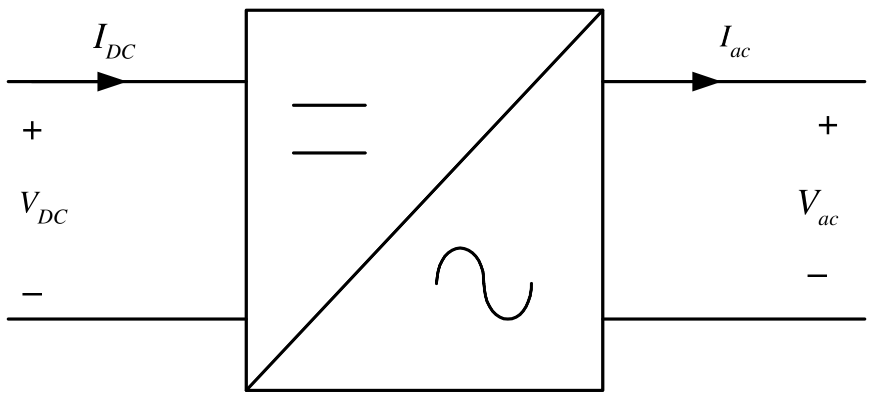
Chapter 4
DC to AC Conversion
(INVERTER)

- General concept
- Single-phase inverter
- Harmonics
- Modulation
- Three-phase inverter

DC to AC Converter (Inverter)

- DEFINITION: Converts DC to AC power by switching the *DC input voltage (or current)* in a pre-determined sequence so as to generate *AC voltage (or current) output*.

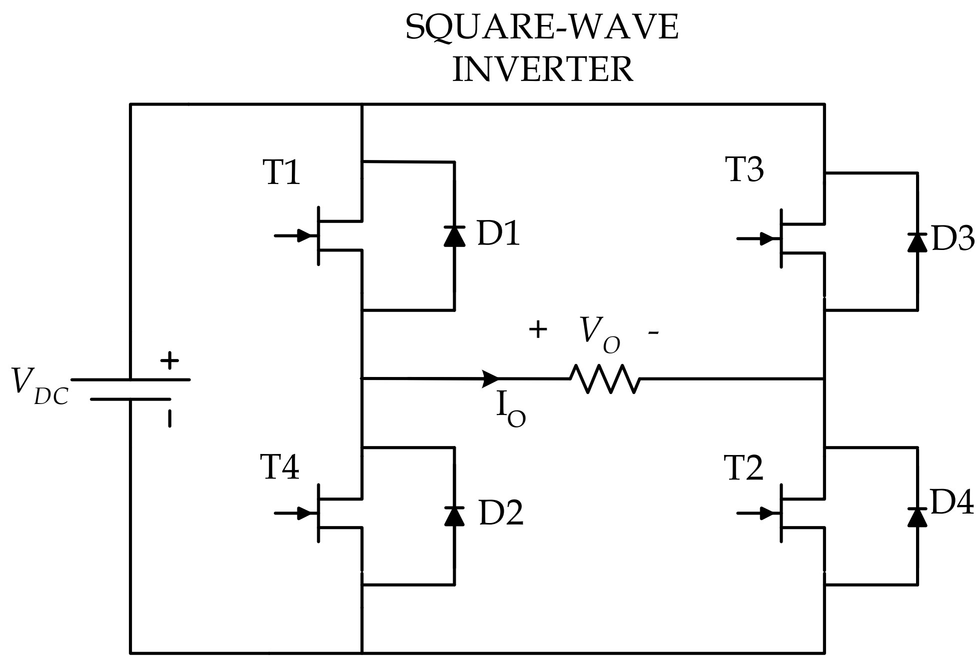
- General block diagram

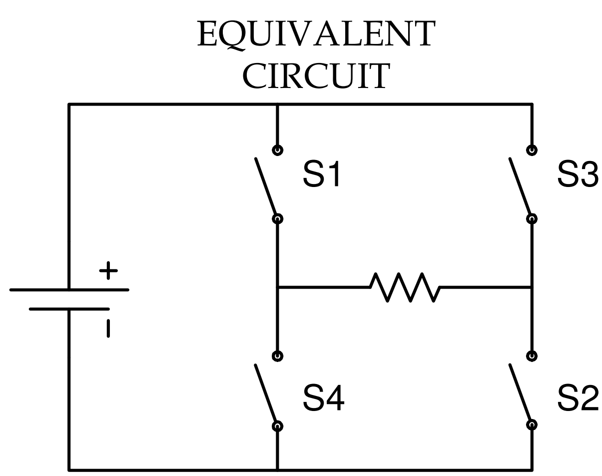


- TYPICAL APPLICATIONS:
  – Un-interruptible power supply (UPS), Industrial (induction motor) drives, Traction, HVDC

Simple square-wave inverter (1)

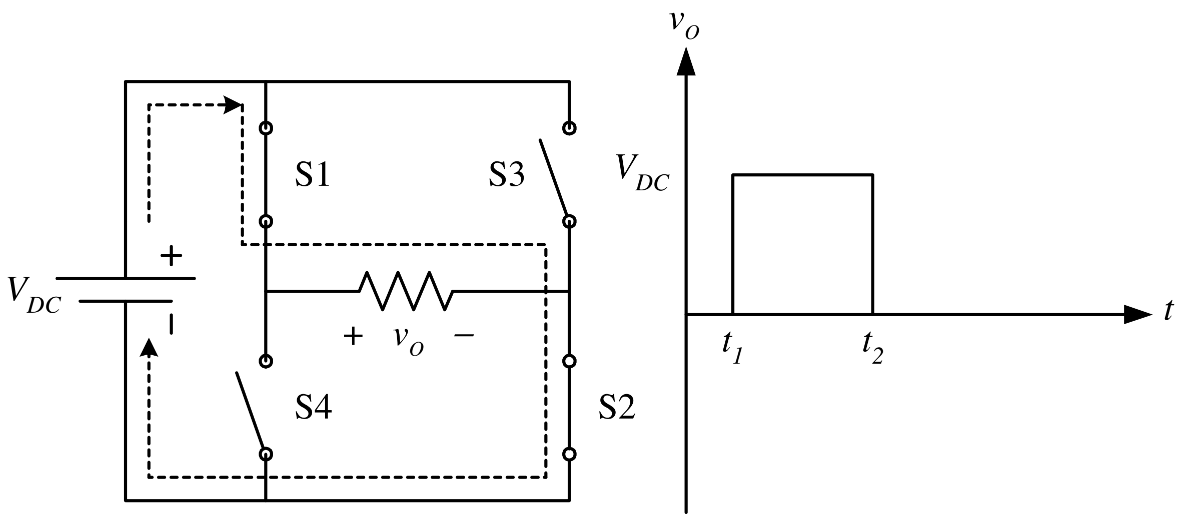
- To illustrate the concept of AC waveform generation



