, R-L load,
  - Controlled
  - Continuous and discontinuous current mode
- Three-phase rectifier
  - uncontrolled
  - controlled

Rectifiers

- **DEFINITION**: Converting AC (from mains or other AC source) to DC power by using power diodes or by controlling the firing angles of thyristors/controllable switches.

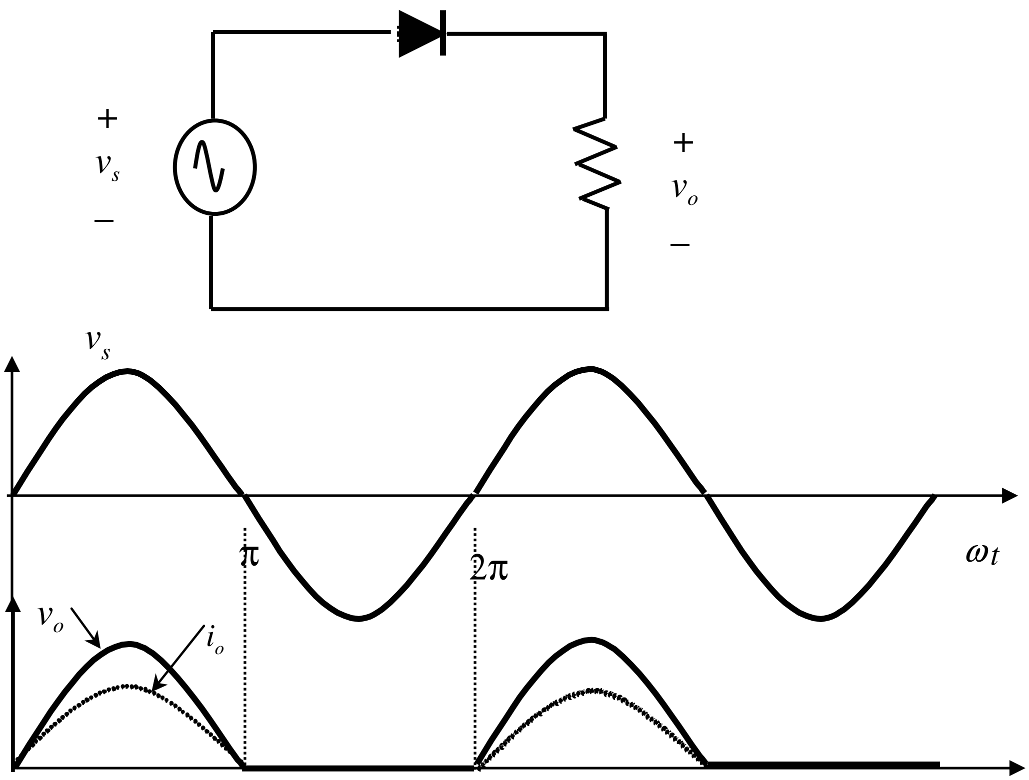
- Basic block diagram



- Input can be single or multi-phase (e.g. 3-phase).
- Output can be made fixed or variable

- **Applications**: DC welder, DC motor drive, Battery charger,DC power supply, HVDC

Single-phase, half-wave, R-load



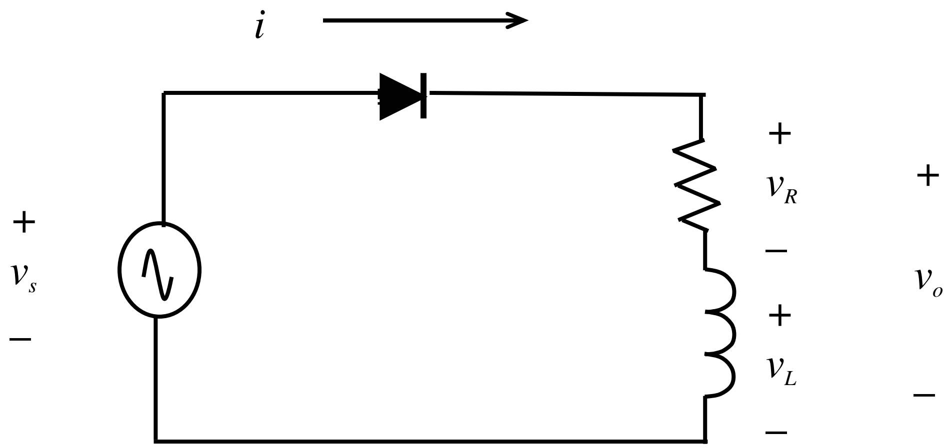
Output voltage (DC or average),

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{\pi} = 0.318 V_m$$

Output voltage (rms),

$$V_{o,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \left(V_m \sin(\omega t)\, d\omega t\right)^2} = \frac{V_m}{2} = 0.5 V_m$$

Half-wave with R-L load



$$\text{KVL}: \quad v_s = v_R + v_L$$

$$V_m \sin(\omega t) = i(\omega t)R + L\frac{di(\omega t)}{d\omega t}$$

First order differential eqn. Solution :

$$i(\omega t) = i_f(\omega t) + i_n(\omega t)$$

i_f : forced response; i_n natural response,

From diagram, forced response is :

$$i_f(\omega t) = \left(\frac{V_m}{Z}\right) \cdot \sin(\omega t - \theta)$$

where :

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

R-L load

Natural response is when source = 0,

$$i(\omega t)R + L\frac{di(\omega t)}{d\omega t} = 0$$

which results in :

$$i_n(\omega t) = Ae^{-\omega t/\omega\tau} \quad ; \tau = L/R$$

Hence

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \left(\frac{V_m}{Z}\right)\cdot\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

A can be solved by realising inductor current is zero before the diode starts conducting, i.e :

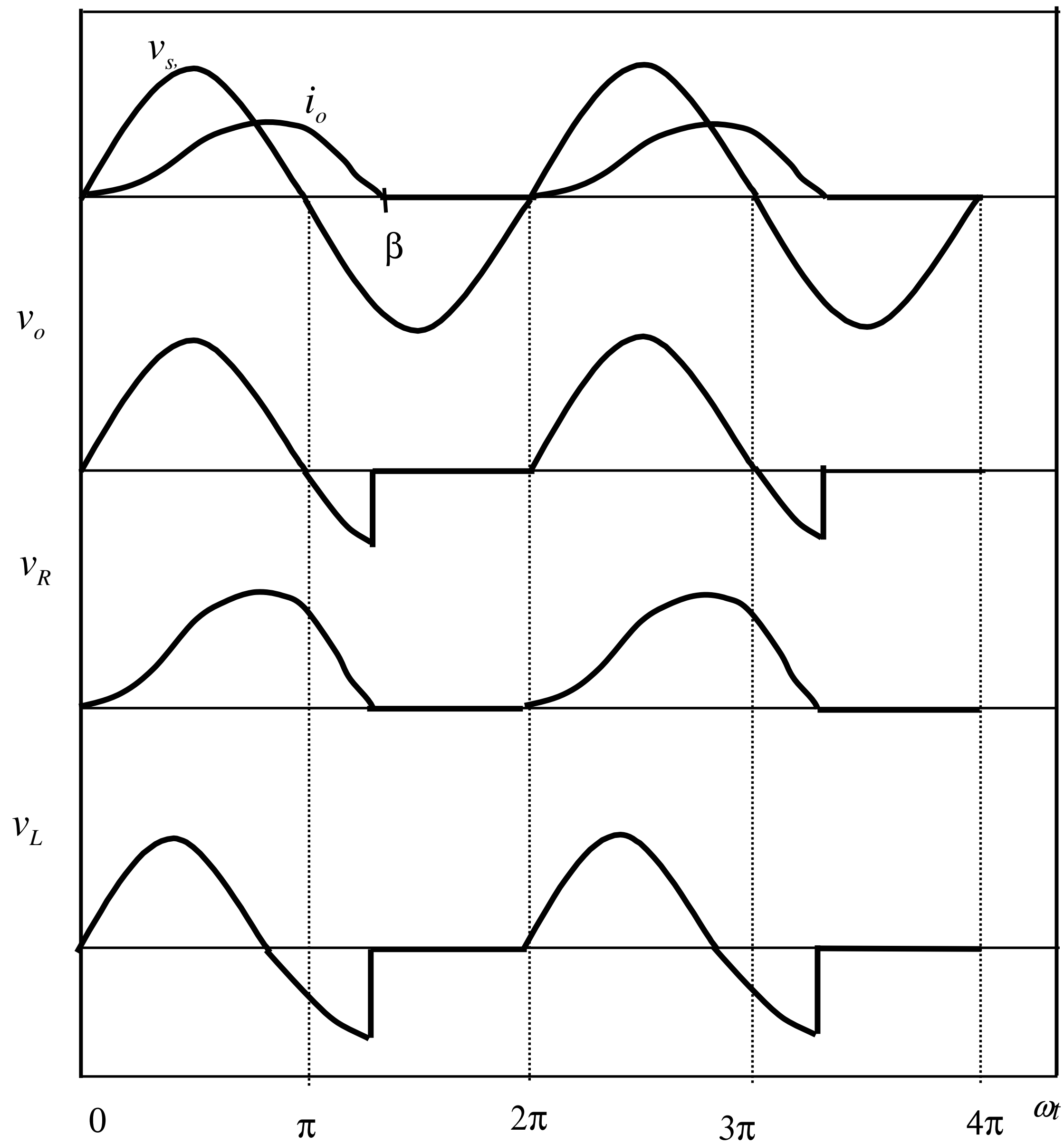
$$i(0) = \left(\frac{V_m}{Z}\right)\cdot\sin(0 - \theta) + Ae^{-0/\omega\tau}$$

$$\Rightarrow A = \left(\frac{V_m}{Z}\right)\cdot\sin(-\theta) = \left(\frac{V_m}{Z}\right)\cdot\sin(\theta)$$

Therefore the current is given as,

$$i(\omega t) = \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right]$$

R-L waveform



Note:

v_L is negative because the current is decreasing, i.e:

$$v_L = L\frac{di}{dt}$$

Extinction angle

Note that the diode remains in forward biased longer than π radians (although the source is negative during that duration)The point when current reaches zero is whendiode turns OFF. This point is known as theextinction angle, β.

$$i(\beta) = \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau}\right] = 0$$

which reduces to :

$$\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau} = 0$$

β can only be solved numerically.

