

Chapter 3

DC to DC CONVERTER (CHOPPER)

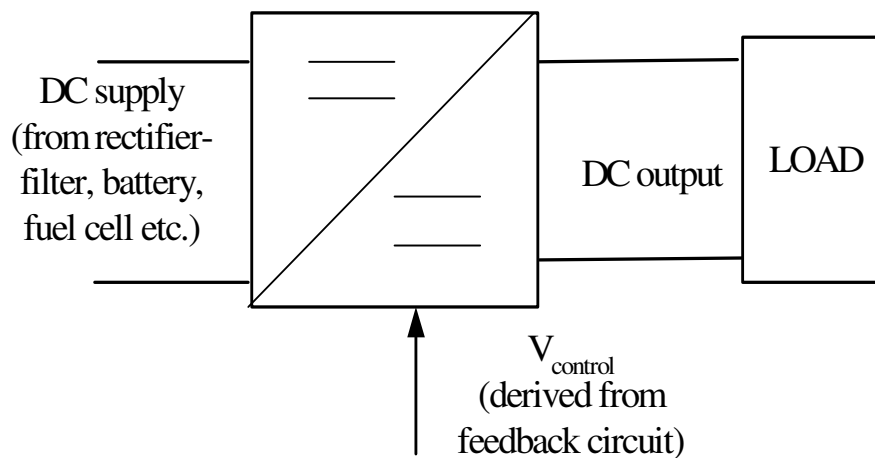
- General
- Buck converter
- Boost converter
- Buck-Boost converter
- Switched-mode power supply
- Bridge converter
- Notes on electromagnetic compatibility (EMC) and solutions.

DC-DC Converter (Chopper)

DEFINITION:

Converting the unregulated DC input to a controlled DC output with a desired voltage level.

- General block diagram:



- APPLICATIONS:

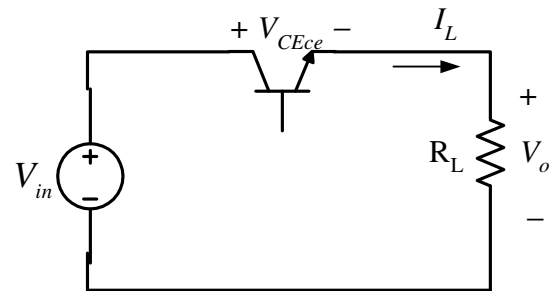
- Switched-mode power supply (SMPS), DC motor control, battery chargers

Linear regulator

- Transistor is operated in linear (active) mode.

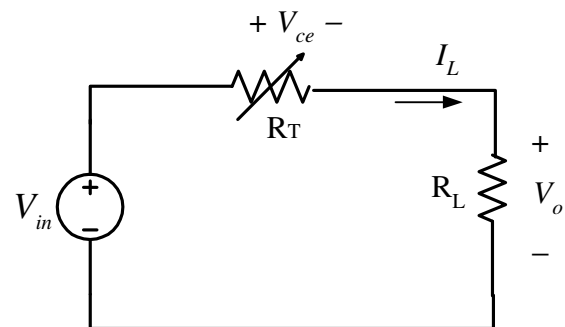
- Output voltage

$$V_o = V_{in} - V_{ce}$$



LINEAR REGULATOR

- The transistor can be conveniently modelled by an equivalent variable resistor, as shown.



EQUIVALENT
CIRCUIT

- Power loss is high at high current due to:

$$P_o = I_L^2 \times R_T$$

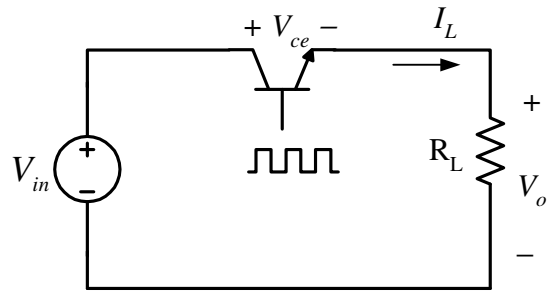
or

$$P_o = V_{ce} \times I_L$$

Switching Regulator

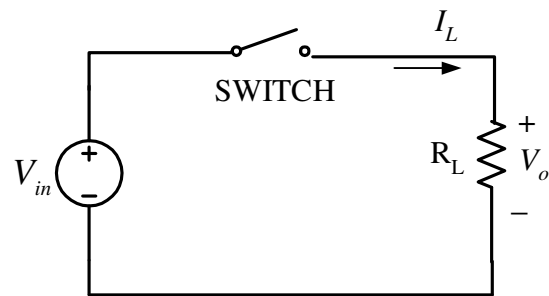
- Transistor is operated in **switched-mode**:

- Switch closed:
Fully on (saturated)
- Switch opened:
Fully off (cut-off)



SWITCHING REGULATOR

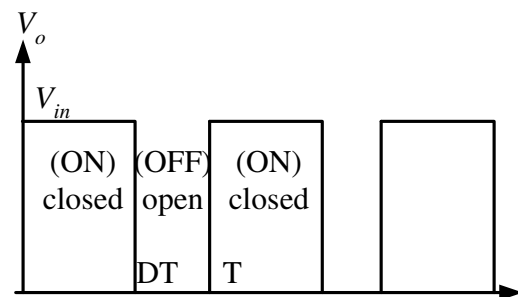
- When switch is open, no current flow in it
- When switch is closed no voltage drop across it.