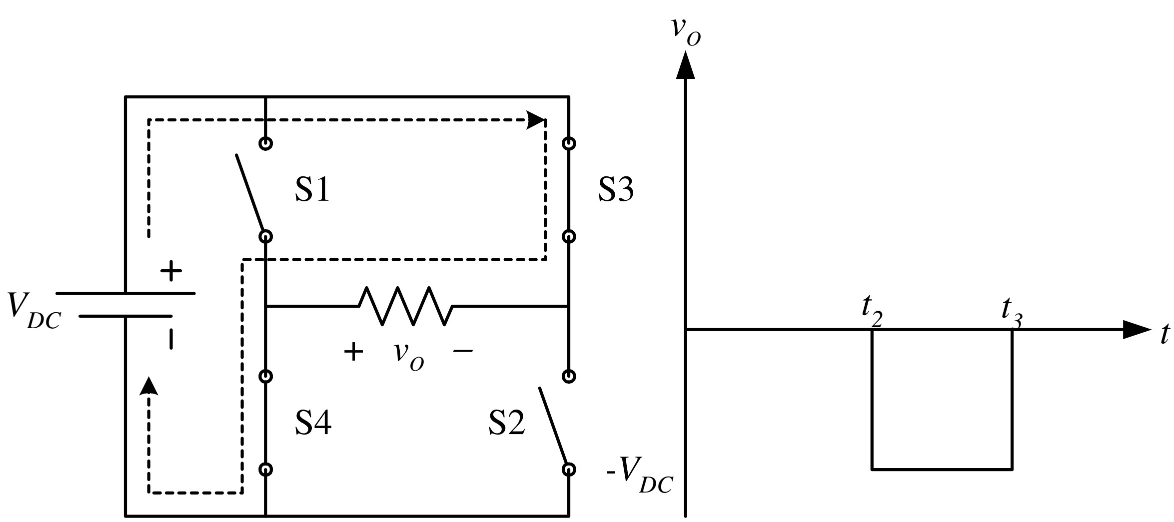


AC Waveform Generation

S1,S2 ON; S3,S4 OFF for $t_1 < t < t_2$

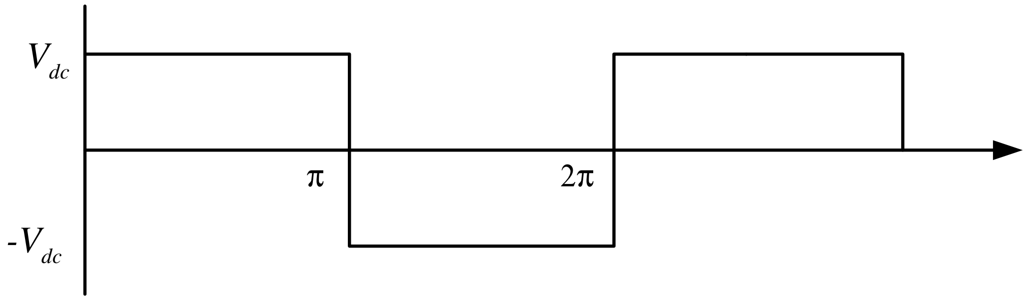


S3,S4 ON ; S1,S2 OFF for $t_2 < t < t_3$

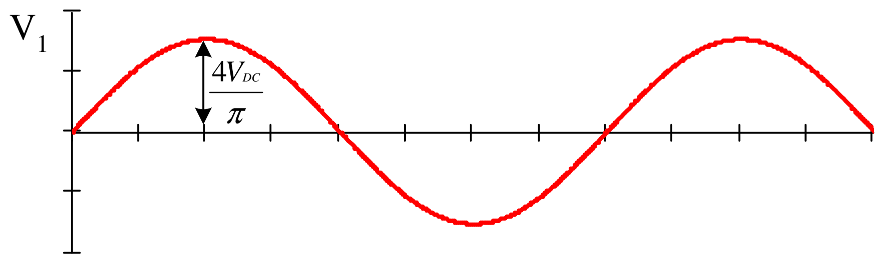


AC Waveforms

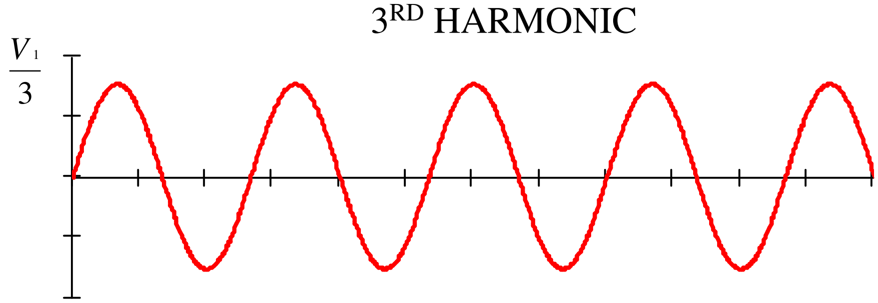
INVERTER OUTPUT VOLTAGE



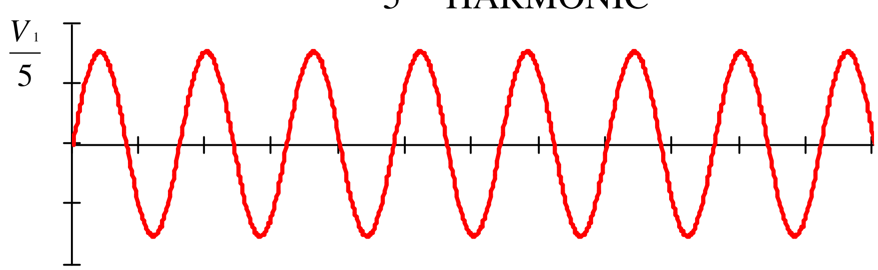
FUNDAMENTAL COMPONENT



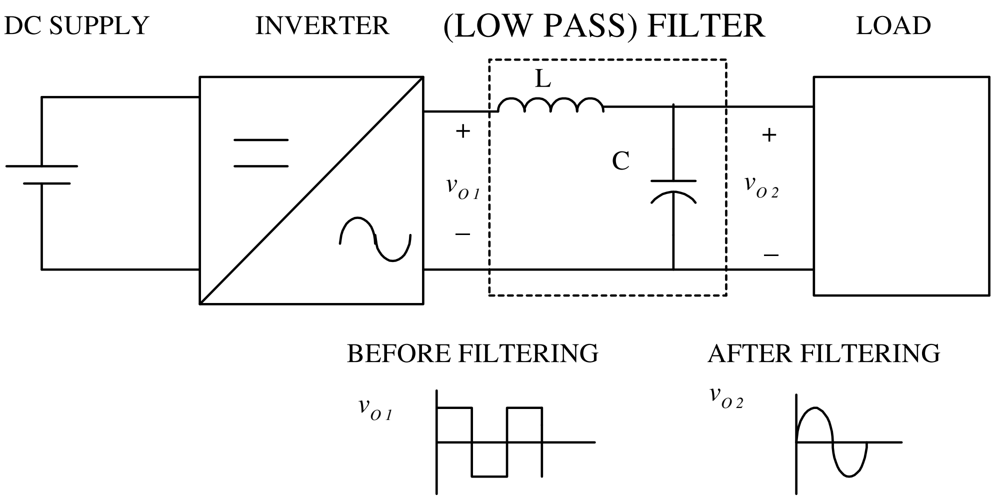
3^{RD} HARMONIC



5^{RD} HARMONIC



Harmonics Filtering

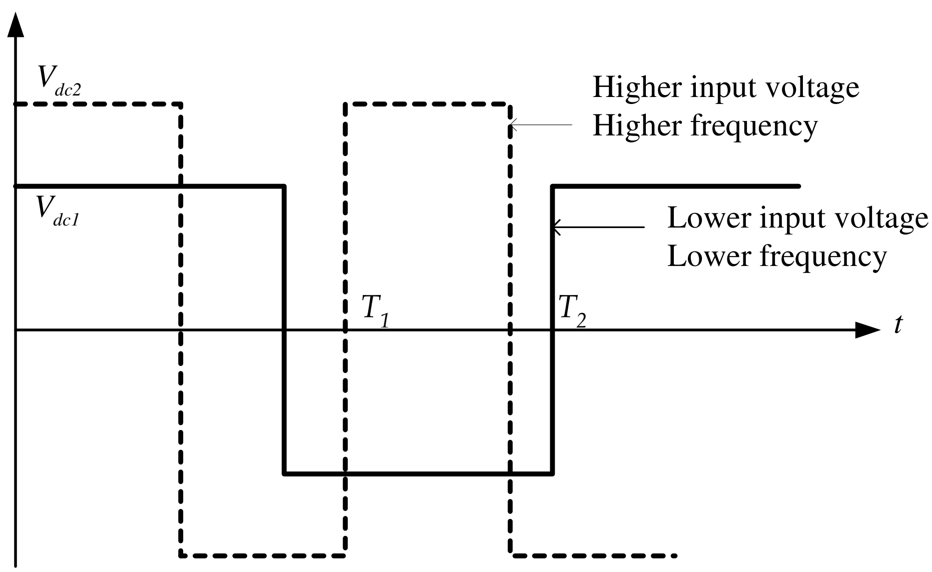


- Output of the inverter is "chopped AC voltage with zero DC component". It contain **harmonics.**

- An **LC section low-pass filter** is normally fitted at the inverter output to reduce the high frequency harmonics.

- In some applications such as UPS, "*high purity*" sine wave output is required. Good filtering is a must.
- In some applications such as AC motor drive, filtering is not required.

Variable Voltage Variable Frequency Capability



- Output voltage frequency can be varied by "period" of the square-wave pulse.
- Output voltage amplitude can be varied by varying the "magnitude" of the DC input voltage.
- Very useful: e.g. variable speed induction motor drive

Output voltage harmonics/ distortion

- Harmonics cause distortion on the output voltage.
- Lower order harmonics (3^{rd}, 5^{th} etc) are very difficult to filter, due to the filter size and high filter order. They can cause serious voltage distortion.
- Why need to consider harmonics?
  - Sinusoidal waveform quality must match TNB supply.
  - "Power Quality" issue.
  - Harmonics may cause degradation of equipment. Equipment need to be "de-rated".
- Total Harmonic Distortion (THD) is a measure to determine the "quality" of a given waveform.

Total Harmonics Distortion (THD)

Voltage THD: If V_n is the nth harmonic voltage,

$$THDv = \frac{\sqrt{\sum_{n=2}^{\infty}\left(V_{n,RMS}\right)^2}}{V_{1,RMS}}$$

$$= \frac{\sqrt{V_{2,RMS}{}^2 + V_{3,RMS}{}^2 + ... + V_{2,RMS}{}^2}}{V_{1,RMS}}$$

If the rms voltage for the vaveform is known,

$$THDv = \frac{\sqrt{\sum_{n=2}^{\infty}\left(V_{RMS}\right)^2 - \left(V_{1,RMS}\right)^2}}{V_{1,RMS}}$$

Current THD:

$$THDi = \frac{\sqrt{\sum_{n=2}^{\infty}\left(I_{n,RMS}\right)^2}}{I_{1,RMS}}$$

$$I_n = \frac{V_n}{Z_n}$$

Z_n is the impedance at harmonic frequency.

Fourier Series

- Study of harmonics requires understanding of wave shapes. Fourier Series is a tool to analyse wave shapes.

Fourier Series

$$a_o = \frac{1}{\pi}\int_0^{2\pi} f(v)d\theta \quad (\text{"DC" term})$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\cos(n\theta)d\theta \quad (\text{"cos" term})$$

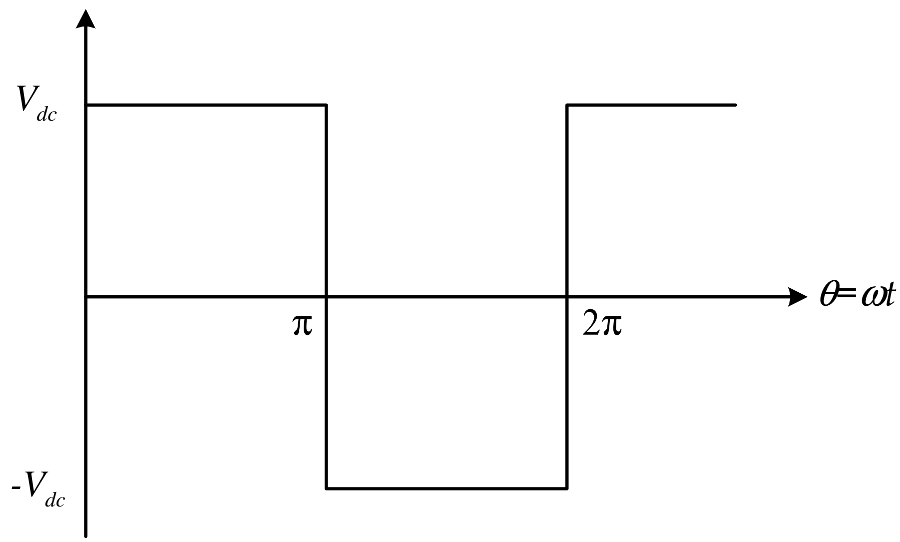
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\sin(n\theta)d\theta \quad (\text{"sin" term})$$

Inverse Fourier

$$f(v) = \frac{1}{2}a_o + \sum_{n=1}^{\infty}\left(a_n \cos n\theta + b_n \sin n\theta\right)$$

where $\theta = \omega t$

Harmonics of square-wave (1)



$$a_o = \frac{1}{\pi}\left[\int_0^{\pi} V_{dc}\, d\theta + \int_{\pi}^{2\pi} -V_{dc}\, d\theta\right] = 0$$

$$a_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} \cos(n\theta)\, d\theta - \int_{\pi}^{2\pi} \cos(n\theta)\, d\theta\right] = 0$$

$$b_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} \sin(n\theta)\, d\theta - \int_{\pi}^{2\pi} \sin(n\theta)\, d\theta\right]$$

Harmonics of square wave (2)

Solving,

$$b_n = \frac{V_{dc}}{n\pi}\left[-\cos(n\theta)\Big|_0^{\pi} + \cos(n\theta)\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{V_{dc}}{n\pi}\left[(\cos 0 - \cos n\pi) + (\cos 2n\pi - \cos n\pi)\right]$$

$$= \frac{V_{dc}}{n\pi}\left[(1 - \cos n\pi) + (1 - \cos n\pi)\right]$$

$$= \frac{2V_{dc}}{n\pi}\left[(1 - \cos n\pi)\right]$$

When n is even, $\cos n\pi = 1$

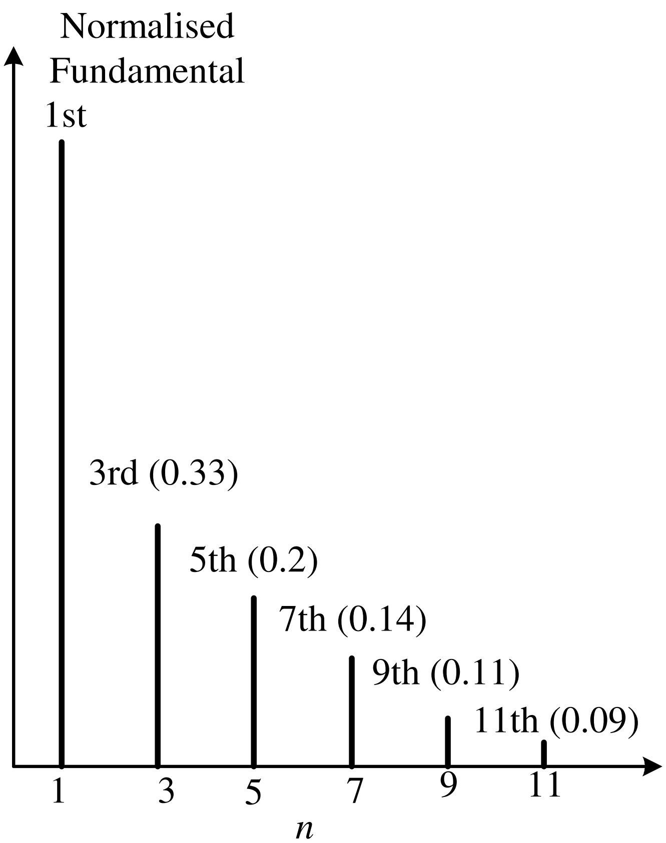
$b_{n=}0$

(i.e. even harmonics do not exist)

When n is odd, $\cos n\pi = -1$

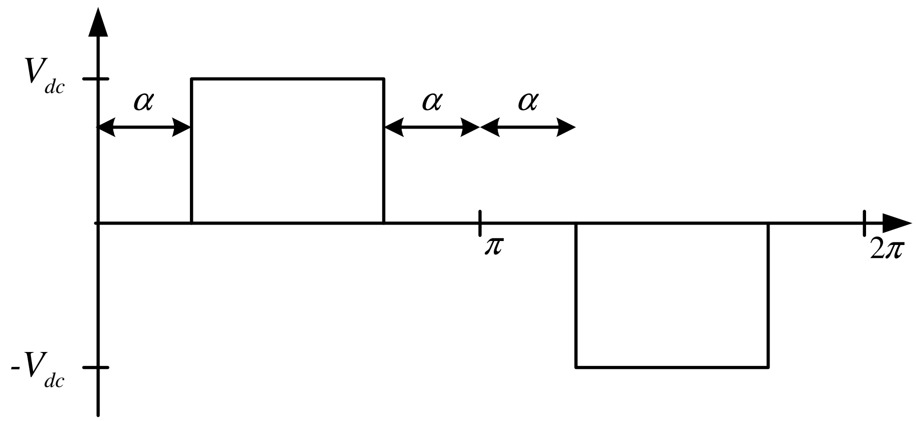
$$b_{n=} \frac{4V_{dc}}{n\pi}$$

Spectra of square wave



- Spectra (harmonics) characteristics:
  - Harmonic decreases with a factor of (1/n).
  - Even harmonics are absent
  - Nearest harmonics is the 3rd. If fundamental is 50Hz, then nearest harmonic is 150Hz.
  - Due to the small separation between the fundamental an harmonics, output low-pass filter design can be very difficult.