Therefore, the diode conducts between 0 and β

To summarise the rectfier with R - L load,

$$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] \\ \text{for } 0 \le \omega t \le \beta \\ 0 \\ \text{otherwise} \end{cases}$$

RMS current, Power

The average (DC) current is :

$$I_o = \frac{1}{2\pi}\int_0^{2\pi} i(\omega t)\,d\omega t = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d\omega t$$

The RMS current is :

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d\omega t}$$

POWER CALCULATION

Power absorbed by the load is :

$$\mathrm{P_o} = (I_{RMS})^2 \cdot R$$

Power Factor is computed from definition :

$$pf = \frac{P}{S}$$

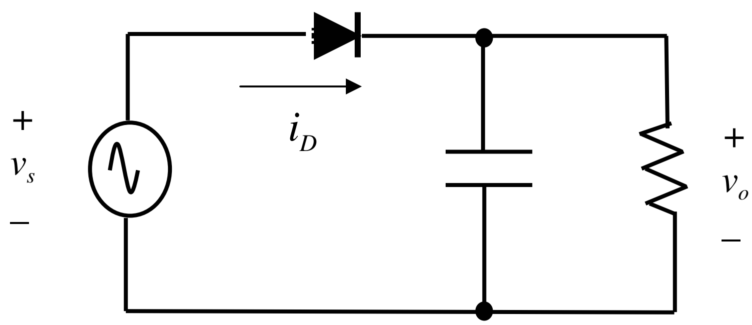
where P is the real power supplied by the source, which equal to the power absorbed by the load.

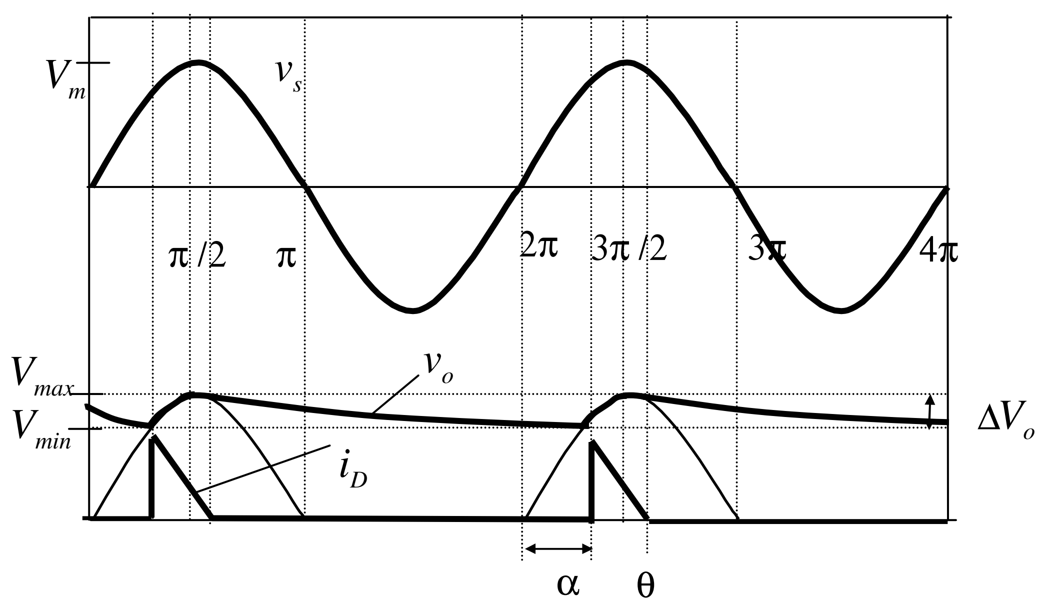
S is the apparent power supplied by the source, i.e

$$S = (V_{s,RMS}).(I_{RMS})$$

$$\Rightarrow pf = \frac{P}{(V_{s,RMS}).(I_{RMS})}$$

Half wave rectifier, R-C Load





$$v_o = \begin{cases} V_m \sin(\omega t) & \text{when diode is ON} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{when diode is OFF} \end{cases}$$

$$v_\theta = V_m \sin\theta$$

Operation

- Let C initially uncharged. Circuit is energised at $\omega t=0$
- Diode becomes forward biased as the source become positive
- When diode is ON the output is the same as source voltage. C charges until V_m
- After $\omega t=\pi/2$, C discharges into load (R).
- The source becomes less than the output voltage
- Diode reverse biased; isolating the load from source.
- The output voltage decays exponentially.

Estimation of θ

The slope of the functions are:

$$\frac{d\left(V_m \sin \omega t\right)}{d(\omega t)} = V_m \cos \omega t$$

and

$$\frac{d\left(V_m \sin\theta \cdot e^{-(\omega t-\theta)/\,\omega RC}\right)}{d(\omega t)}$$

$$= V_m \sin\theta \cdot \left(-\frac{1}{\omega RC}\right) \cdot e^{-(\omega t-\theta)/\,\omega RC}$$

At $\omega t = \theta$, the slopes are equal,

$$V_m \cos\theta = V_m \sin\theta \cdot \left(-\frac{1}{\omega RC}\right) \cdot e^{-(\theta-\theta)/\,\omega RC}$$

$$\Rightarrow \frac{V_m \cos\theta}{V_m \sin\theta \cdot} = -\frac{1}{\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

$$\theta = \tan^{-1}\left(-\omega RC\right) = -\tan^{-1}\left(\omega RC\right) + \pi$$

For practical circuits, ωRC is large, then:

$$\theta = \text{-}\tan(\infty) + \pi = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

θ is very close to the peak of the sine wave. Therefore

and $V_m \sin\theta = V_m$

Estimation of α

At $\omega t = 2\pi + \alpha$,

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta) e^{-(2\pi+\alpha-\theta)/\omega RC}$$

or

$$\sin(\alpha - (\sin\theta) e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$$

This equation must be solved numerically for α

Ripple Voltage

Max output voltage is $V_{\max}$.

Min output voltage occurs at $\omega t = 2\pi + \alpha$

$$\Delta V_o = V_{\max} - V_{\min}$$

$$= V_m - V_m \sin(2\pi + \alpha) = V_m - V_m \sin\alpha$$

If $V_\theta = V_m$ and $\theta = \pi/2$, and C is large such that DC output voltage is constant, then $\alpha \approx \pi/2$.

The output voltage evaluated at $\omega t = 2\pi + \alpha$ is :

$$v_o(2\pi + \alpha) = V_m e^{-\left(\frac{2\pi + \pi/2 - \pi/2}{\omega RC}\right)} = V_m e^{-\left(\frac{2\pi}{\omega RC}\right)}$$

The ripple voltage is approximated as :

$$\Delta V_o \approx V_m - V_m e^{-\left(\frac{2\pi}{\omega RC}\right)} = V_m\left(1 - e^{-\left(\frac{2\pi}{\omega RC}\right)}\right)$$

Using Series expansoin : $\quad e^{-\left(\frac{2\pi}{\omega RC}\right)} = 1 - \frac{2\pi}{\omega RC}$

$$\Rightarrow \Delta V_o = V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

Capacitor Current

The current in the capacitor can be expressed as :

$$i_c(t) = C\frac{dv_o(t)}{d(t)}$$

In terms of ωt,:

$$i_c(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

But

$$v_o(\omega t) = \begin{cases} V_m \sin(\omega t) & \text{when diode is ON} \\ V_m \sin\theta \cdot e^{-(\omega t-\theta)/\omega RC} & \text{when diode is OFF} \end{cases}$$

Then, substituting $v_o(\omega t)$,

$$i_c(\omega t) = \begin{cases} \omega C V_m \cos(\omega t) \\ \text{when diode is ON,} \\ \text{i.e } (2\pi+\alpha) \le \omega t \le (2\pi+\theta) \\ \\ -\frac{V_m \sin\theta}{R} \cdot e^{-(\omega t-\theta)/\omega RC} \\ \text{when diode is OFF,} \\ \text{i.e } (\theta) \le \omega t \le (2\pi+\alpha) \end{cases}$$

Peak Diode Current

Note that :

$$i_s = i_D = i_R + i_C$$

The peak diode current occurs at $(2\pi + \alpha)$. Hence.

$$I_{c,peak} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

Resistor current at $(2\pi + \alpha)$ can be obtained :

.

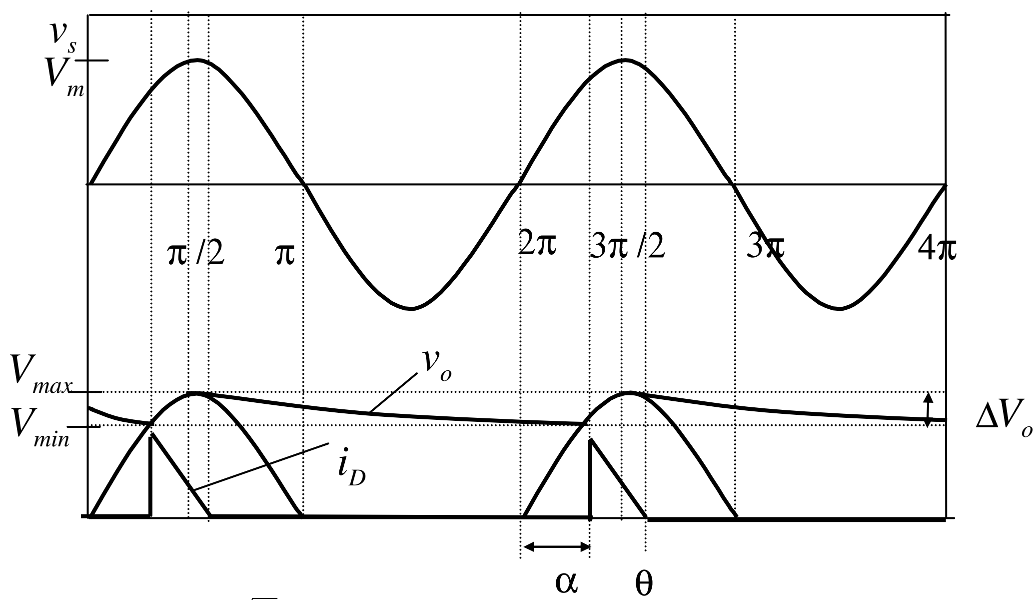
$$i_R(2\pi + \alpha) = \frac{V_m \sin(2\pi + \alpha)}{R} = \frac{V_m \sin\alpha}{R}$$

The diode peak current is :

$$i_{D,peak} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R}$$

Example

A half-wave rectifier has a 120V rms source at 60Hz. The load is =500 Ohm, C=100uF. Assume α and θ are calculated as 48 and 93 degrees respectively. Determine (a) Expression for output voltage (b) peak-to peak ripple (c) capacitor current (d) peak diode current.