EQUIVALENT CIRCUIT

- Since $P=V.I$, **no losses** occurs in the switch.

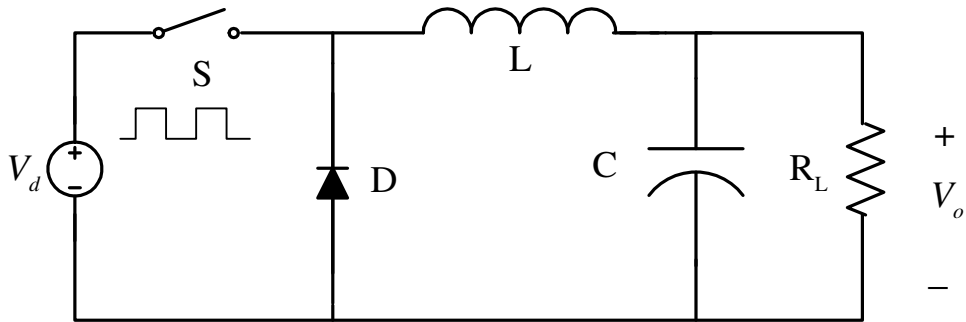
- Power is 100% transferred from source to load.
- Power loss is **zero** (for ideal switch):



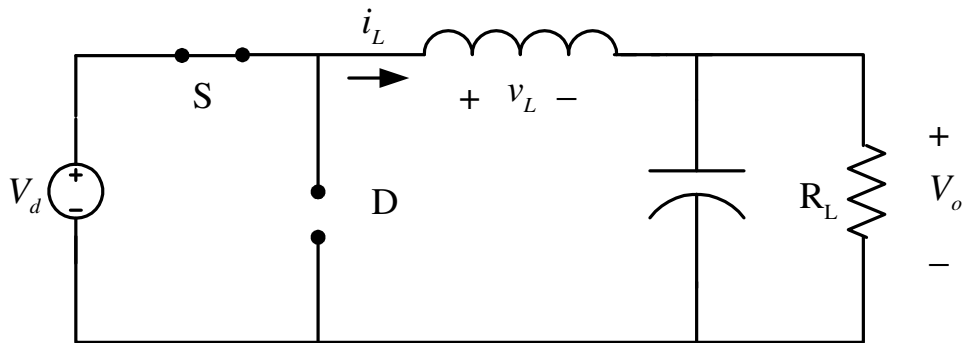
OUTPUT VOLTAGE

- Switching regulator is the basis of all DC-DC converters

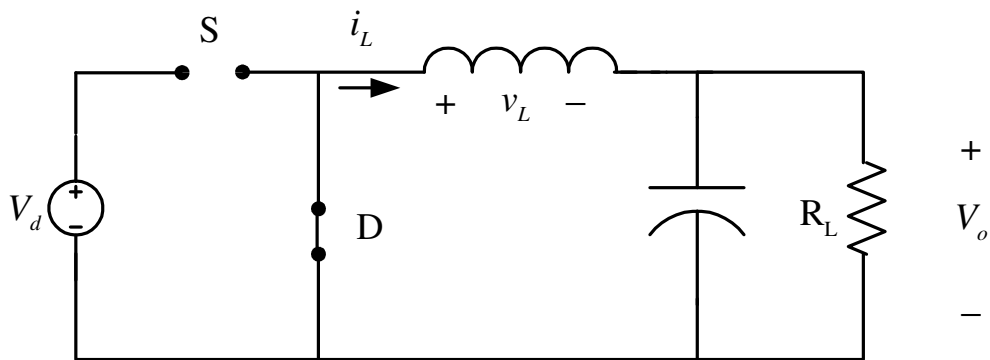
Buck (step-down) converter



CIRCUIT OF BUCK CONVERTER



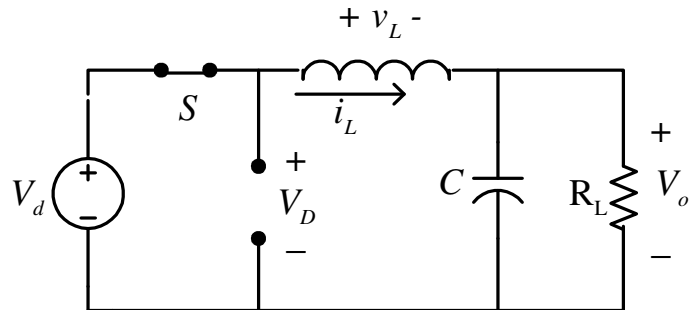
CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