Quasi-square wave (QSW)



Note that $a_n = 0.$ (due to half - wave symmetry)

$$b_n = 2\left[\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}\sin(n\theta)d\theta\right] = \frac{2V_{dc}}{n\pi}\left[-\cos n\theta\Big|_{\alpha}^{\pi-\alpha}\right]$$

$$= \frac{2V_{dc}}{n\pi}\left[\cos(n\alpha) - \cos n(\pi-\alpha)\right]$$

Expanding :

$$\cos n(\pi-\alpha) = \cos(n\pi - n\alpha)$$

$$= \cos n\pi \cos n\alpha + \sin n\pi \sin n\alpha = \cos n\pi \cos n\alpha$$

$$\Rightarrow b_n = \frac{2V_{dc}}{n\pi}\left[\cos(n\alpha) - \cos n\pi \cos n\alpha\right]$$

$$= \frac{2V_{dc}}{n\pi}\cos(n\alpha)\left[1 - \cos n\pi\right]$$

Harmonics control

If n is even, $\Rightarrow b_n = 0,$

If n is odd, $\Rightarrow b_n = \frac{4V_{dc}}{n\pi}\cos(n\alpha)$

In particular, amplitude of the fundamental is :

$$b_1 = \frac{4V_{dc}}{\pi}\cos(\alpha)$$

Note :

The fundamental , b_1, is controlled by varying α

Harmonics can also be controlled by adjusting α,

Harmonics Elimination :

For example if $\alpha = 30^o$, then $b_3 = 0$, or the third harmonic is eliminated from the waveform. In general, harmonic n will be eliminated if :

$$\alpha = \frac{90^{\text{o}}}{n}$$

Example

A full - bridge single phase inverter is fed by square wave signals. The DC link voltage is 100V. The load is R = 10R and L = 10mH in series. Calculate:

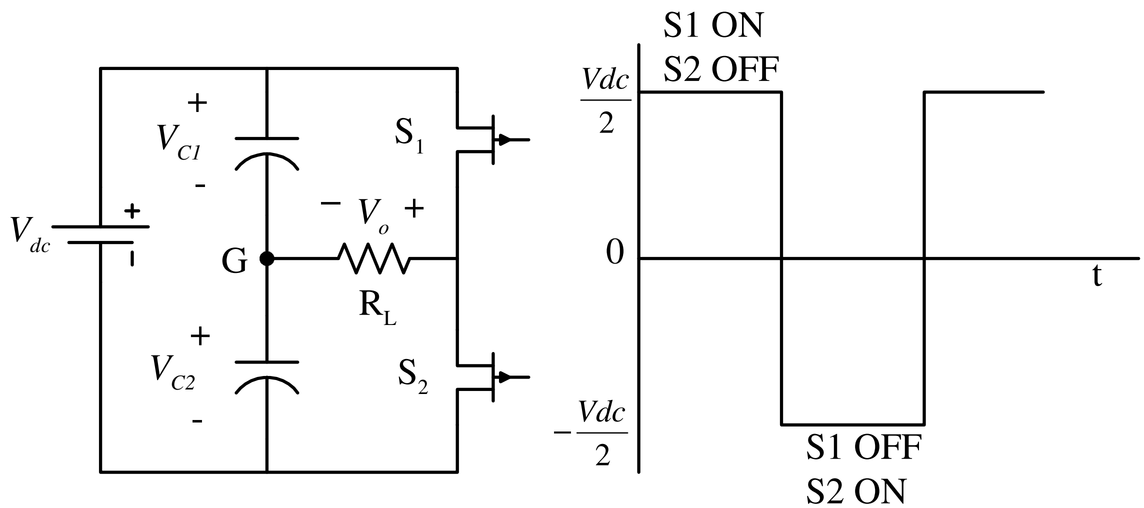
a) the THDv using the "exact" formula.

b) the THDv by using the first three non - zero harmonics

c) the THDi by using the first three non - zero harmonics

Repeat (b) and (c) for quasi - square wave case with $\alpha = 30$ degrees

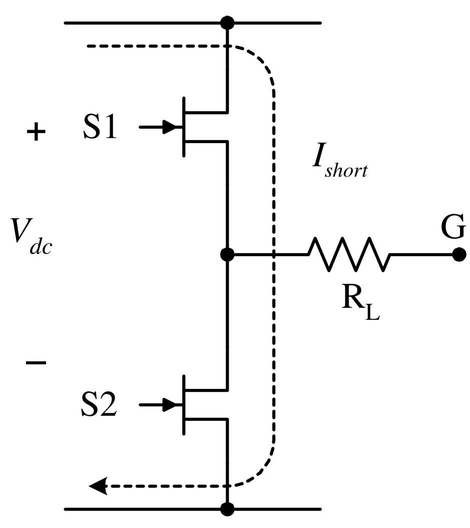
Half-bridge inverter (1)



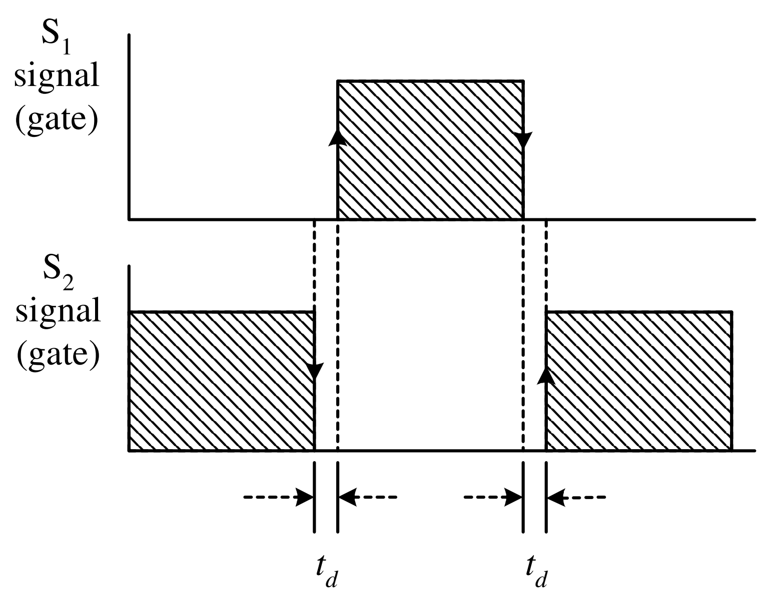
- Also known as the "inverter leg".
- Basic building block for full bridge, three phase and higher order inverters.
- G is the "centre point".
- Both capacitors have the same value. Thus the DC link is equally "spilt" into two.
- The top and bottom switch has to be "complementary", i.e. If the top switch is closed (on), the bottom must be off, and vice-versa.

Shoot through fault and "Dead-time"

- In practical, a dead time as shown below is required to avoid "shoot-through" faults, i.e. short circuit across the DC rail.
- Dead time creates "low frequency envelope". Low frequency harmonics emerged.
- This is the main source of distortion for high-quality sine wave inverter.



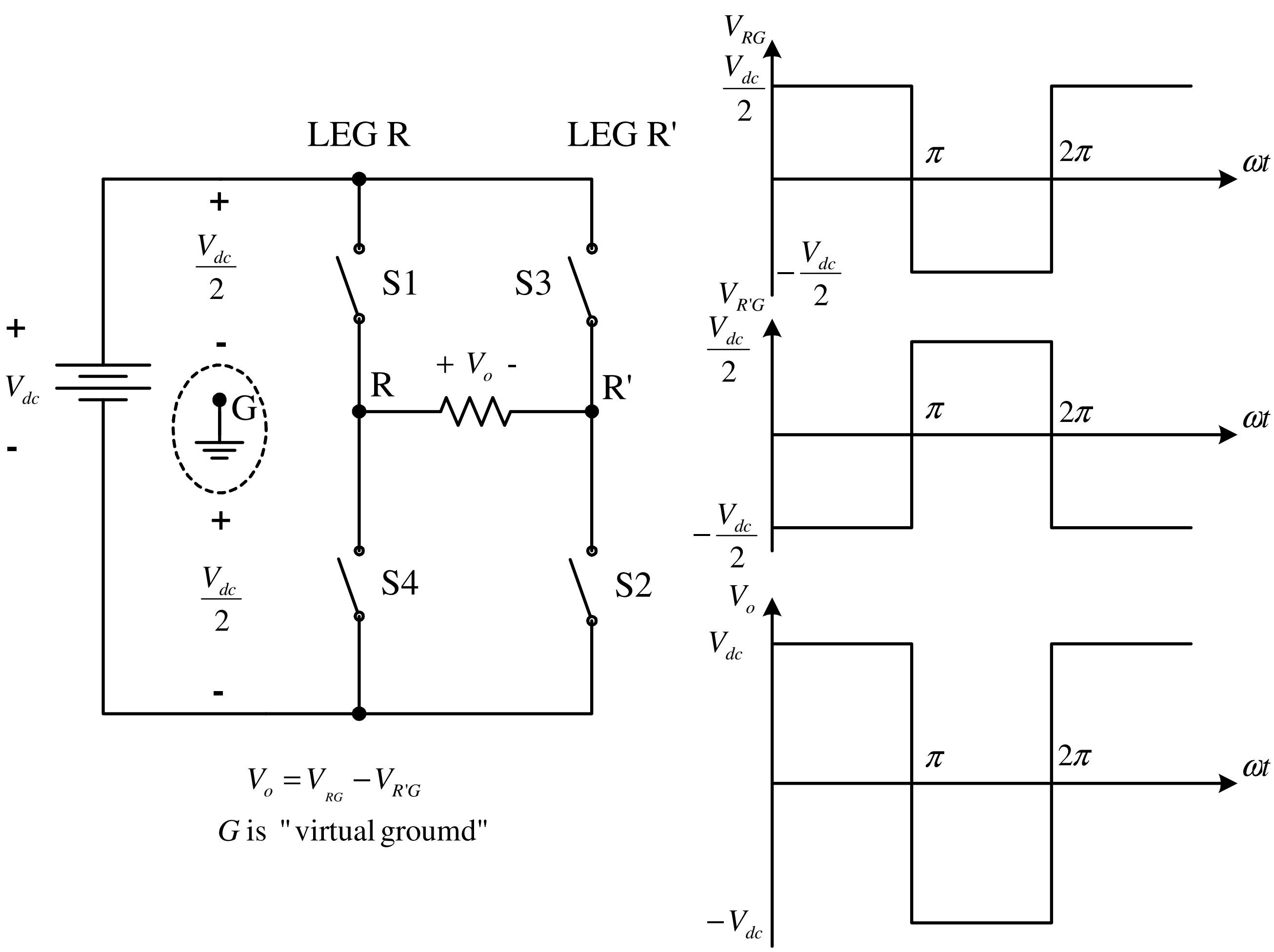
"Shoot through fault" . I_{short} is very large