$$V_m = 120\sqrt{2} = 169.7V;$$

$$\theta = 93^o = 1.62rad;$$

$$\alpha = 48^o = 0.843rad$$

$$V_m \sin\theta = 169.7\sin(1.62rad) = 169.5V;$$

(a) Output voltage:

$$v_o(\omega t) = \begin{cases} V_m \sin(\omega t) = 169.7\sin(\omega t) & \text{(ON)} \\ V_m \sin\theta \cdot e^{-(\omega t-\theta)/\omega RC} & \text{(OFF)} \end{cases}$$

$$= \begin{cases} 169.7\sin(\omega t) & \text{(ON)} \\ 169.5e^{-(\omega t-1.62)/(18.85)} & \text{(OFF)} \end{cases}$$

Example (cont')

(b)Ripple:

Using: $\Delta V_o = V_{\max} - V_{\min}$

$$\Delta V_o = V_m - V_m \sin(2\pi + \alpha) = V_m - V_m \sin\alpha = 43V$$

Using Approximation:

$$\Delta V_o = V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC} = \frac{169.7}{60 \times 500 \times 100u} = 56.7V$$

(c) Capacitor current:

$$i_c(\omega t) = \begin{cases} \omega C V_m \cos(\omega t) & \text{(ON)} \\ -\dfrac{V_m \sin(\theta)}{R} \cdot e^{-(\omega t - \theta)/(\omega RC)} & \text{(OFF)} \end{cases}$$

$$= \begin{cases} 6.4\cos(\omega t)\ \text{A} & \text{(ON)} \\ -0.339 \cdot e^{-(\omega t - 1.62)/(18.85)}\ \ \text{A} & \text{(OFF)} \end{cases}$$

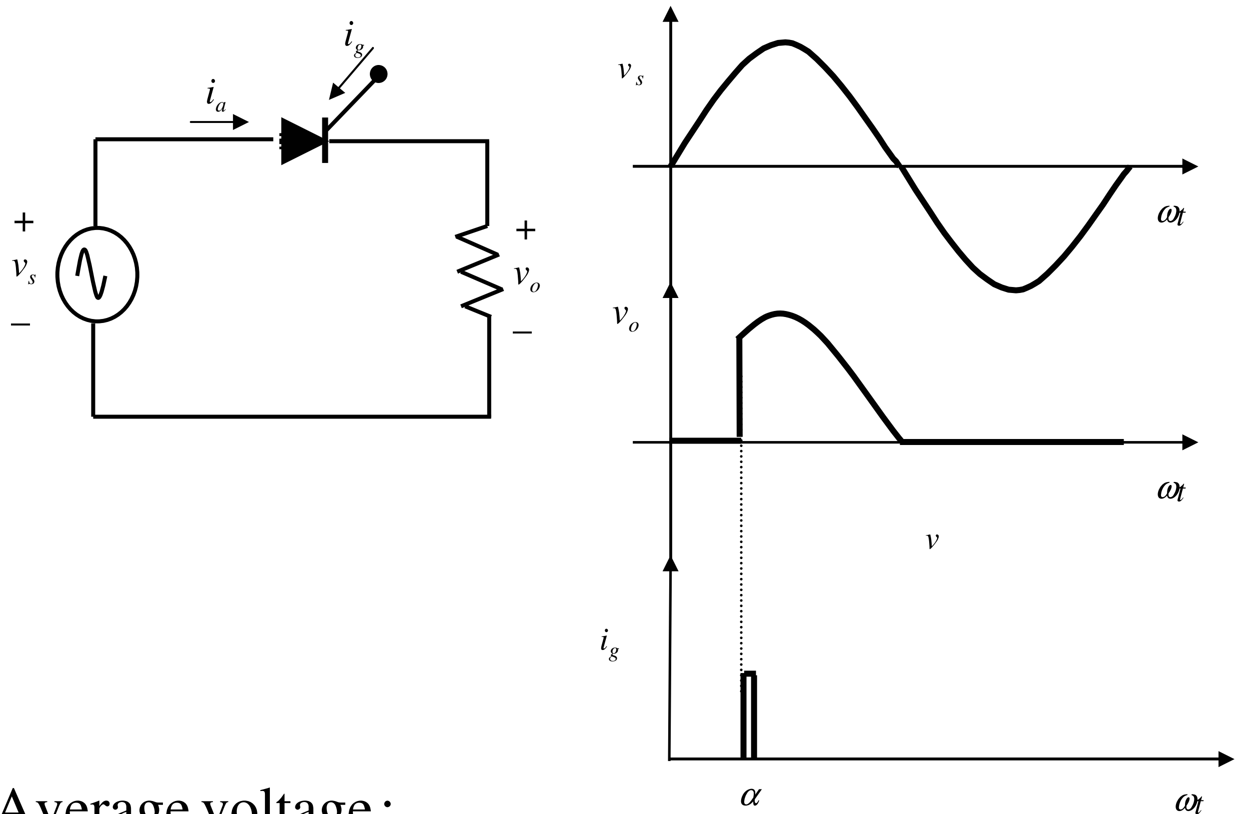
(d) Peak diode current:

$$i_{D,peak} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R}$$

$$= (2 \times \pi \times 60)(100u)169.7\cos(0.843rad) + \frac{169.7\sin(1.62rad)}{500}$$

$$= (4.26 + 0.34) = 4.50A$$

Controlled half-wave



Average voltage:

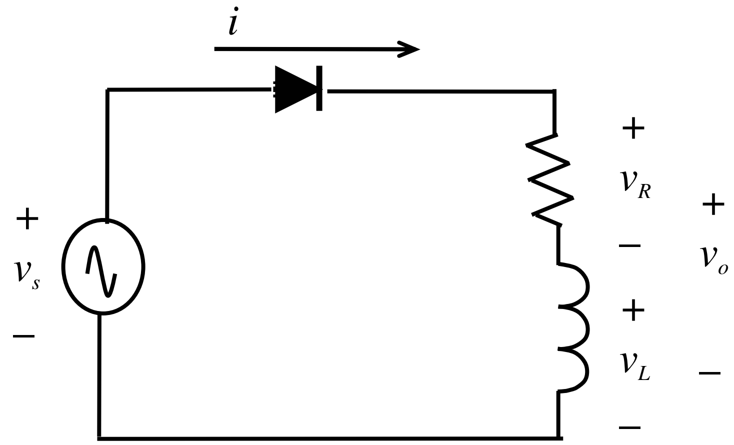
$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t = \frac{V_m}{2\pi}[1+\cos\alpha]$$

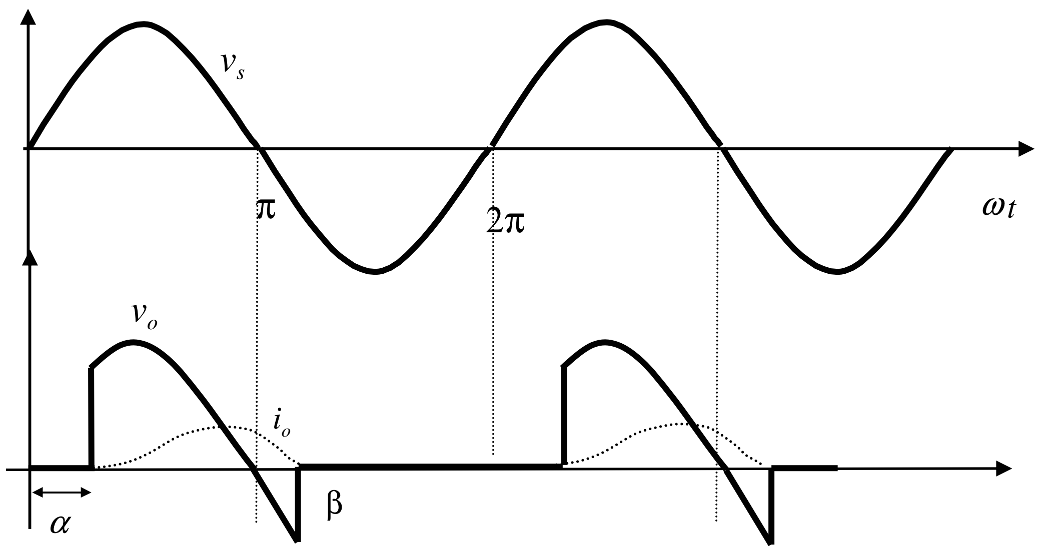
RMS voltage

$$V_{o,RMS} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}[V_m \sin(\omega t)]^2 \, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_{\alpha}^{\pi}[1-\cos(2\omega t] \, d\omega t} = \frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}$$

Controlled h/w, R-L load





$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \left(\frac{V_m}{Z}\right)\cdot \sin(\omega t - \theta) + Ae^{\frac{-\omega t}{\omega\tau}}$$

Initial condition : $i(\alpha) = 0$,

$$i(\alpha) = 0 = \left(\frac{V_m}{Z}\right)\cdot \sin(\alpha - \theta) + Ae^{\frac{-\alpha}{\omega\tau}}$$

$$\Rightarrow A = -\left[\left(\frac{V_m}{Z}\right)\cdot \sin(\alpha - \theta)\right]e^{\frac{-\alpha}{\omega\tau}}$$

Controlled R-L load

Substituting for *A* and simplifying,

$$i(\omega t)=\begin{cases}\left(\dfrac{V_m}{Z}\right)\cdot\left[\sin(\omega t-\theta)-\sin(\alpha-\theta)e^{\frac{-(\alpha-\omega t)}{\omega\tau}}\right] & \text{for } \alpha\le\omega t\le\beta\\ 0 \quad \text{otherwise}\end{cases}$$

Extinction angle β must be solved numerically

$$i(\beta)=0=\left(\frac{V_m}{Z}\right)\left[\sin(\beta-\theta)-\sin(\beta-\theta)e^{\frac{-(\alpha-\beta)}{\omega\tau}}\right]$$

Angle $\gamma=(\beta-\theta)$ is called the conduction angel.