Switch is turned on (closed)

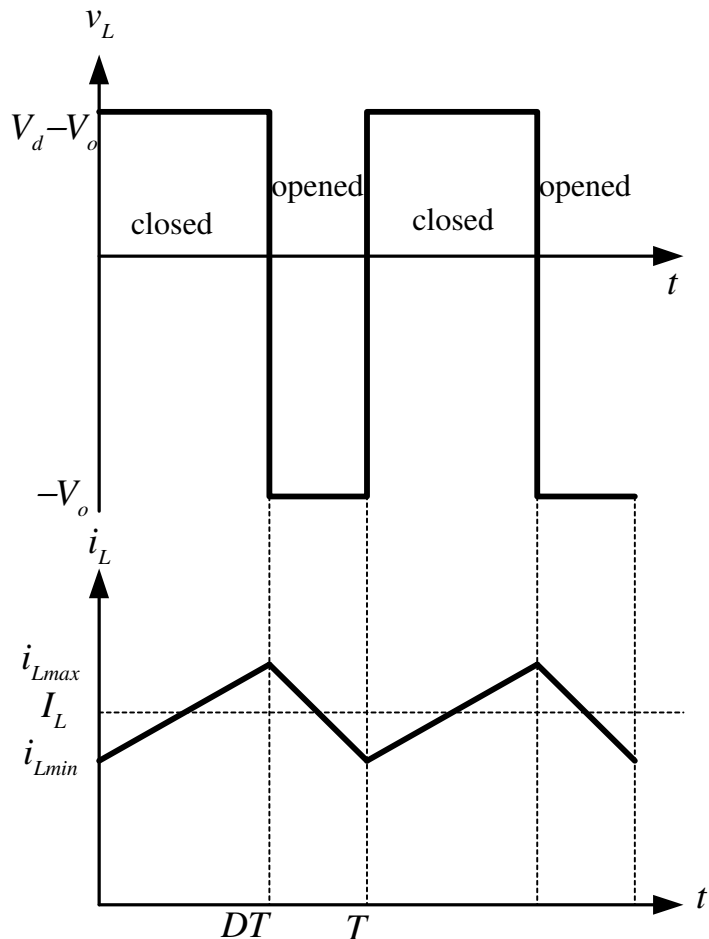
- Diode is reversed biased.
- Switch conducts inductor current
- This results in positive inductor voltage, i.e:



- $$v_L = V_d - V_o$$
- It causes linear increase in the inductor current

$$v_L = L \frac{di_L}{dt}$$

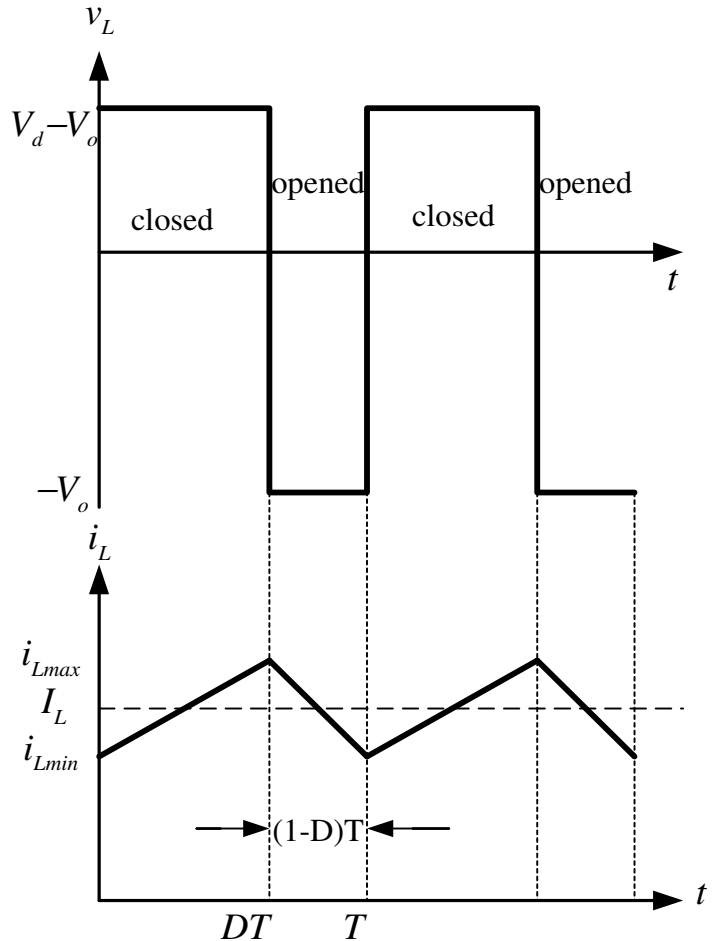
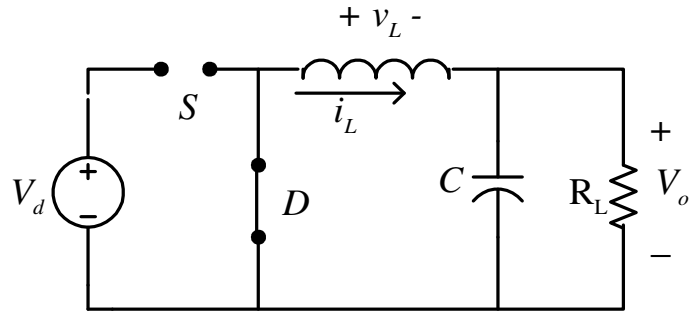
$$\Rightarrow i_L = \frac{1}{L} \int v_L dt$$