"Dead time' $= t_d$

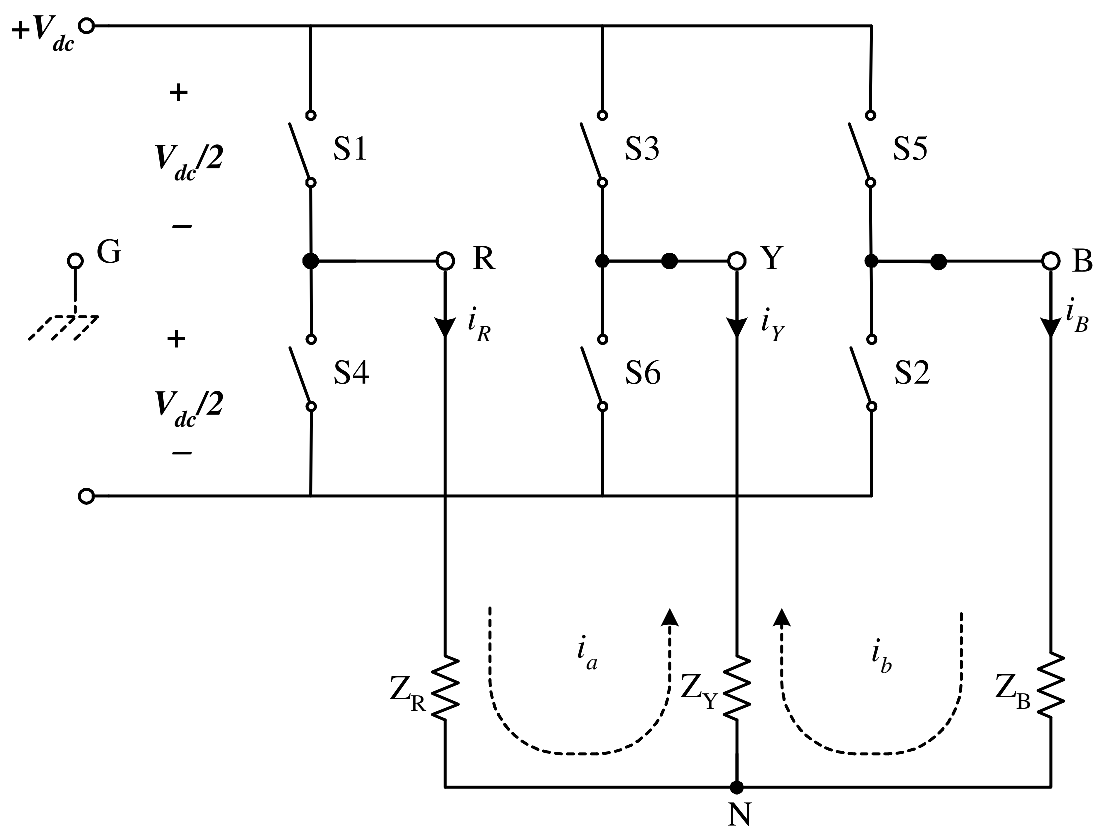
Single-phase, full-bridge (1)

- Full bridge (single phase) is built from two half-bridge leg.
- The switching in the second leg is "delayed by 180 degrees" from the first leg.

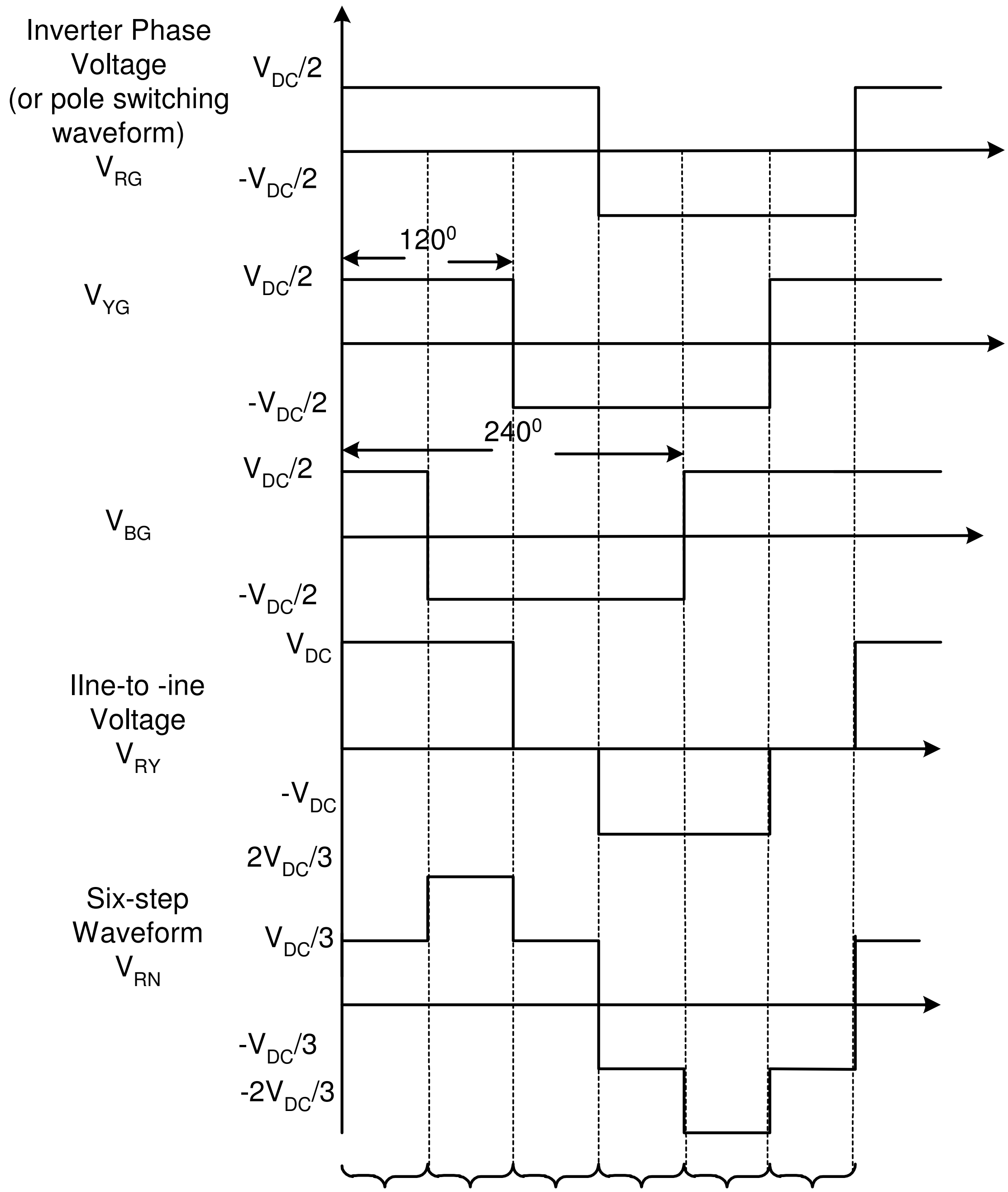


Three-phase inverter

- Each leg (Red, Yellow, Blue) is delayed by 120 degrees.
- A three-phase inverter with star connected load is shown below



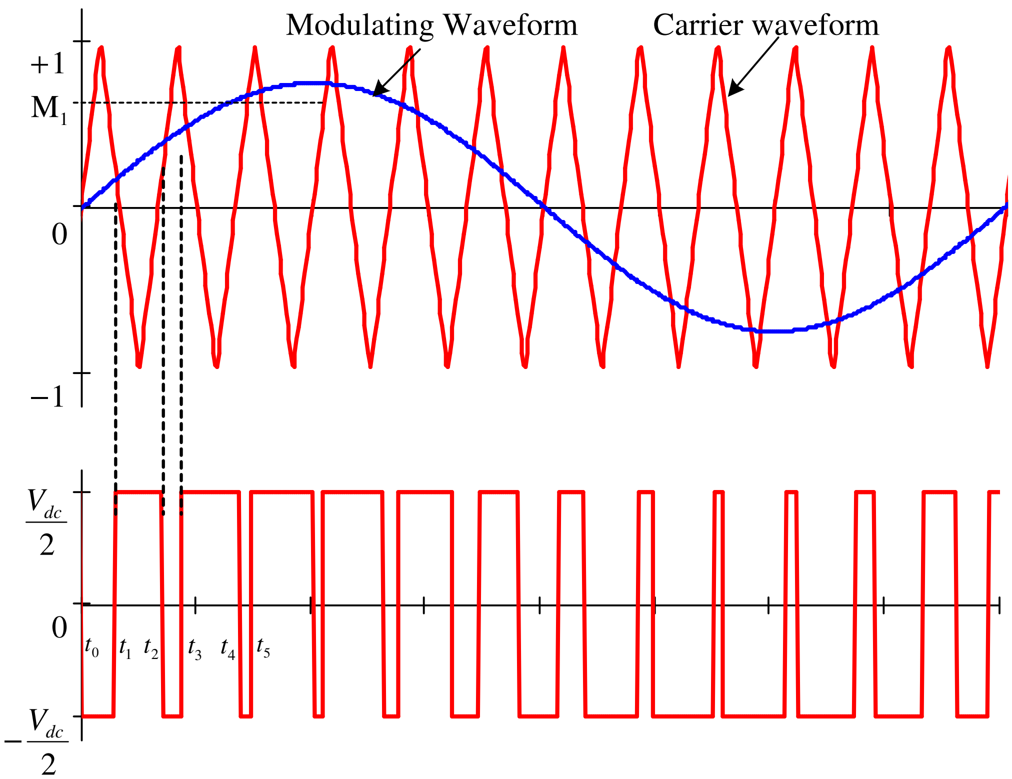
Three phase inverter waveforms



Interval	1	2	3	4	5	6
Positive device(s) on	3	3,5	5	1,5	1	1,3
Negative device(s) on	2,4	4	4,6	6	2,6	2

Quasi-square wave operation voltage waveforms

Pulse Width Modulation (PWM)

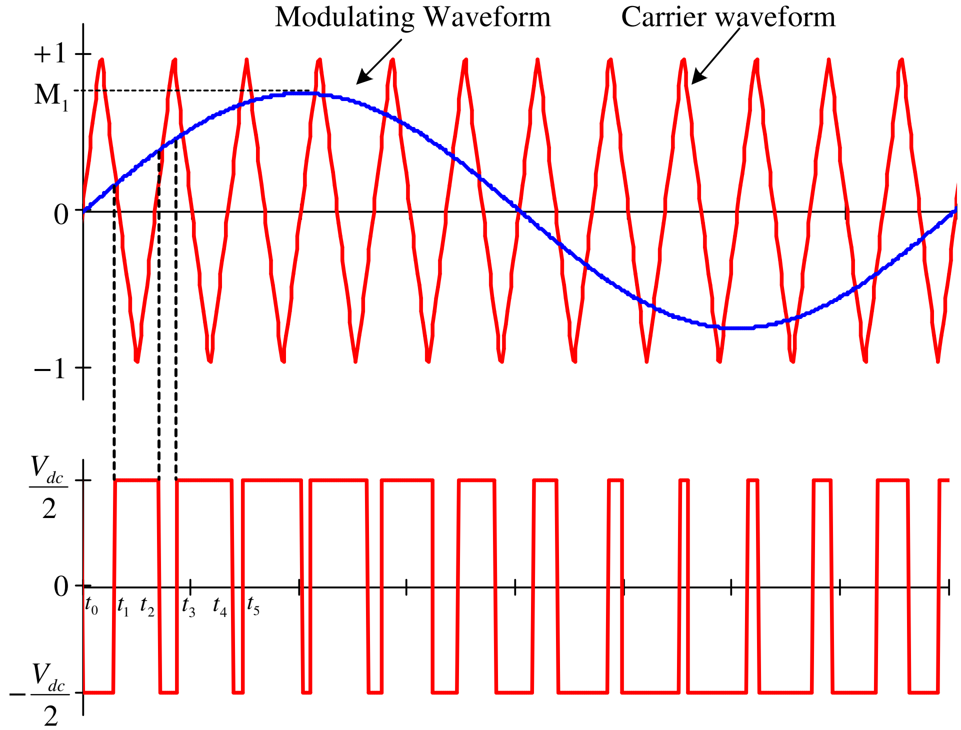


- Triangulation method (Natural sampling)
  - Amplitudes of the triangular wave (carrier) and sine wave (modulating) are compared to obtain PWM waveform. Simple analogue comparator can be used.
  - Basically an analogue method. Its digital version, known as REGULAR sampling is widely used in industry.