Average voltage:

$$V_o=\frac{1}{2\pi}\int_{\alpha}^{\beta}V_m\sin(\omega t)d\omega t=\frac{V_m}{2\pi}[\cos\alpha-\cos\beta]$$

Average current:

$$I_o=\frac{1}{2\pi}\int_{\alpha}^{\beta}i(\omega t)d\omega$$

RMS current:

$$I_{RMS}=\sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta}i^2(\omega t)d\omega}$$

The power absorbed by the load:

$$P_o=I_{RMS}{}^2\cdot R$$

Examples

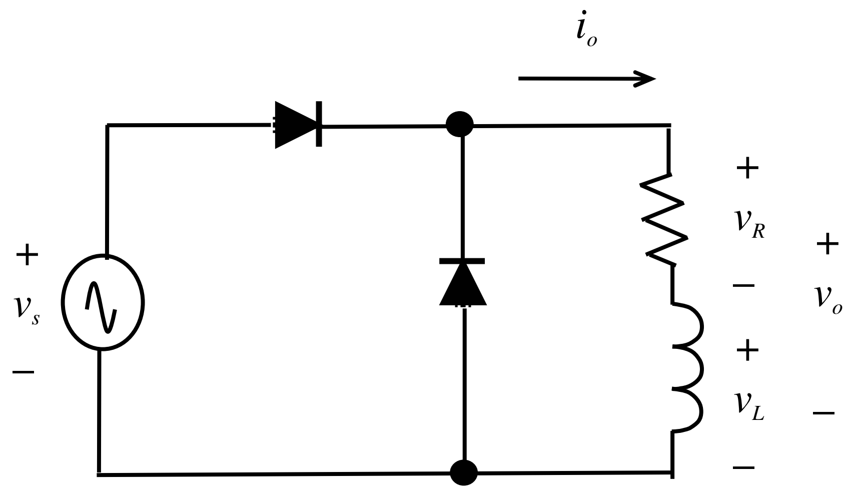
1. A half wave rectifier has a source of 120V RMS at 60Hz. R=20 ohm, L=0.04H, and the delay angle is 45 degrees. Determine: (a) the expression for $i(\omega t)$, (b) average current, (c) the power absorbed by the load.

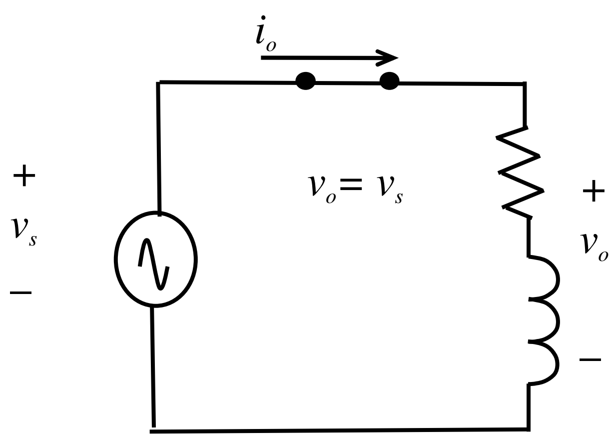
2. Design a circuit to produce an average voltage of 40V across a 100 ohm load from a 120V RMS, 60Hz supply. Determine the power factor absorbed by the resistance.

Freewheeling diode (FWD)

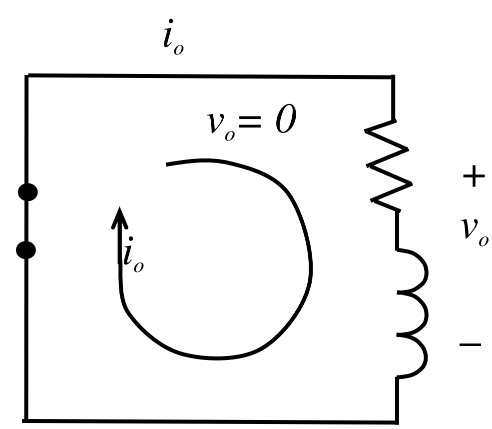
- Note that for single-phase, half wave rectifier with R-L load, the load (output) current is NOT continuos.

- A FWD (sometimes known as commutation diode) can be placed as shown below to make it continuos





D_1 is on, D_2 is off



D_2 is on, D_1 is off

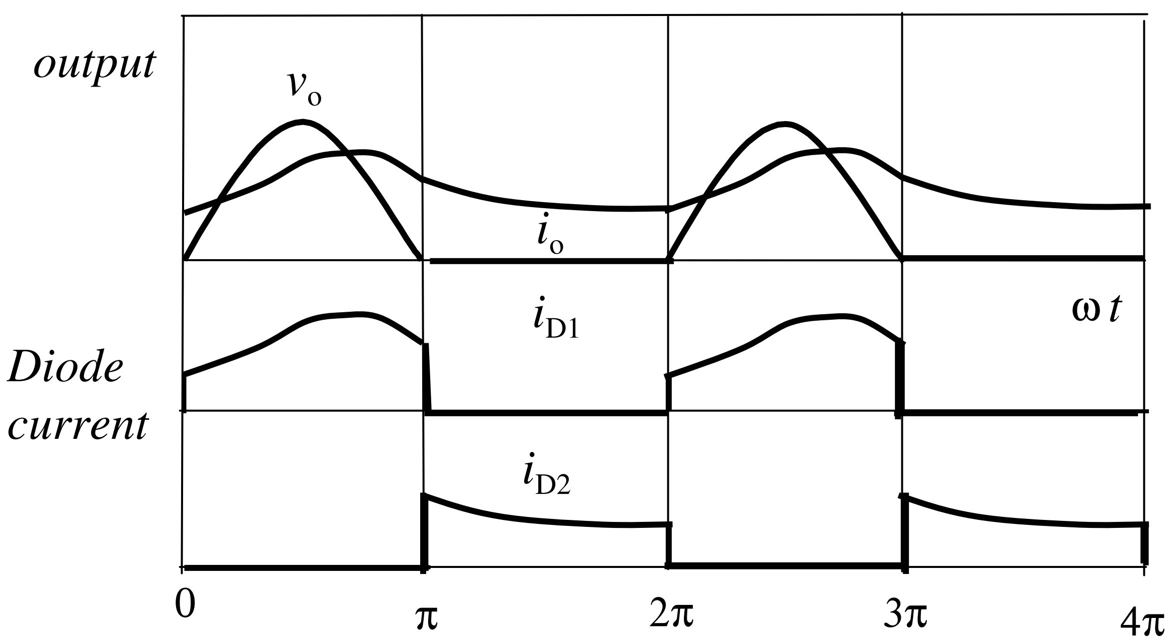
Operation of FWD

- Note that both D_1 and D_2 cannot be turned on at the same time.
- For a positive cycle voltage source,
  - D_1 is on, D_2 is off
  - The equivalent circuit is shown in Figure (b)
  - The voltage across the R-L load is the same as the source voltage.

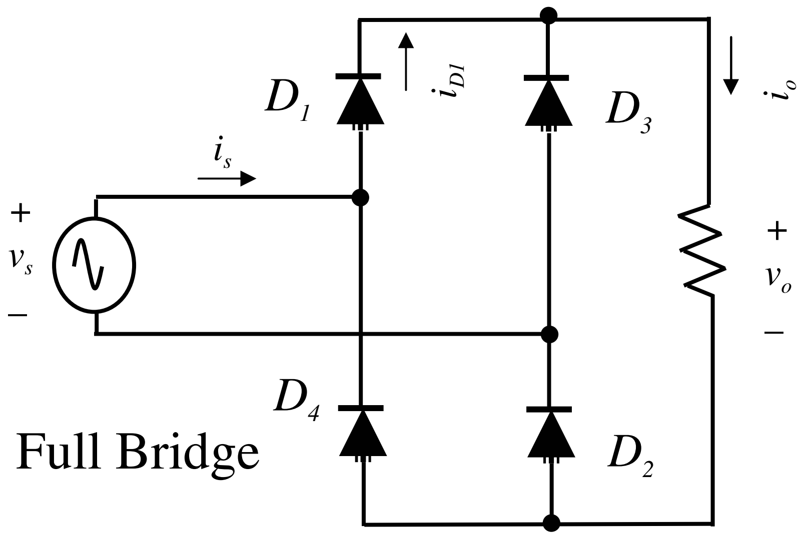
- For a negative cycle voltage source,
  - D_1 is off, D_2 is on
  - The equivalent circuit is shown in Figure (c)
  - The voltage across the R-L load is zero.
  - However, the inductor contains energy from positive cycle. The load current still circulates through the R-L path.
  - But in contrast with the normal half wave rectifier, the circuit in Figure (c) does not consist of supply voltage in its loop.
  - Hence the "negative part" of v_o as shown in the normal half-wave disappear.

FWD- Continuous load current

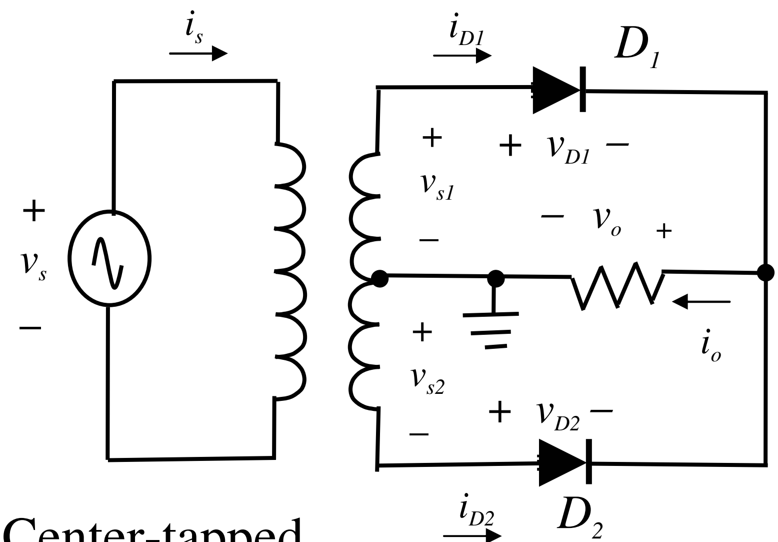
- The inclusion of FWD results in continuos load current, as shown below.
- Note also the output voltage has no negative part.