Switch turned off (opened)

- Because of inductive energy storage, i_L continues to flow.
- Diode is forward biased
- Current now flows (freewheeling) through the diode.
- The inductor voltage can be derived as:

$$v_L = -V_o$$



Analysis

When the switch is closed (on):

$$v_L = V_d - V_o = L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

Derivative of i_L is a positive constant. Therefore i_L must increase linearly.

From Figure

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_d - V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{closed} = \left(\frac{V_d - V_o}{L} \right) \cdot DT$$

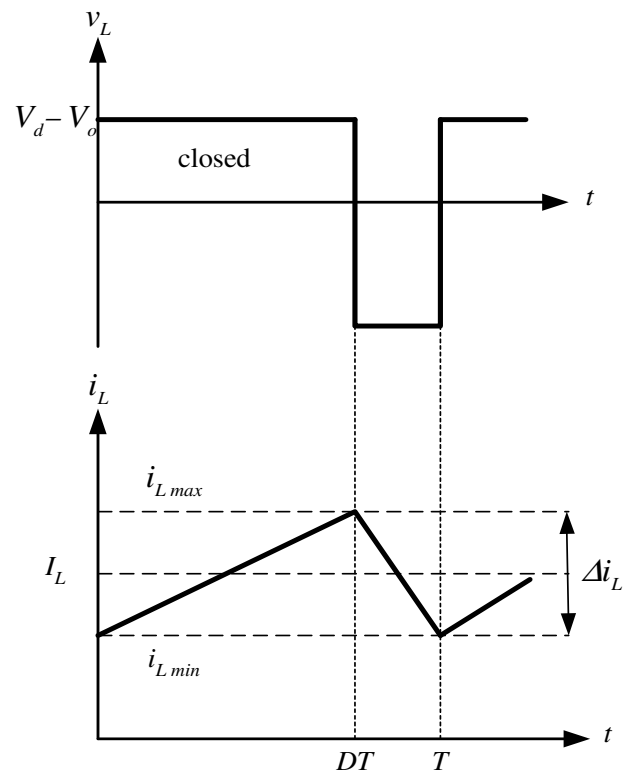
For switch opened,

$$v_L = -V_o = L \frac{di_L}{dt}$$

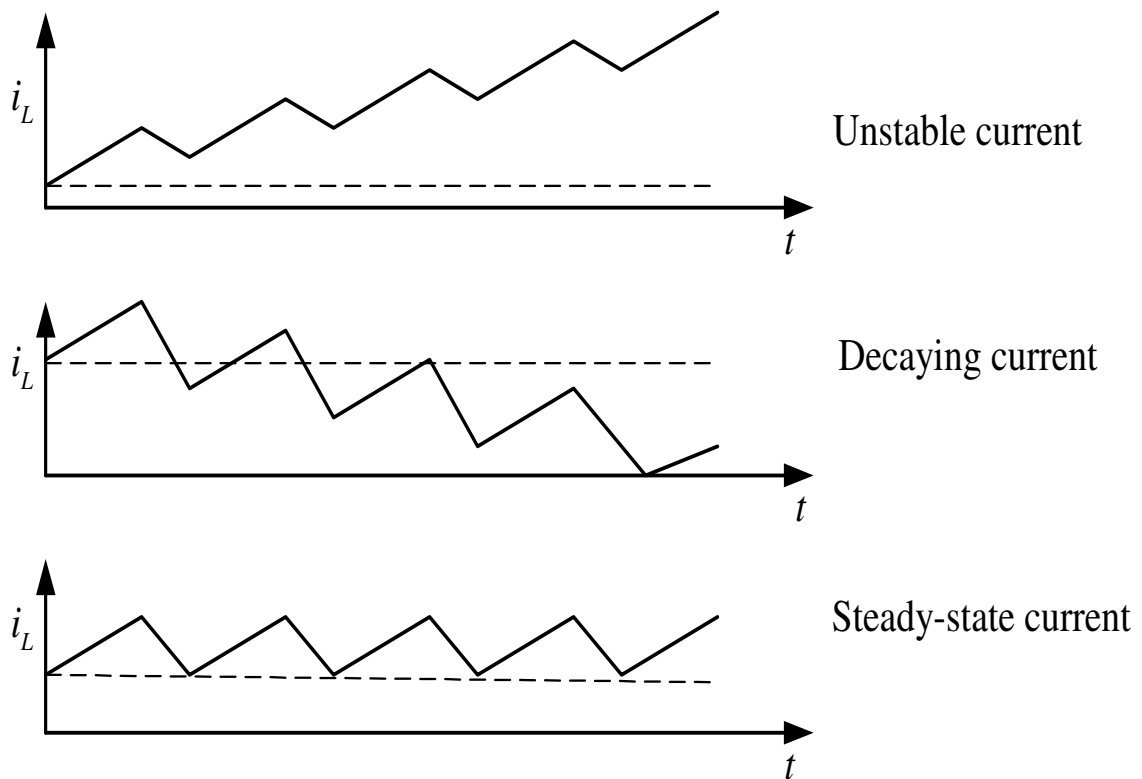
$$\Rightarrow \frac{di_L}{dt} = \frac{-V_o}{L}$$