PWM types

- Natural (sinusoidal) sampling (as shown on previous slide)
  - Problems with analogue circuitry, e.g. Drift, sensitivity etc.
- Regular sampling
  - simplified version of natural sampling that results in simple digital implementation
- Optimised PWM
  - PWM waveform are constructed based on certain performance criteria, e.g. THD.
- Harmonic elimination/minimisation PWM
  - PWM waveforms are constructed to eliminate some undesirable harmonics from the output waveform spectra.
  - Highly mathematical in nature
- Space-vector modulation (SVM)
  - A simple technique based on volt-second that is normally used with three-phase inverter motor-drive

Modulation Index, Ratio



$$\text{Modulation Index (Modulation Depth)} = M_I :$$

$$M_I = \frac{\text{Amplitude of the modulating waveform}}{\text{Amplitude of the carrier waveform}}$$

$$\text{Modulation Ratio (Frequency Ratio)} = M_R \; (= p)$$

$$M_R = p = \frac{\text{Frequency of the carrier waveform}}{\text{Frequency of the modulating waveform}}$$

Modulation Index, Ratio

Modulation Index deterrmines the output voltage fundamental component

If $0 < M_I < 1$,

$V_1 = M_I V_{in}$

where V_1, V_{in} are fundamental of the output voltage and input (DC) voltage, respectively.

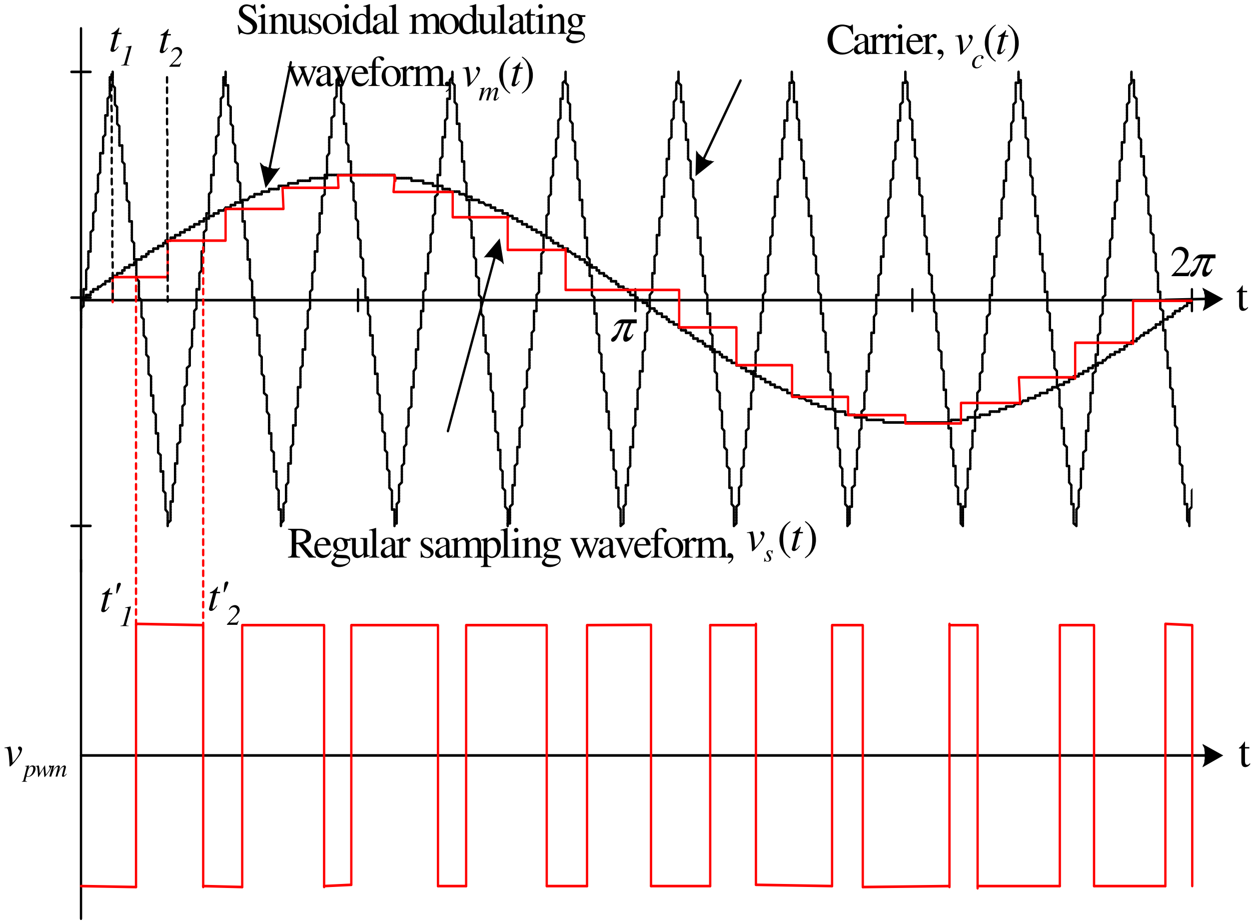
Modulation ratio determines the incident (location) of harmonics in the spectra.

The harmonics are normally located at :

$f = kM_R(f_m)$

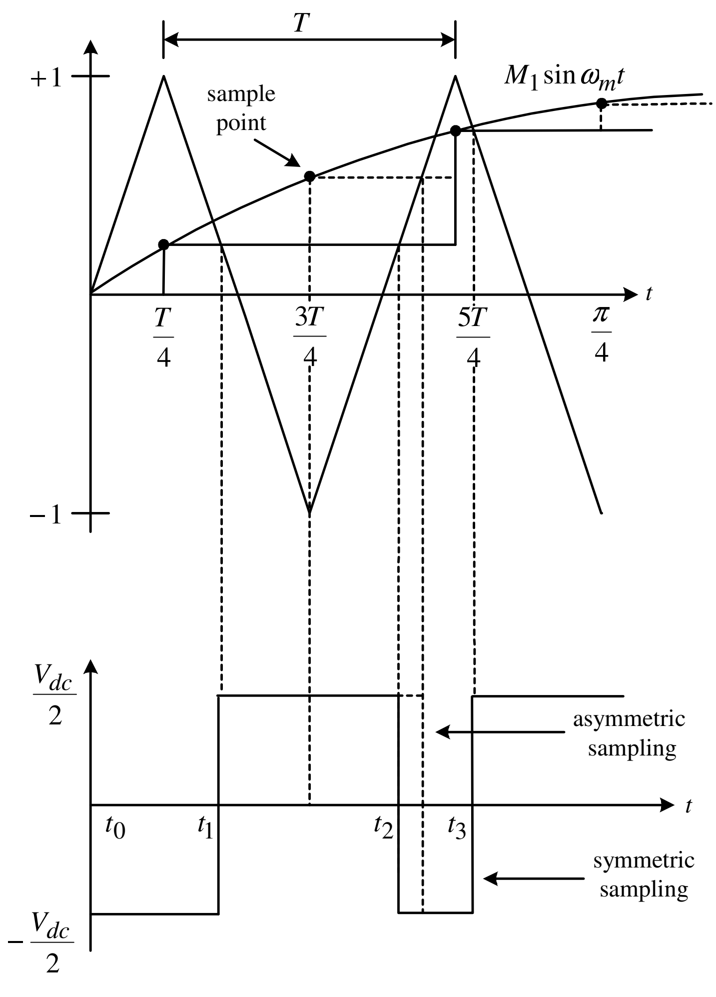
where f_m is the frequency of the modulating signal and k is an integer (1,2,3...)