Full wave rectifier



Full Bridge



Center-tapped

- Center-tapped (CT) rectifier requires center-tap transformer. Full Bridge (FB) does not.
- CT: 2 diodes
- FB: 4 diodes. Hence, CT experienced only one diode volt-drop per half-cycle
- Conduction losses for CT is half.
- Diodes ratings for CT is twice than FB

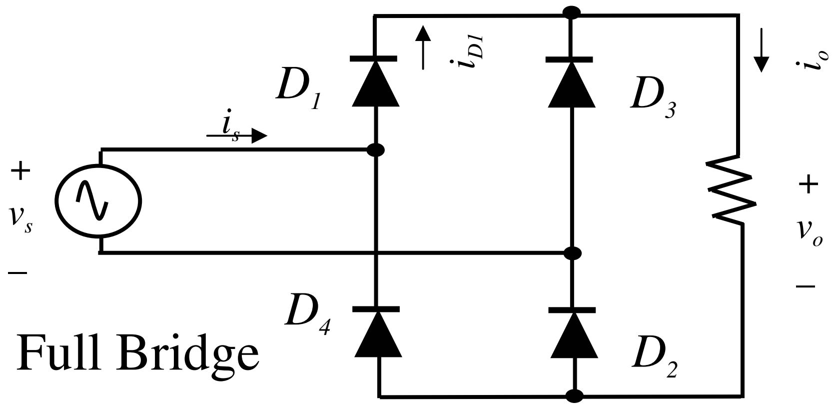
For both circuits,

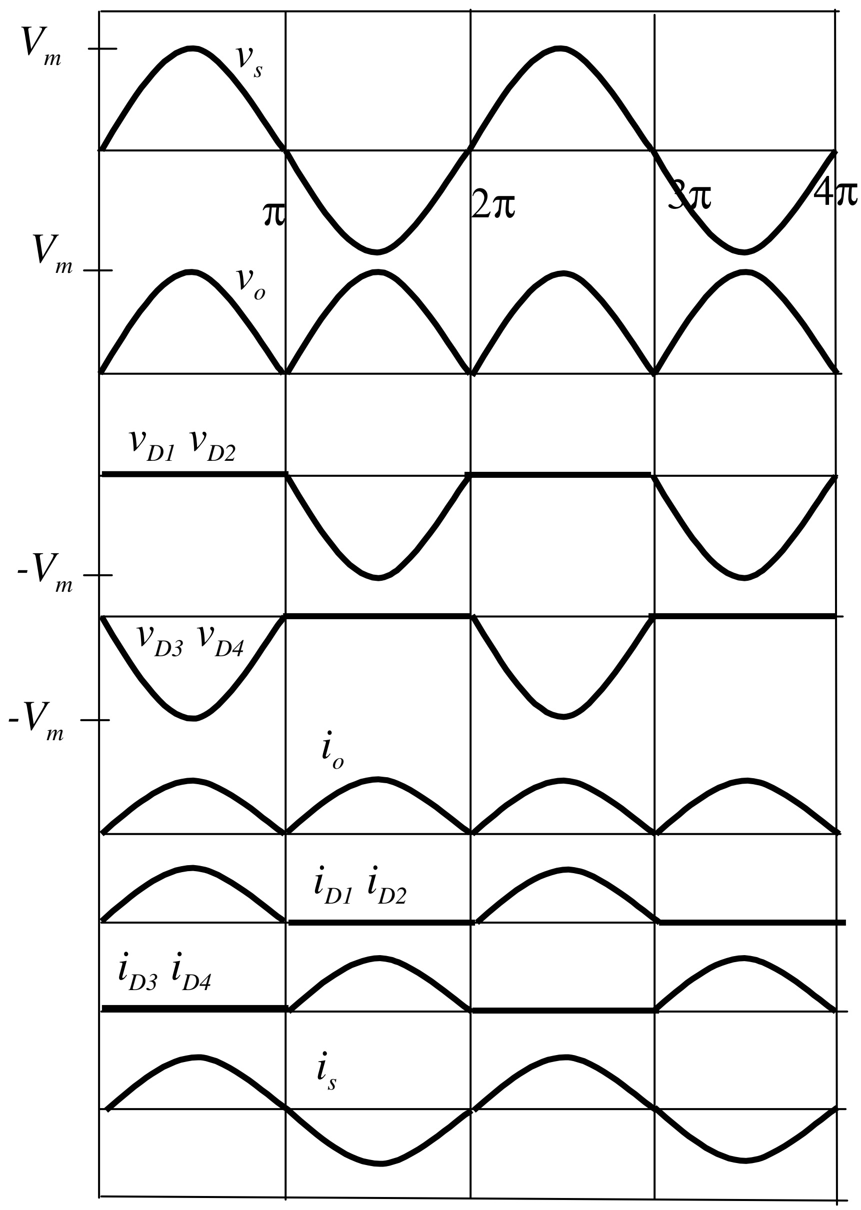
$$v_o = \begin{cases} V_m \sin \omega t & 0 \le \omega t \le \pi \\ -V_m \sin \omega t & \pi \le \omega t \le 2\pi \end{cases}$$

Average (DC) voltage:

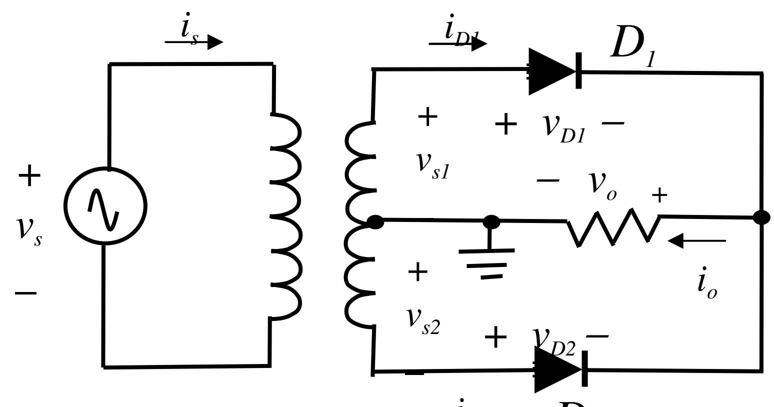
$$V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{2V_m}{\pi} = 0.637 V_m$$