$$\therefore \frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{-V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \left(\frac{-V_o}{L} \right) \cdot (1-D)T$$



Steady-state operation



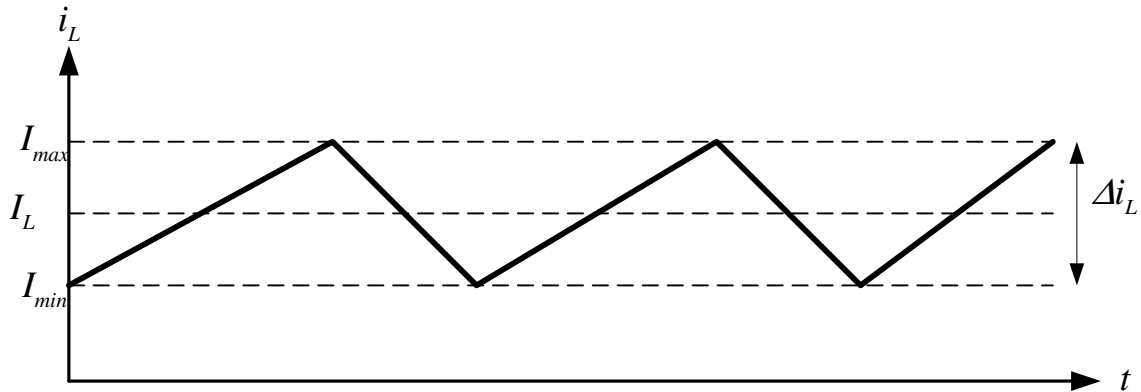
Steady - state operation requires that i_L at the end of switching cycle is the same at the beginning of the next cycle. That is the change of i_L over one period is zero, i.e :

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\left(\frac{V_d - V_o}{L} \right) \cdot DT_s - \left(\frac{-V_o}{L} \right) \cdot (1 - D)T_s = 0$$

$$\Rightarrow V_o = DV_d$$

Average, Maximum and Minimum Inductor Current



Average inductor current = Average current in R_L

$$\Rightarrow I_L = I_R = \frac{V_o}{R}$$

Maximum current :

$$\begin{aligned} I_{max} &= I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2} \left(\frac{V_o}{L} (1-D)T \right) \\ &= V_o \left(\frac{1}{R} + \frac{(1-D)}{2Lf} \right) \end{aligned}$$

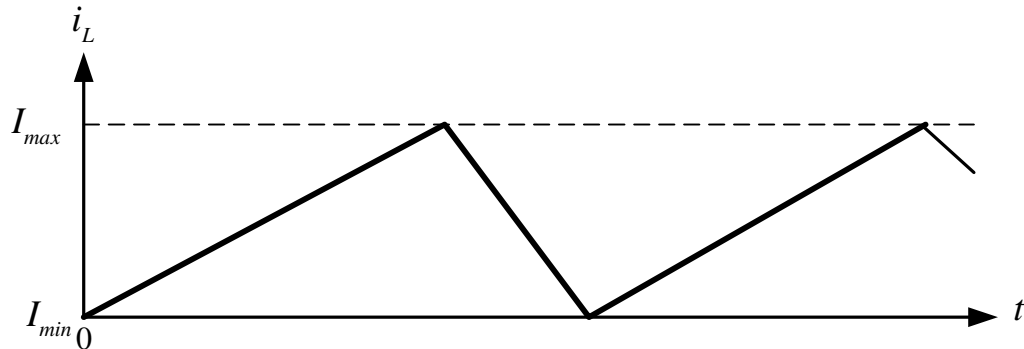
Minimum current :

$$I_{min} = I_L - \frac{\Delta i_L}{2} = V_o \left(\frac{1}{R} - \frac{(1-D)}{2Lf} \right)$$

Inductor current ripple :

$$\Delta i_L = I_{max} - I_{min}$$

Continuous Current Mode (CCM)



From previous analysis,

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_o \left(\frac{1}{R} - \frac{(1-D)}{2Lf} \right)$$

For continuous operation, $I_{\min} \geq 0$,

$$V_o \left(\frac{1}{R} - \frac{(1-D)}{2Lf} \right) \geq 0$$

$$\Rightarrow L \geq L_{\min} = \frac{(1-D)}{2f} \cdot R$$

This is the minimum inductor current to ensure continuous mode of operation.