Regular sampling



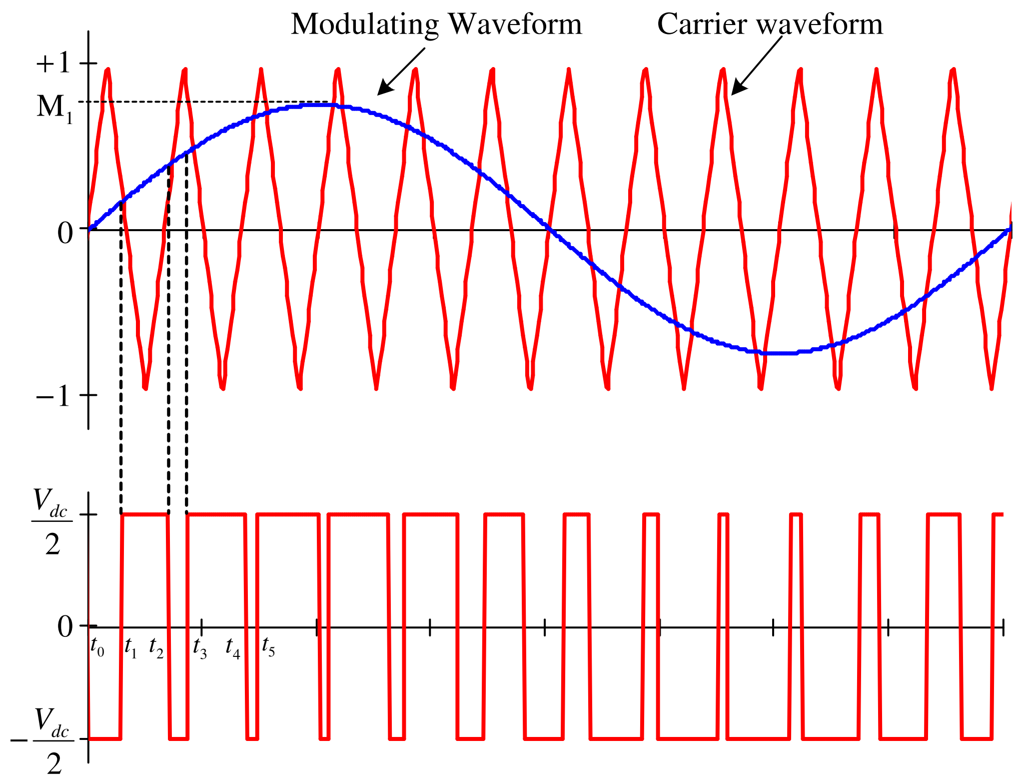
Regular sampling PWM

Asymmetric and symmetric regular sampling

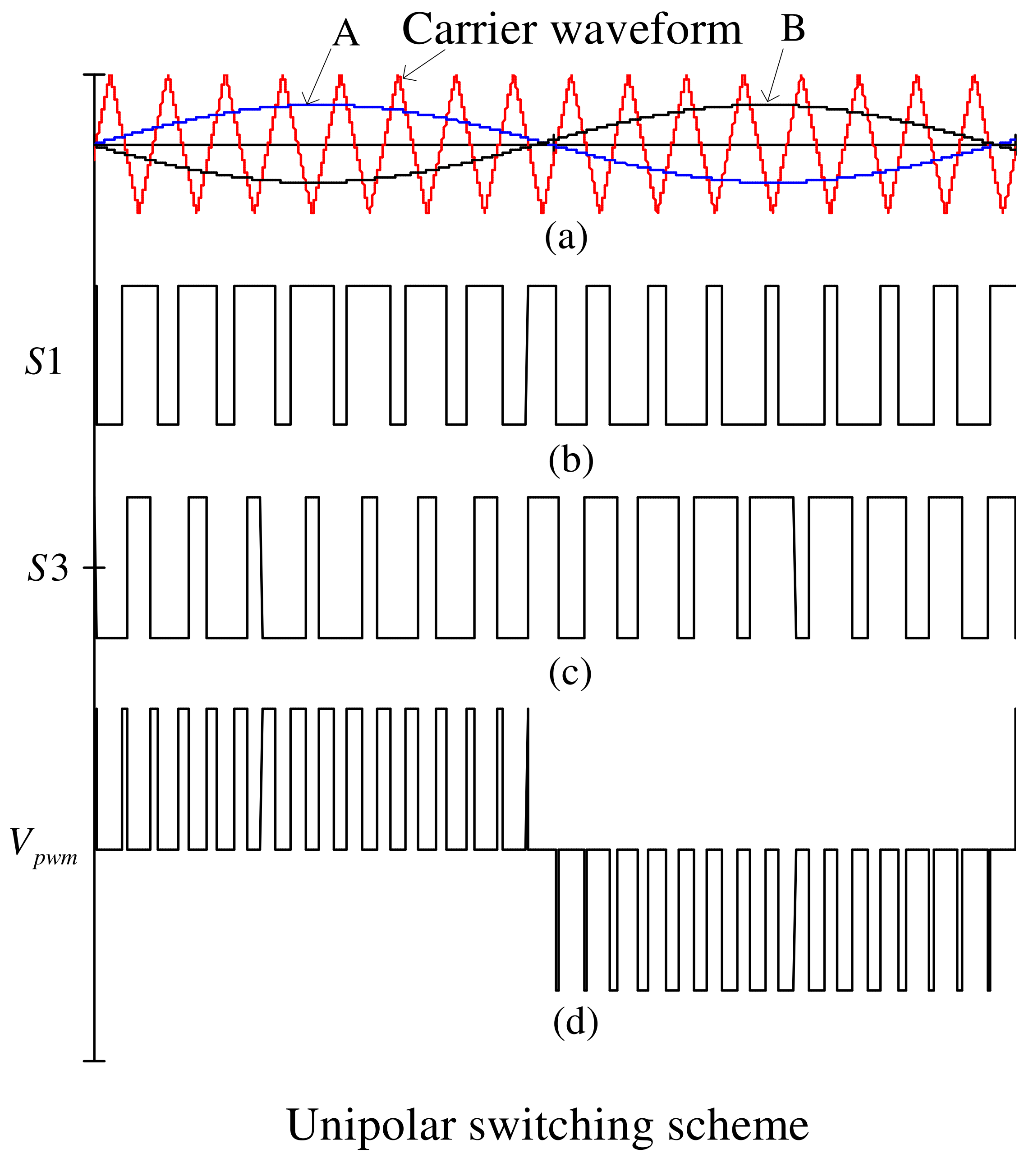


Generating of PWM waveform regular sampling

Bipolar Switching

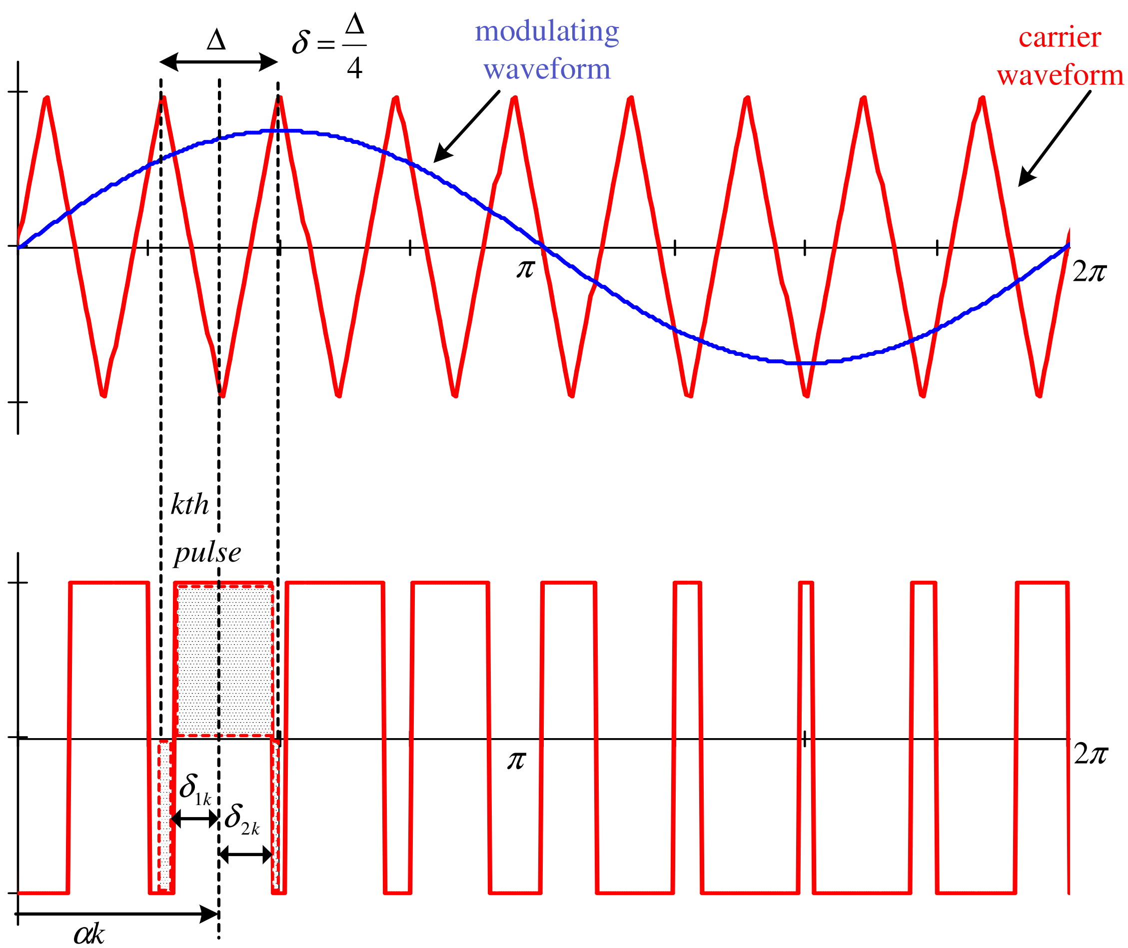


Unipolar switching

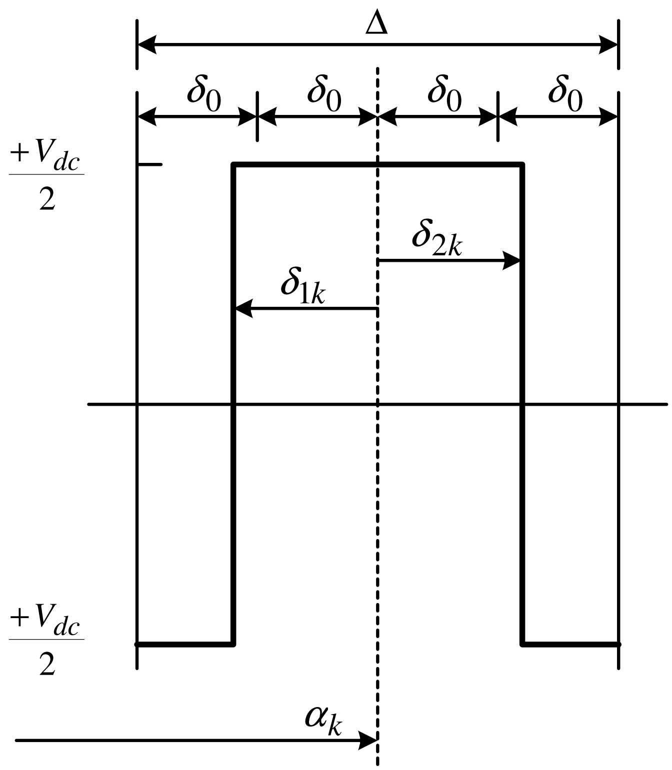


Unipolar switching scheme

Bipolar PWM switching: Pulse-width characterization

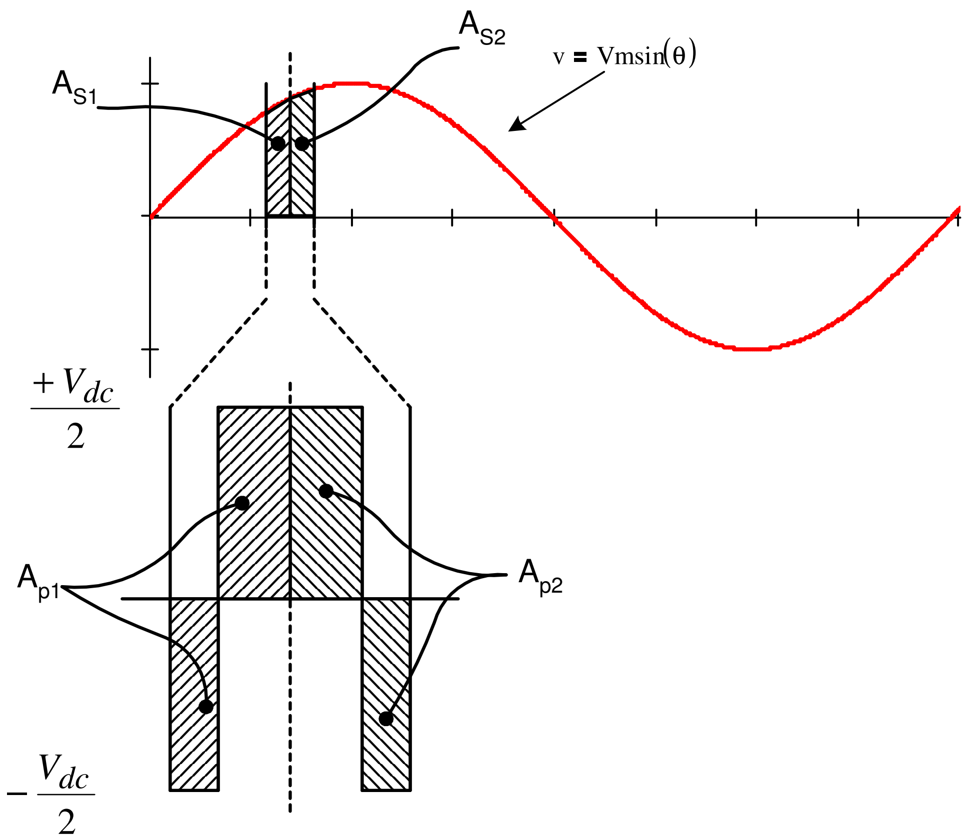


The k^{th} Pulse



The kth PWM pulse

Determination of switching angles for kth PWM pulse (1)



Equating the volt - second,

$$A_{s1} = A_{p1}$$

$$A_{s2} = A_{p2}$$

PWM Switching angles (2)