Bridge waveforms

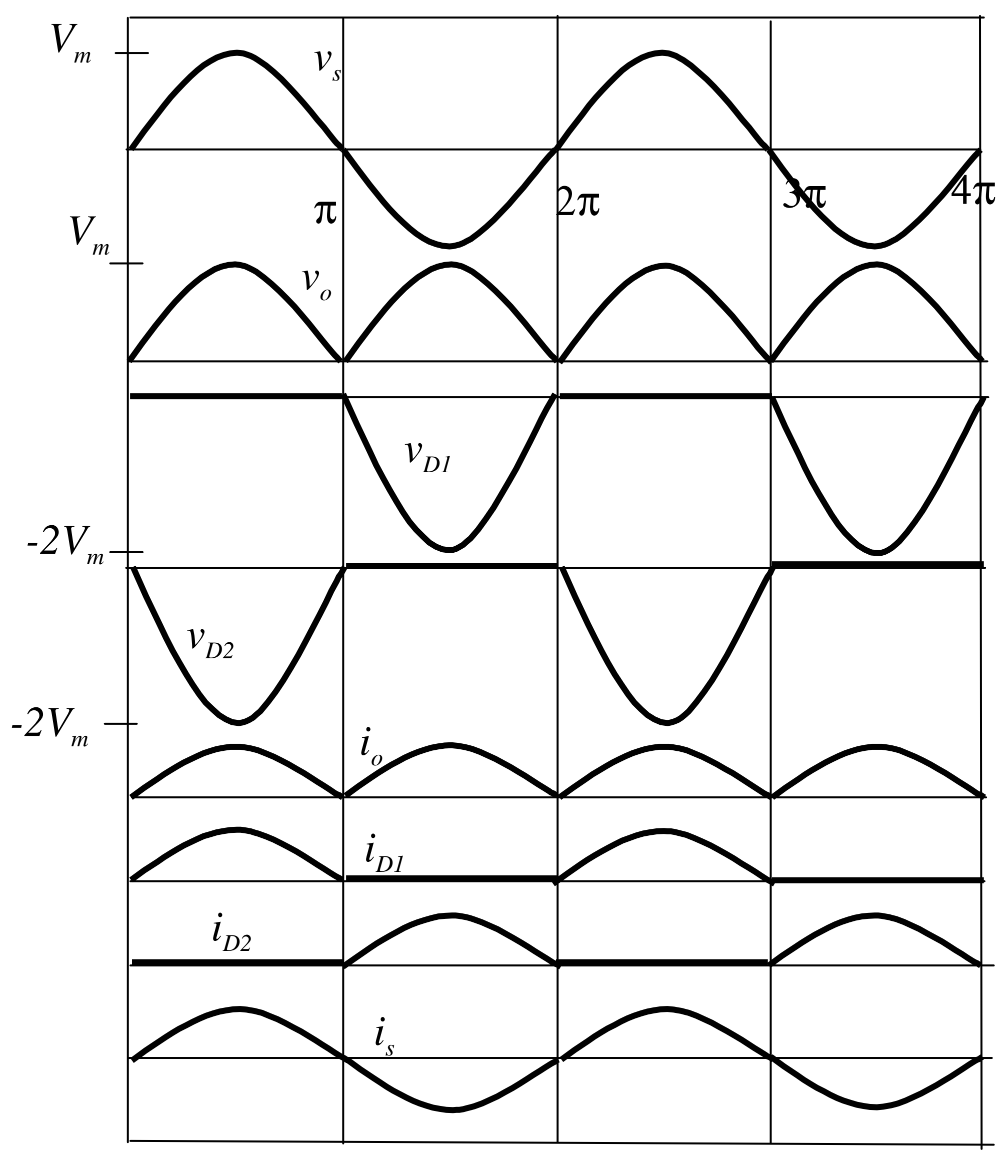




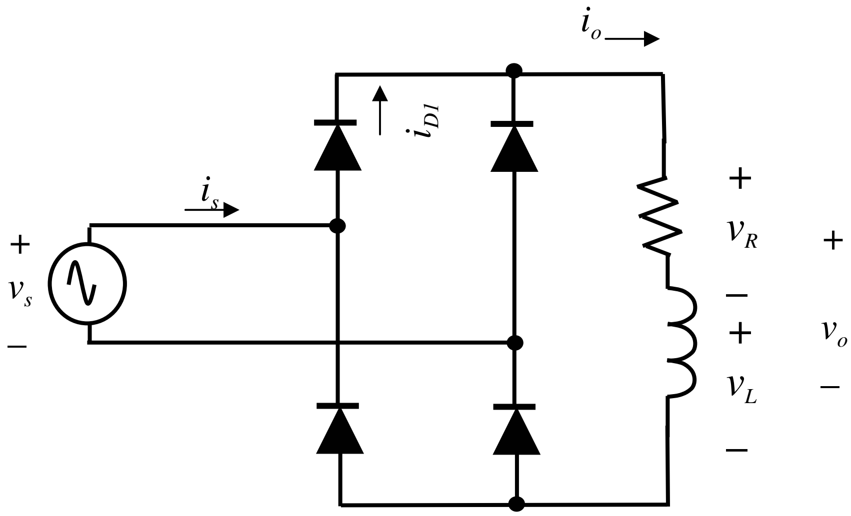
Center-tapped waveforms

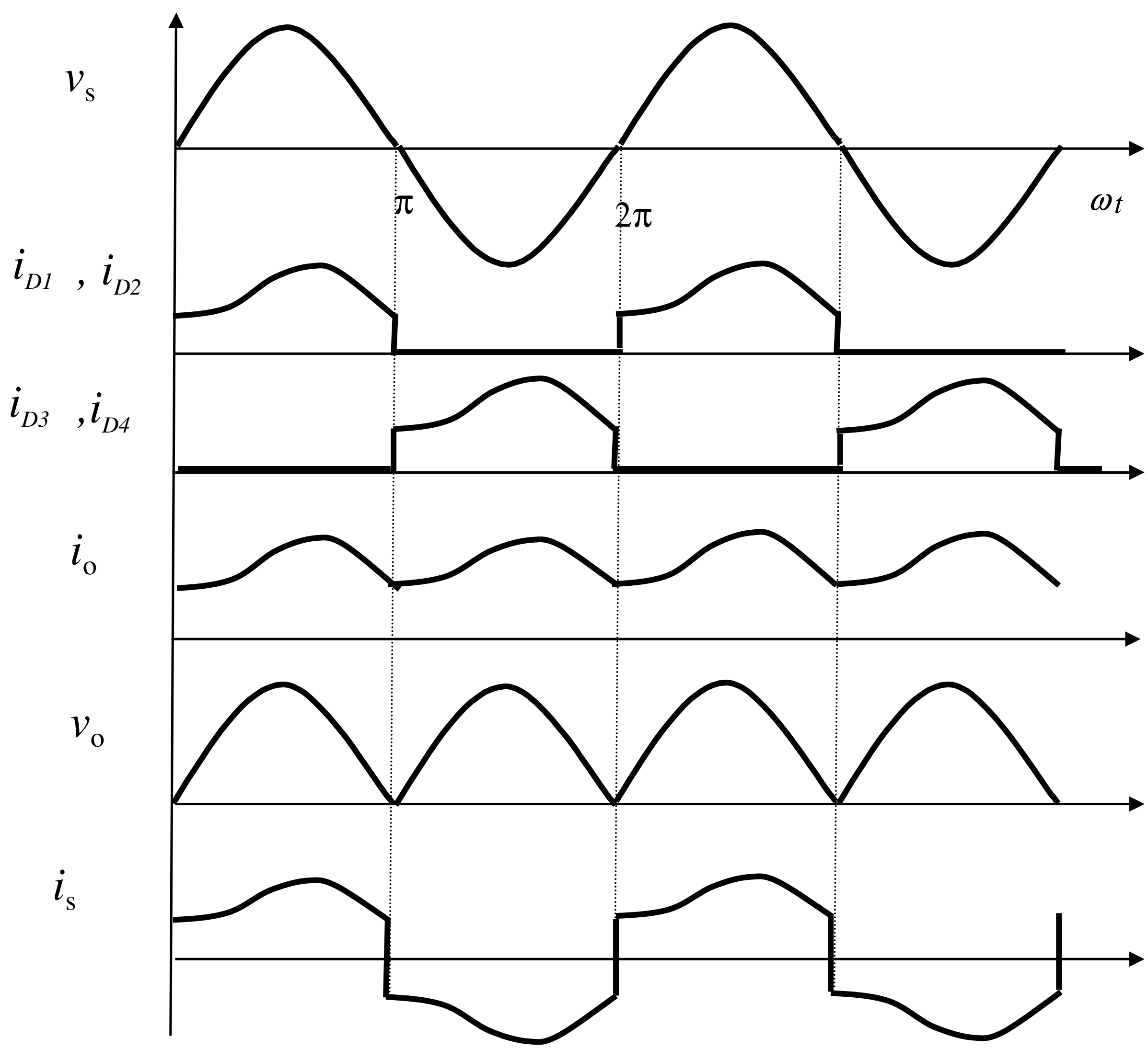


Center-tapped



Full wave bridge, R-L load





Approximation with large L

Using Fourier Series,

$$v_o(\omega t) = V_o + \sum_{n=2,4...}^{\infty} V_n \cos(n\omega t + \pi)$$

where the DC term

$$V_o = \frac{2V_m}{\pi}$$

and the harmonics terms

$$V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

The DC curent

$$I_o = \frac{V_o}{R}$$

The harmonic currents:

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\left|R + jn\omega L\right|}$$

As n increases, V_n harmonic decreases.

Thus I_n decreases rapidly very increasing n.

If ωL is large enough, it is possible to drop all the harmonic terms, i.e.:

$$i(\omega t) \approx I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R}, \quad \text{for } \omega L >> R,$$

R-L load approximation

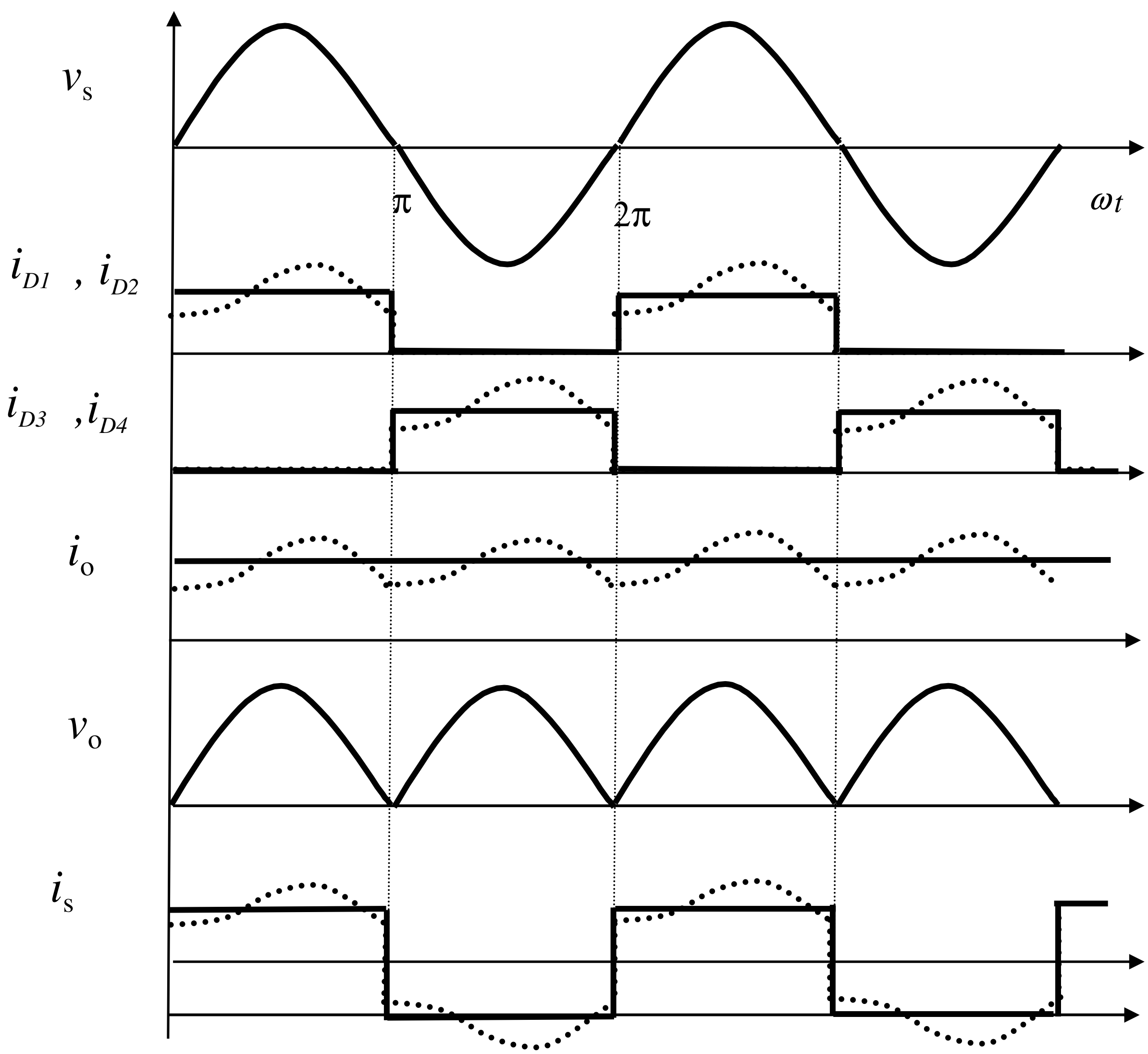
Approximate current

$$I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R},$$

$$I_{RMS} = \sqrt{I_o^{\,2} + \sum \left(I_{n,RMS}^{\,2}\right)} = I_o$$

Power delivered to the load :

$$P_o = I_{RMS}^{\,2} R$$



Examples

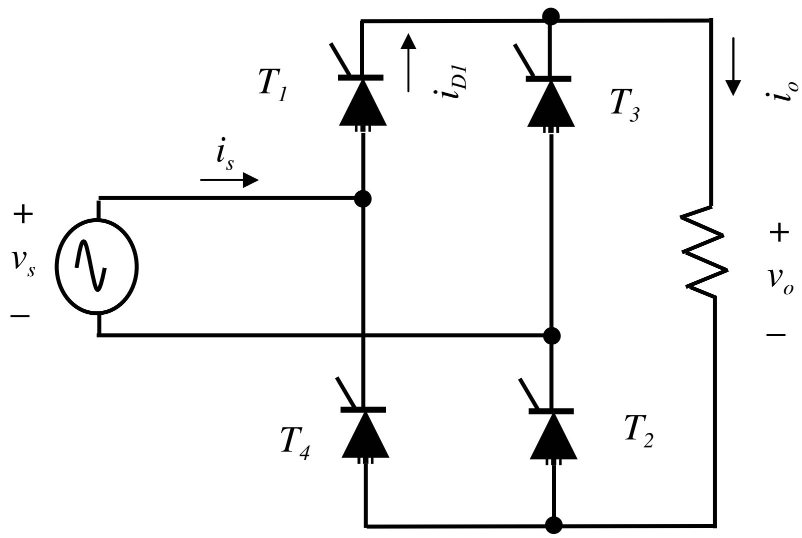
Given a bridge rectifier has an AC source V_m=100V at 50Hz, and R-L load with R=100ohm, L=10mH

a) determine the average current in the load

b) determine the first two higher order harmonics of the load current

c) determine the power absorbed by the load

Controlled full wave, R load



Average (DC) voltage:

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t = \frac{V_m}{\pi}[1+\cos\alpha]$$