Normally L is chosen to be $\gg L_{\min}$

Output voltage ripple

KCL, Capacitor current :

$$i_c = i_L + i_R$$

The charge can be written as :

$$Q = CV_o$$

$$\Rightarrow \Delta Q = C\Delta V$$

$$\Rightarrow \Delta V_o = \frac{\Delta Q}{C}$$

Use triangle area formula :

$$\Delta Q = \frac{1}{2} \left(\frac{T}{2} \right) \left(\frac{\Delta i_L}{2} \right)$$

$$= \frac{T\Delta i_L}{8}$$

Ripple voltage (Peak - to peak)

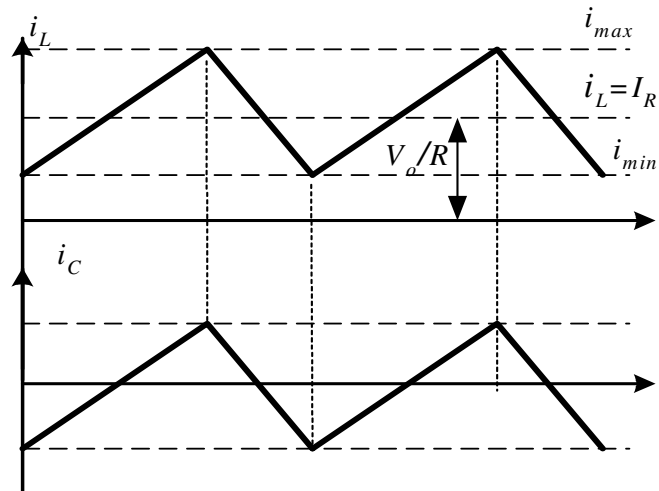
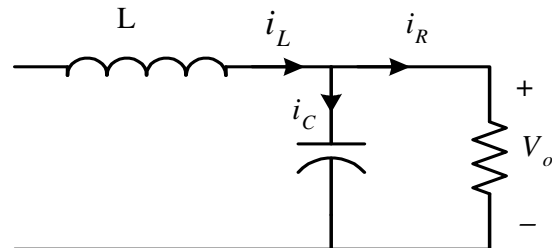
$$\therefore \Delta V_o = \frac{T\Delta i_L}{8C} = \frac{(1-D)}{8LCf^2}$$

So, the ripple factor,

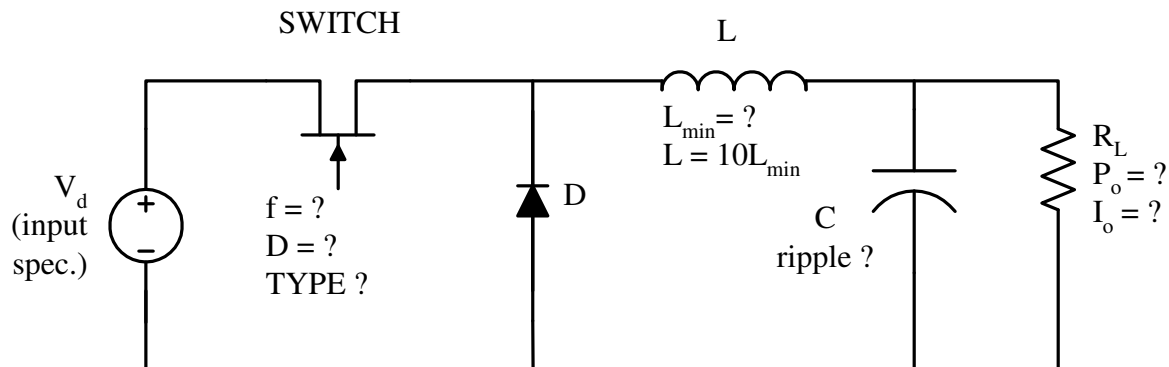
$$r = \frac{\Delta V_o}{V_o} = \frac{(1-D)}{8LCf^2}$$

Note : Ripple can be reduced by :

- 1) Increasing switching frequency
- 2) Increasing inductor size
- 3) Increasing capacitor size.