The Volt - second during the first half cycle of the PWM pulse is given as:

$$A_{p1} = \left(\frac{V_{dc}}{2}\right)(\delta_{1k}) - \left(\frac{V_{dc}}{2}\right)(2\delta_o - \delta_{1k})$$

$$= (V_{dc})(\delta_{1k} - \delta_o)$$

Similarly for the second half,

$$A_{p2} = \left(\frac{V_{dc}}{2}\right)(\delta_{2k}) - \left(\frac{V_{dc}}{2}\right)(2\delta_o - \delta_{2k})$$

$$= (V_{dc})(\delta_{2k} - \delta_o)$$

The volt - second supplied by the sinusoid,

$$A_{s1} = \int_{\alpha_k - 2\delta_o}^{\alpha_k} V_m \sin\theta d\theta = V_m[\cos(\alpha_k - 2\delta_o) - \cos\alpha_k]$$

$$= 2V_m \sin\delta_o \sin(\alpha_k - \delta_o)$$

Similarly,

$$A_{s2} = 2\delta_o V_m \sin(\alpha_k + \delta_o)$$

Switching angles (3)

For small angle δ_o

$\sin\delta_o \rightarrow \delta_o$,

$\Rightarrow A_{s1} = 2\delta_o V_m \sin(\alpha_k - \delta_o)$

$\Rightarrow A_{s2} = 2\delta_o V_m \sin(\alpha_k - \delta_o)$

To derive the modulation strategy,

$A_{p1} = A_{s1};$

$A_{p2} = A_{s2}$

Hence, for the the first half cycle of PWM pulse,

$$(V_{dc})(\delta_{1k} - \delta_o) = 2\delta_o V_m \sin(\alpha_k - \delta_o)$$

$$\Rightarrow (\delta_{1k} - \delta_o) = \frac{2V_m}{V_{dc}}(\delta_o \sin(\alpha_k - \delta_o)$$

By definition, the Modulation Ratio,

$M_I = \dfrac{V_m}{(V_{dc}/2)}$ is known as modulation

Thus, the pulse width for the first half cycle of the PWM waveform is given by :

$$\Rightarrow \delta_{1k} = \delta_o[1 + M_I \sin(\alpha_k - \delta_o)]$$

PWM switching angles (4)

Thus the leading edge switching angle of the kth pulse is :

$$\alpha_k - \delta_{1k}$$

Using similar method, pulse width of the second half cycle of PWM waveform :

$$\delta_{2k} = \delta_o [1 + M_I \sin(\alpha_k + \delta_o)]$$

And the trailing edge angle :

$$\alpha_k + \delta_{2k}$$

The above equation is valid for Asymmetric Modulation, i.e $\delta_{1\text{k}}$ and $\delta_{2\text{k}}$ are different.

For Symmetric Modulation,

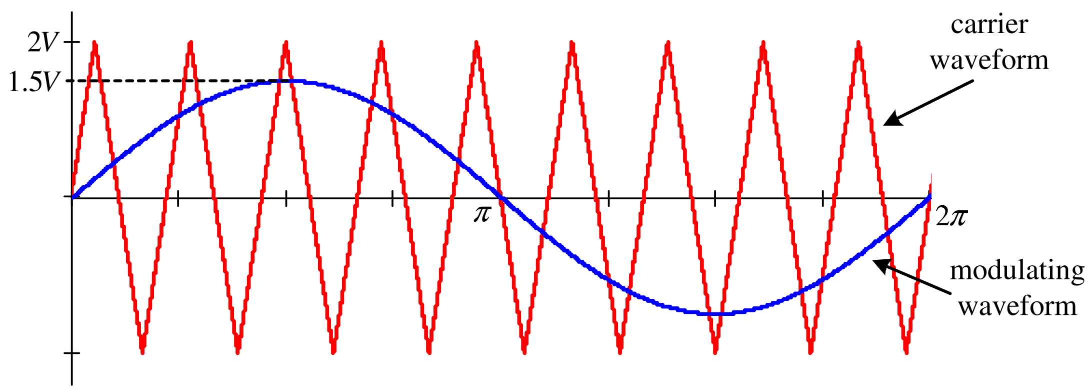
$$\delta_{1\text{k}} = \delta_{2\text{k}} = \delta_{\text{k}}$$

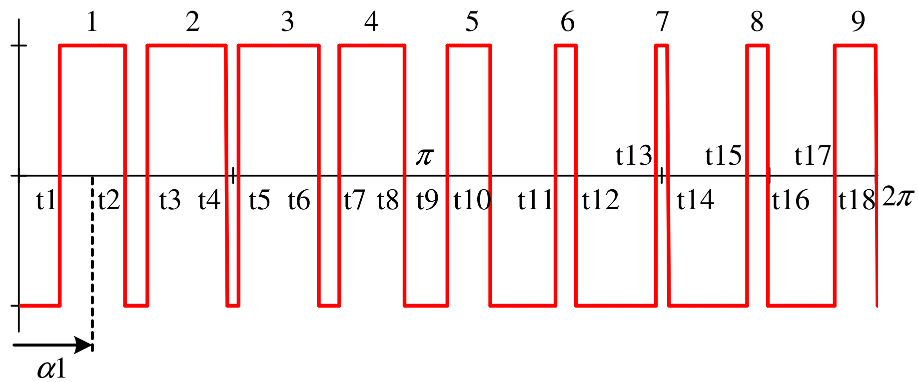
Hence

$$\Rightarrow \delta_k = \delta_o [1 + M_I \sin \alpha_k]$$

Example

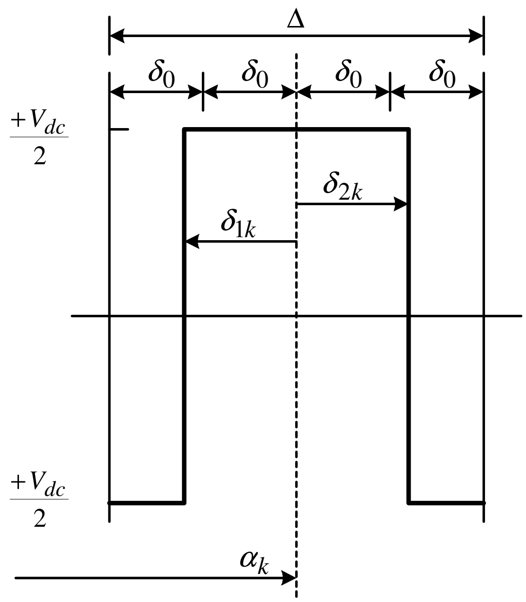
- For the PWM shown below, calculate the switching angles pulses no. 2.





Harmonics of bipolar PWM

Assuming the PWM waveform is half wave symmetry, harmonic content of each (kth) PWM pulse can be computed as :



$$b_{nk} = 2\left(\frac{1}{\pi} \int_0^T f(v) \sin n\theta d\theta \right)$$

$$= \frac{2}{\pi} \left\{ \int_{\alpha_k - 2\delta_o}^{\alpha_k - \delta_{1k}} \left(-\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

$$+ \frac{2}{\pi} \left\{ \int_{\alpha_k - \delta_{1k}}^{\alpha_k + \delta_{2k}} \left(\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

$$+ \frac{2}{\pi} \left\{ \int_{\alpha_k + \delta_{2k}}^{\alpha_k + 2\delta_o} \left(-\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

Harmonics of Bipolar PWM

Which can be reduced to :

$$b_{nk} = -\frac{V_{dc}}{n\pi}\{\cos n(\alpha_k - 2\delta_o) - \cos n(\alpha_k - \delta_{1k})$$
$$+ \cos n(\alpha_k + \delta_{2k}) - \cos n(\alpha_k - \delta_{1k})$$
$$+ \cos n(\alpha_k + \delta_{2k}) - \cos n(\alpha_k + 2\delta_o)\}$$

Yeilding,

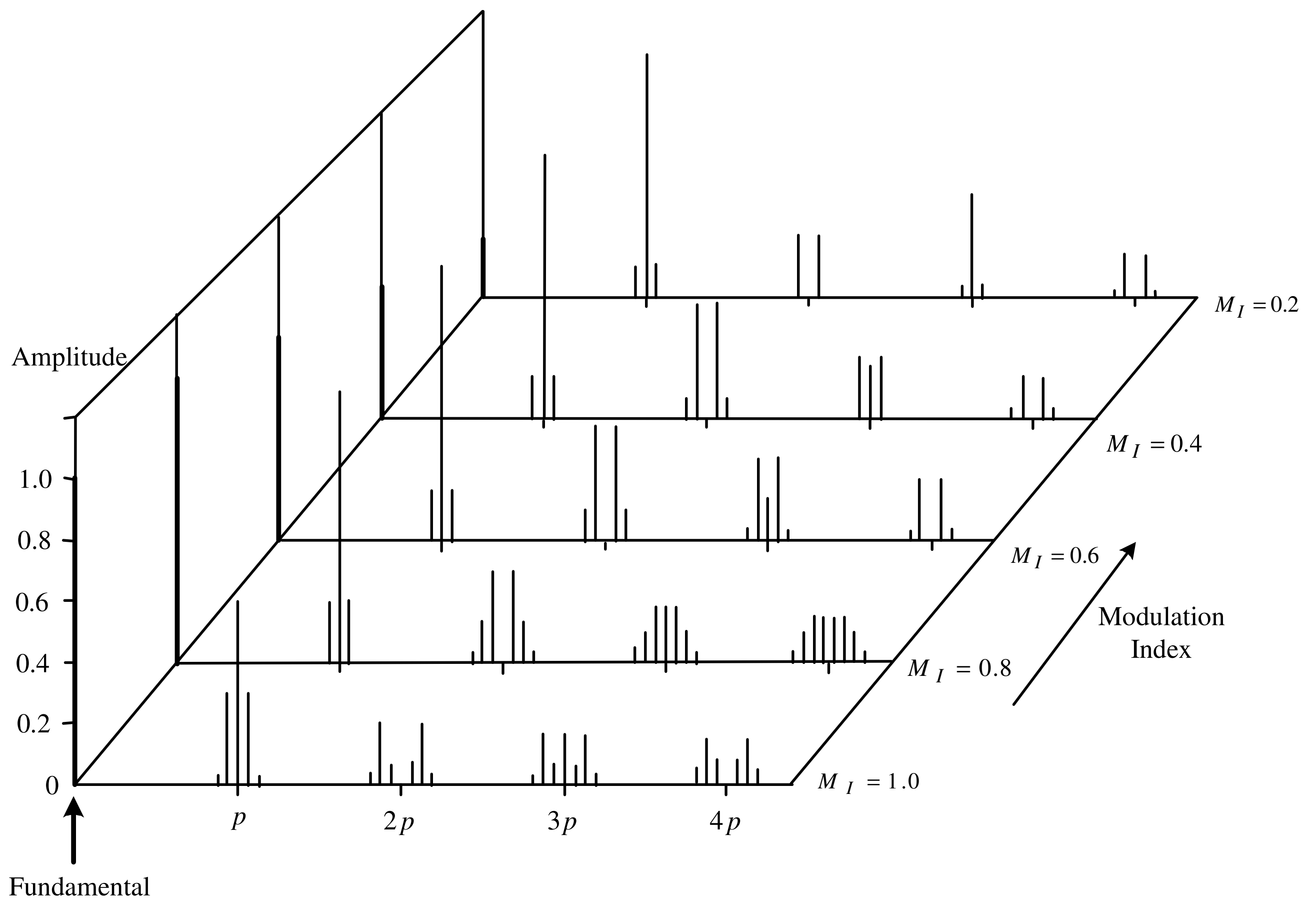
$$b_{nk} = \frac{2V_{dc}}{n\pi}[\cos n(\alpha_k - \delta_{1k}) - \cos n(\alpha_k - 2_{1k})$$
$$+ 2\cos n\alpha_k \cos n2\delta_o]$$

This equation cannot be simplified productively.The Fourier coefficent for the PWM waveform isthe sum of b_{nk} for the p pulses over one period, i.e.:

$$b_n = \sum_{k=1}^{p} b_{nk}$$

Next slide shows the computation of this equation.