RMS Voltage

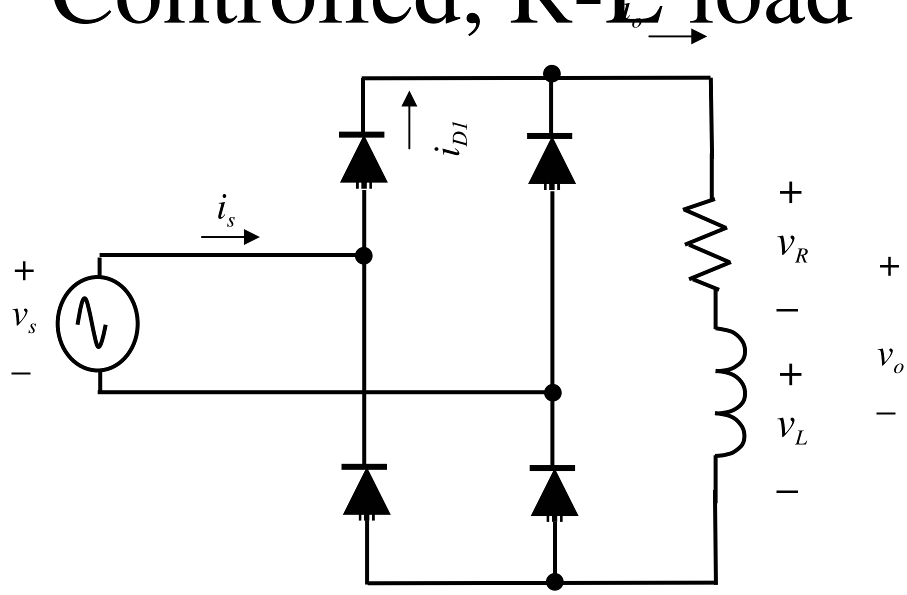
$$V_{o,RMS} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}[V_m \sin(\omega t)]^2 \, d\omega t}$$

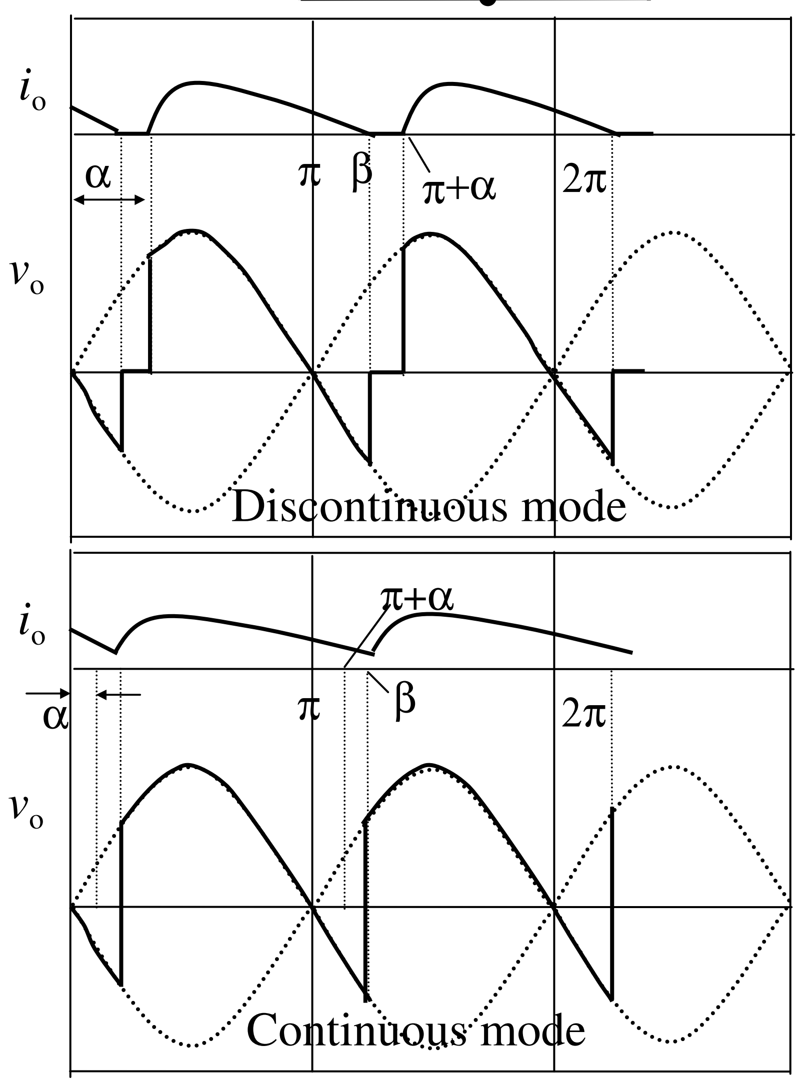
$$= V_m\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The power absorbed by the R load is:

$$P_o = \frac{{V_{RMS}}^2}{R}$$

Controlled, R-L load





Discontinuous mode

Analysis similar to controlled half wave with R - L load :

$$i(\omega t) = \left(\frac{V_m}{Z}\right) \cdot \left[\sin(\omega t - \theta) - \sin(\alpha - \theta) e^{-(\omega t - \alpha)/\omega\tau}\right]$$

for $\alpha \le \omega t \le \beta$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$; $\tau = \frac{L}{R}$

For discontinous mode, need to ensure :

$$\beta < (\alpha + \pi)$$

Note that β is the extinction angle and must be solved numerically with condition :

$$i_o(\beta) = 0$$

The boundary between continous and discontinous current mode is when β in the output current expression is $(\pi + \alpha)$.

For continous operation current at $\omega t = (\pi + \alpha)$ must be greater than zero.

Continuous mode

$$i(\pi+\alpha)\geq 0$$

$$\sin(\pi+\alpha-\theta)-\sin(\pi+\alpha-\theta)e^{-(\pi+\alpha-\alpha)/\omega\tau}\geq 0$$

Using Trigonometry identity :

$$\sin(\pi+\alpha-\theta)=\sin(\theta-\alpha),$$

$$\sin(\theta-\alpha)\left[1-e^{-(\pi/\omega\tau)}\right]\geq 0,$$

Solving for α

$$\alpha=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

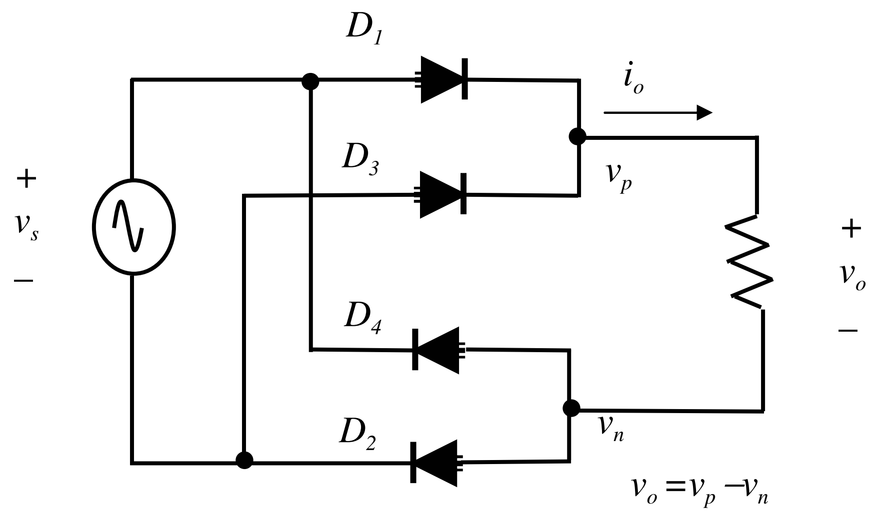
Thus for continuous current mode,

$$\alpha\leq\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Average (DC) output voltage is given as :

$$V_o=\frac{1}{\pi}\int_{\alpha}^{\alpha+\pi}V_m\sin(\omega t)d\omega t=\frac{2V_m}{\pi}\cos\alpha$$

Single-phase diode groups



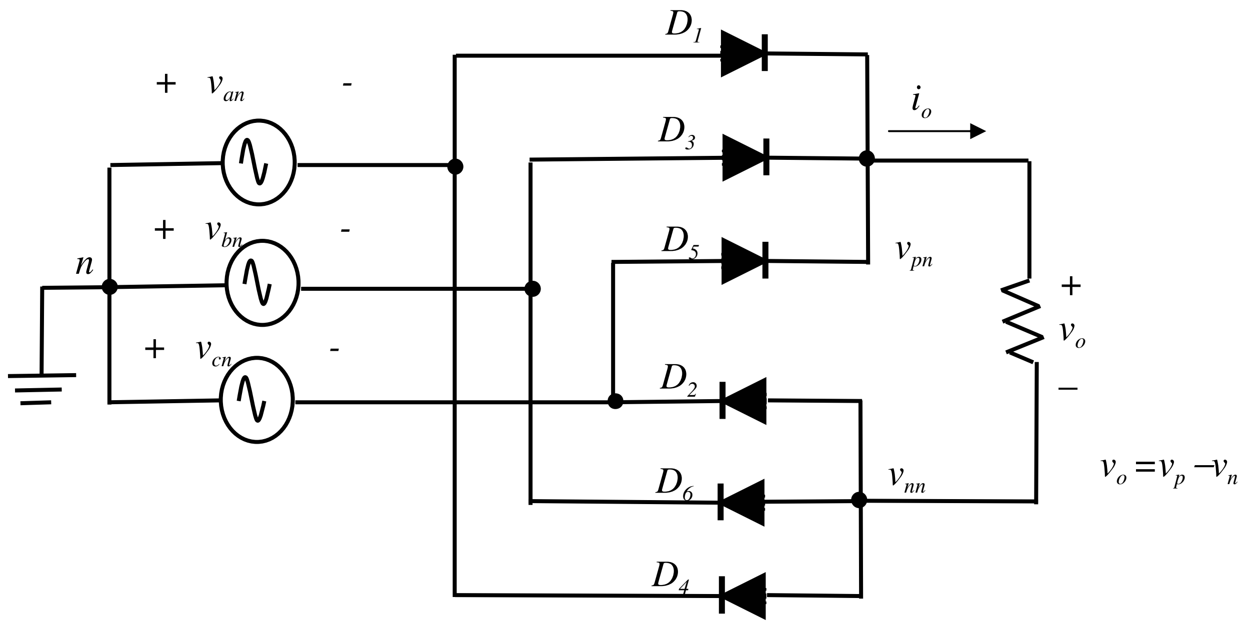
- In the top group (D_1, D_3), the cathodes (-) of the two diodes are at a common potential. Therefore, the diode with its anode (+) at the highest potential will conduct (carry) i_d.