Basic design procedures



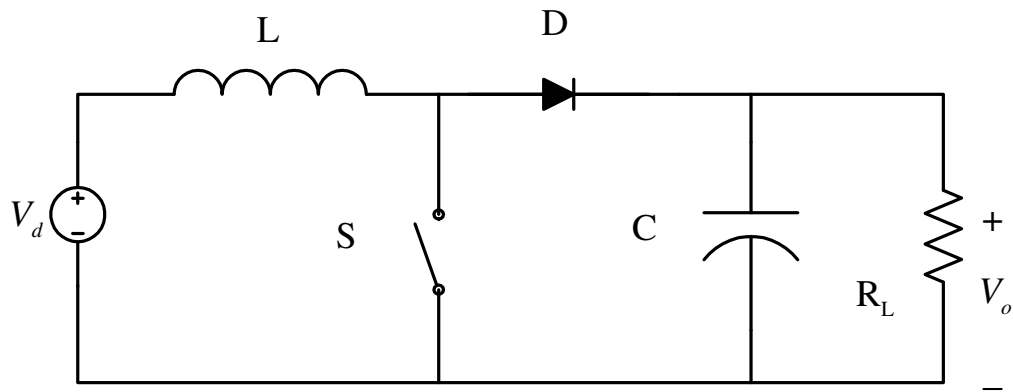
- Calculate D to obtain required output voltage.
- Select a particular switching frequency (f) and device
 - preferably $f > 20\text{KHz}$ for negligible acoustic noise
 - higher fs results in smaller L and C. But results in higher losses. Reduced efficiency, larger heat sink.
 - Possible devices: MOSFET, IGBT and BJT. Low power MOSFET can reach MHz range.
- Calculate L_{\min} . Choose $L \gg 10 L_{\min}$
- Calculate C for ripple factor requirement.
 - Capacitor ratings:
 - must withstand peak output voltage
 - must carry required RMS current. Note RMS current for triangular w/f is $I_p/3$, where I_p is the peak capacitor current given by $\Delta i_L/2$.
 - ECAPs can be used
- Wire size consideration:
 - Normally rated in RMS. But i_L is known as peak. RMS value for i_L is given as:

$$I_{L,RMS} = \sqrt{I_L^2 + \left(\frac{\Delta i_L/2}{\sqrt{3}}\right)^2}$$

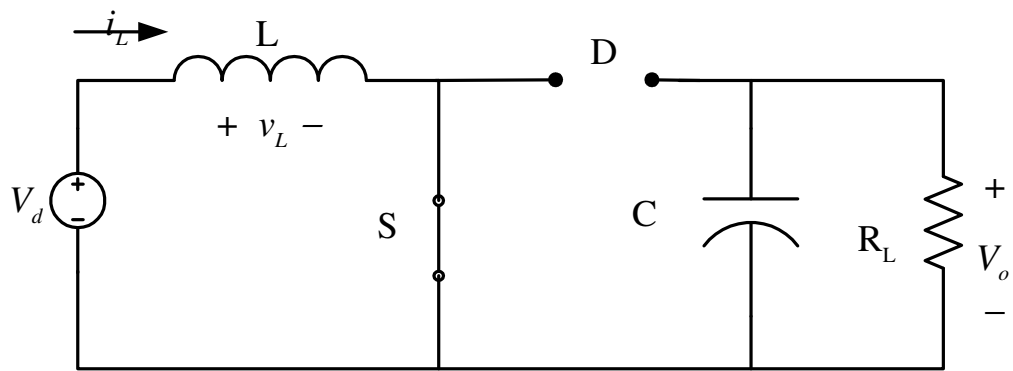
Examples

- A buck converter is supplied from a 50V battery source. Given $L=400\mu\text{H}$, $C=100\mu\text{F}$, $R=20\ \Omega$, $f=20\text{KHz}$ and $D=0.4$. Calculate: (a) output voltage (b) maximum and minimum inductor current, (c) output voltage ripple.
- A buck converter has an input voltage of 50V and output of 25V. The switching frequency is 10KHz. The power output is 125W. (a) Determine the duty cycle, (b) value of L to limit the peak inductor current to 6.25A, (c) value of capacitance to limit the output voltage ripple factor to 0.5%.
- Design a buck converter such that the output voltage is 28V when the input is 48V. The load is 8 Ω . Design the converter such that it will be in continuous current mode. The output voltage ripple must not be more than 0.5%. Specify the frequency and the values of each component. Suggest the power switch also.