PWM Spectra



NORMALISED HARMONIC AMPLITUDES FOR
SINUSOIDAL PULSE-WITDH MODULATION

PWM spectra observations

- The harmonics appear in “clusters” at multiple of the carrier frequencies .
- Main harmonics located at :

  $f = kp\,(f_m);\qquad k=1,2,3....$

  where f_m is the frequency of the modulation (sine) waveform.

- There also exist “side-bands” around the main harmonic frequencies.

- Amplitude of the fundamental is proportional to the modulation index.

  The relation ship is given as:

  $V_1 = M_I V_{in}$

- The amplitude of the harmonic changes with M_I. Its incidence (location on spectra) is not.

- When p>10, or so, the harmonics can be normalised. For lower values of p, the side-bands clusters overlap-normalised results no longer apply.

Tabulated Bipolar PWM Harmonics

n \ M_I	0.2	0.4	0.6	0.8	1.0
1	0.2	0.4	0.6	0.8	1.0
M_R	1.242	1.15	1.006	0.818	0.601
$M_R \pm 2$	0.016	0.061	0.131	0.220	0.318
$M_R \pm 4$					0.018
$2M_R \pm 1$	0.190	0.326	0.370	0.314	0.181
$2M_R \pm 3$		0.024	0.071	0.139	0.212
$2M_R \pm 5$				0.013	0.033
$3M_R$	0.335	0.123	0.083	0.171	0.113
$3M_R \pm 2$	0.044	0.139	0.203	0.716	0.062
$3M_R \pm 4$		0.012	0.047	0.104	0.157
$3M_R \pm 6$				0.016	0.044
$4M_R \pm 1$	0.163	0.157	0.008	0.105	0.068
$4M_R \pm 3$	0.012	0.070	0.132	0.115	0.009
$4M_R \pm 5$			0.034	0.084	0.119
$4M_R \pm 7$				0.017	0.050

Three-phase harmonics

- For three-phase inverters, there is significant advantage if M_R is chosen to be:
  - **Odd:** All even harmonic will be eliminated from the pole-switching waveform.
  - **triplens** (multiple of three (e.g. 3,9,15,21, 27..): All triplens harmonics will be eliminated from the line-to-line output voltage.
- By observing the waveform, it can be seen that with odd M_R, the line-to-line voltage shape looks more "sinusoidal".
- As can be noted from the spectra, the phase voltage amplitude is 0.8 (normalised). This is because the modulation index is 0.8. The line voltage amplitude is square root three of phase voltage due to the three-phase relationship

Effect of odd and "triplens"

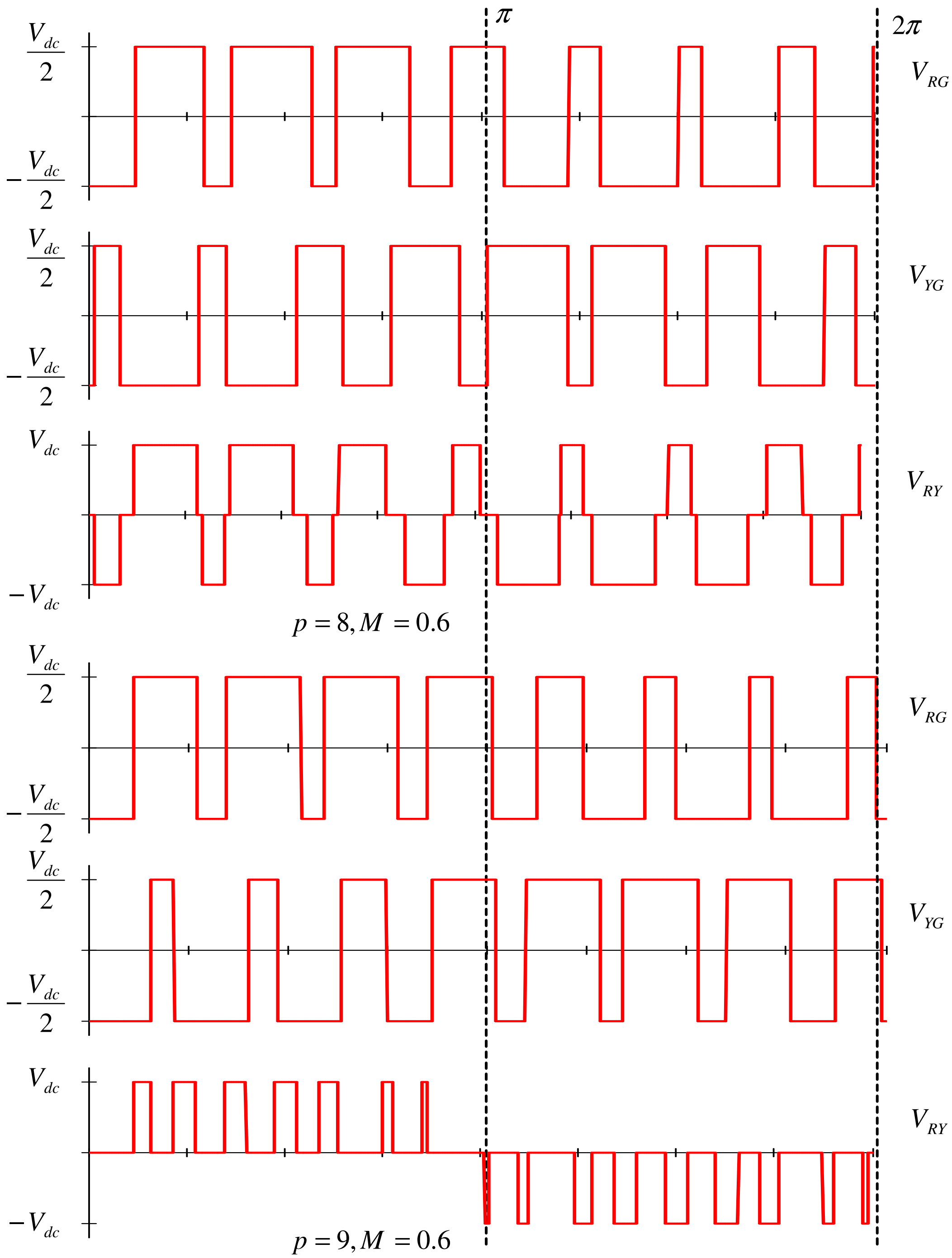
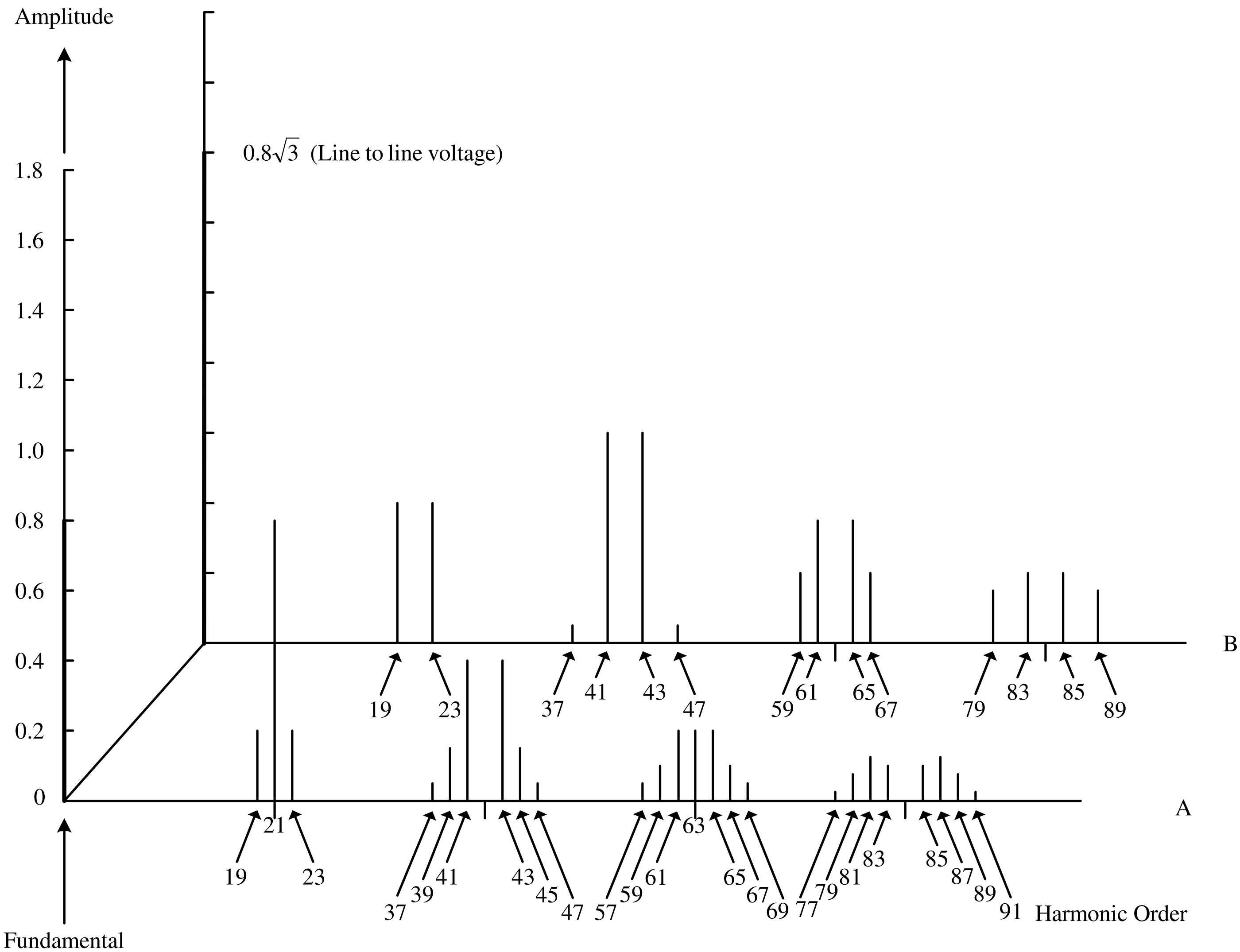


ILLUSTRATION OF BENEFITS OF USING A FREQUENCY RATIO THAT IS A MULTIPLE OF THREE IN A THREE PHASE INVERTER

Spectra: effect of “triplens”



COMPARISON OF INVERTER PHASE VOLTAGE (A) & INVERTER LINE VOLTAGE
(B) HARMONIC (P=21, M=0.8)

Comments on PWM scheme

- It is desirable to have M_R as large as possible.
- This will push the harmonic at higher frequencies on the spectrum. Thus filtering requirement is reduced.
- Although the voltage THD improvement is not significant, but the current THD will improve greatly because the load normally has some current filtering effect.
- However, higher M_R has side effects:
  - Higher switching frequency: More losses.
  - Pulse width may be too small to be constructed. "Pulse dropping" may be required.

Example

The amplitudes of the pole switching waveform harmonics of the red phase of a three-phase inverter is shown in Table below. The inverter uses a symmetric regular sampling PWM scheme. The carrier frequency is 1050Hz and the modulating frequency is 50Hz. The modulation index is 0.8. Calculate the harmonic amplitudes of the line-to-voltage (i.e. red to blue phase) and complete the table.

Harmonic number	Amplitude (pole switching waveform)	Amplitude (line-to line voltage)
1	1	
19	0.3	
21	0.8	
23	0.3	
37	0.1	
39	0.2	
41	0.25	
43	0.25	
45	0.2	
47	0.1	
57	0.05	
59	0.1	
61	0.15	
63	0.2	
65	0.15	
67	0.1	
69	0.05	