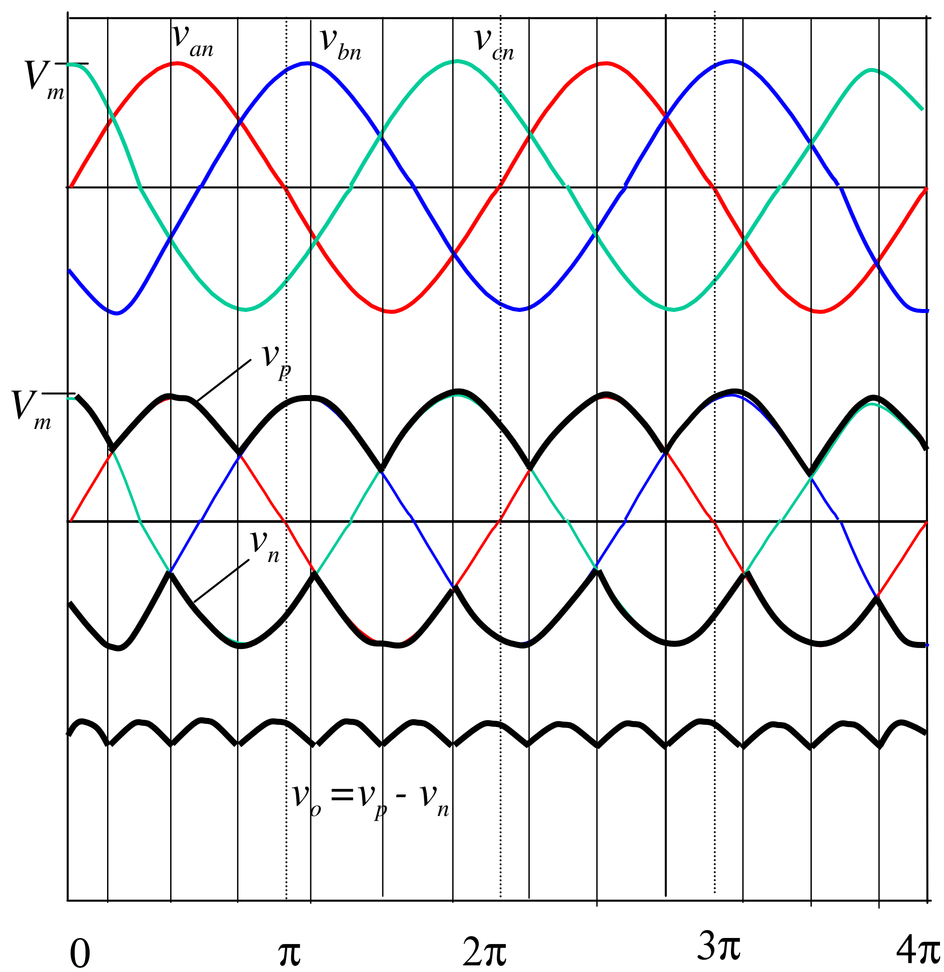
- For example, when v_s is (+), D_1 conducts i_d and D_3 reverses (by taking loop around v_s, D_1 and D_3). When v_s is (-), D_3 conducts, D_1 reverses.

- In the bottom group, the anodes of the two diodes are at common potential. Therefore the diode with its cathode at the lowest potential conducts i_d.

- For example, when v_s (+), D_2 carry i_d. D_4 reverses. When v_s is (-), D_4 carry i_d. D_2 reverses.

Three-phase rectifiers





Three-phase waveforms

- Top group: diode with its anode at the highest potential will conduct. The other two will be reversed.

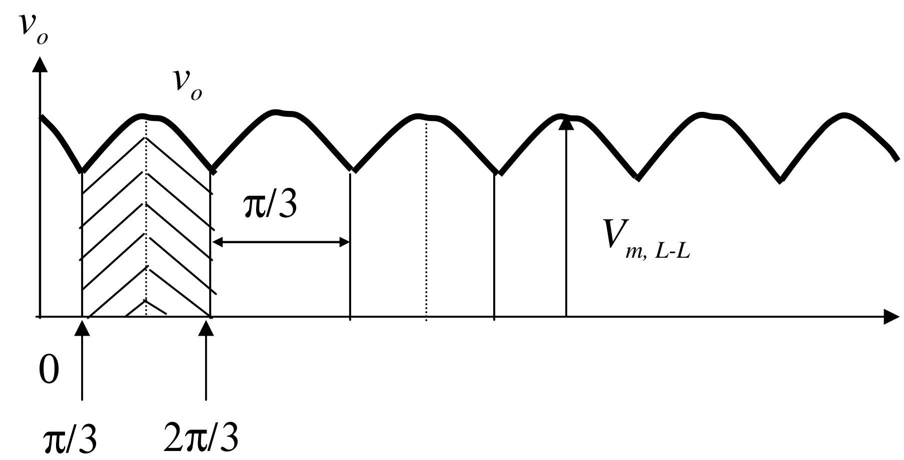
- Bottom group: diode with the its cathode at the lowest potential will conduct. The other two will be reversed.

- For example, if D_1 (of the top group) conducts, v_p is connected to v_{an}. If D_6 (of the bottom group) conducts, v_n connects to v_{bn}. All other diodes are off.

- The resulting output waveform is given as: $v_o = v_p - v_n$

- For peak of the output voltage is equal to the peak of the line to line voltage v_{ab} .

Three-phase, average voltage



Considers only one of the six segments. Obtain its average over 60 degrees or $\pi/3$ radians.

Average voltage:

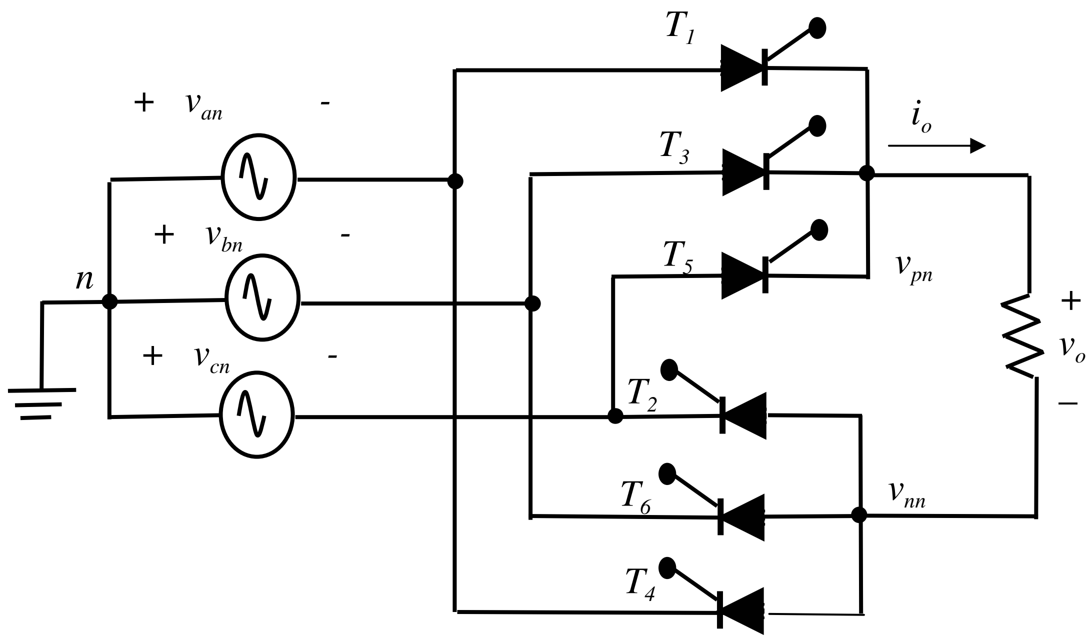
$$V_o = \frac{1}{\pi/3} \int_{\pi/3}^{2\pi/3} V_{m,L-L} \sin(\omega t) d\omega t$$

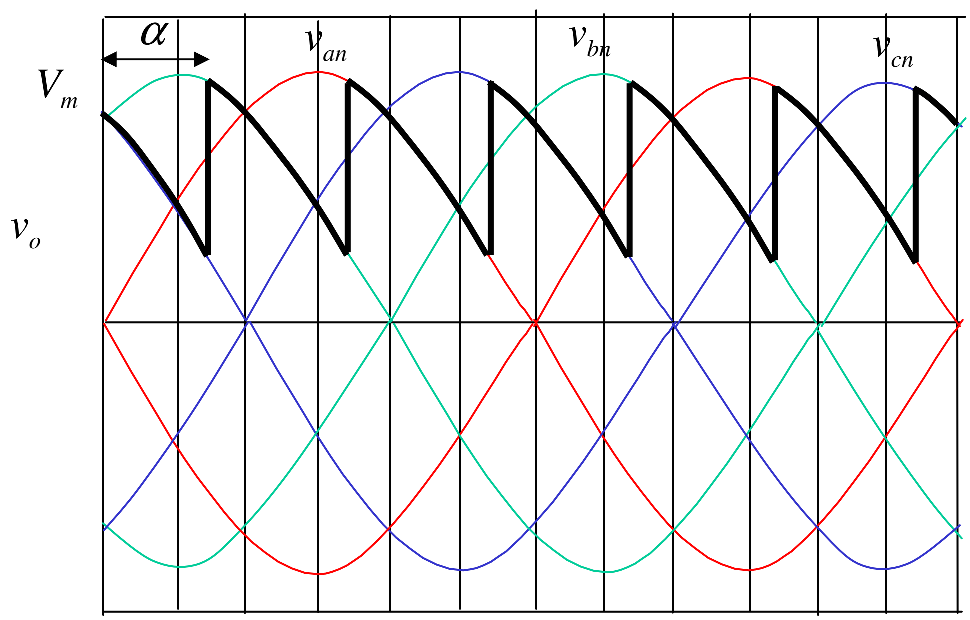
$$= \frac{3V_{m,L-L}}{\pi} [\cos(\omega t)]_{\pi/3}^{2\pi/3}$$

$$= \frac{3V_{m,L-L}}{\pi} = 0.955 V_{m,L-L}$$

Note that the output DC voltage component of a three-phase rectifier is much higher than of a single-phase.

Controlled, three-phase





Output voltage of controlled three phase rectifier

From the previous Figure, let α be the delay angle of the SCR.

Average voltage can be computed as :

$$V_o = \frac{1}{\pi/3} \int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_{m,L-L} \sin(\omega t) d\omega t$$

$$= \left(\frac{3V_{m,L-L}}{\pi} \right) \cdot \cos\alpha$$

- EXAMPLE: A three-phase controlled rectifier has an input voltage of 415V RMS at 50Hz. The load R=10 ohm. Determine the delay angle required to produce current of 50A.