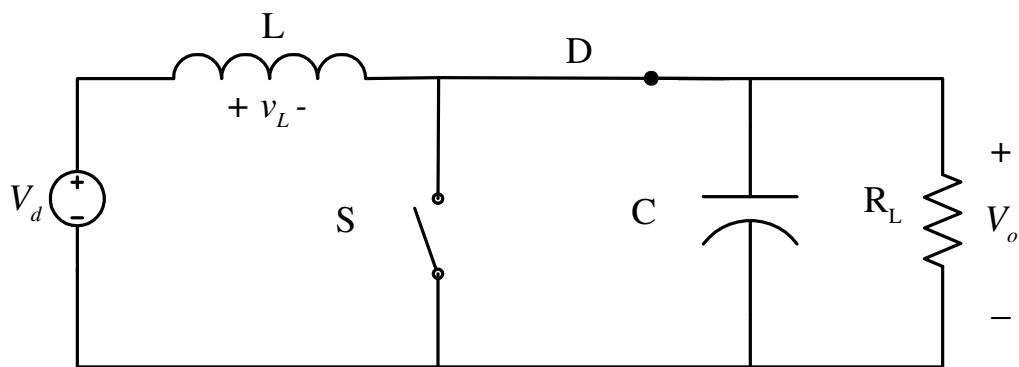
Boost (step-up) converter



CIRCUIT OF BOOST CONVERTER

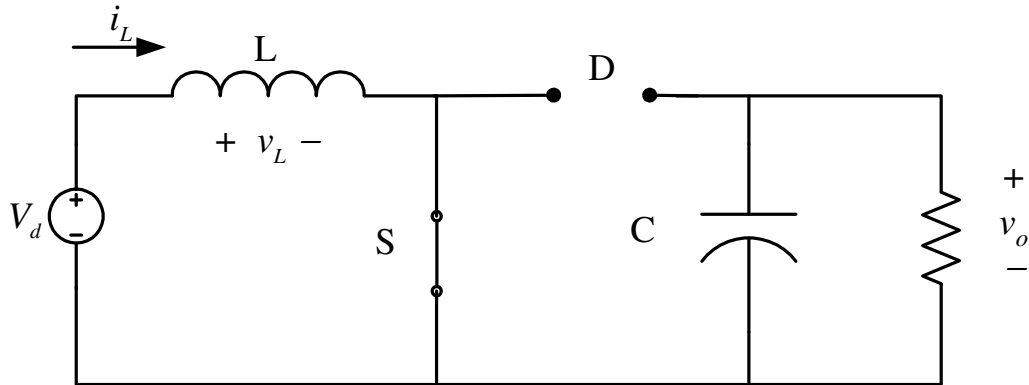


CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

Boost analysis: switch closed



$$v_L = V_d$$

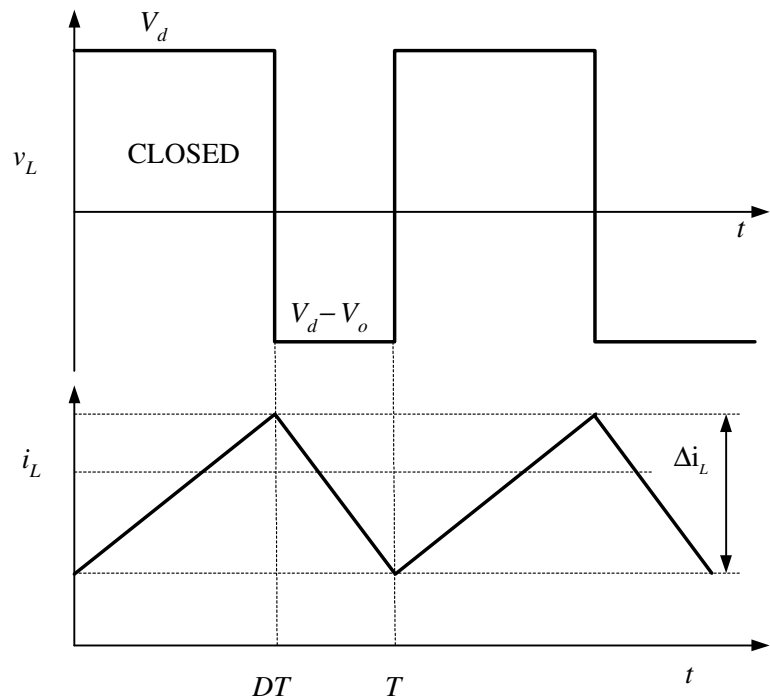
$$= L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

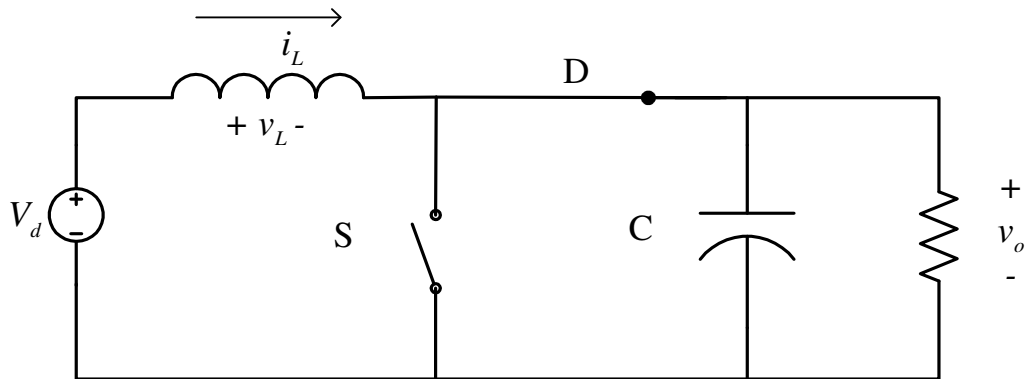
$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$



Switch opened



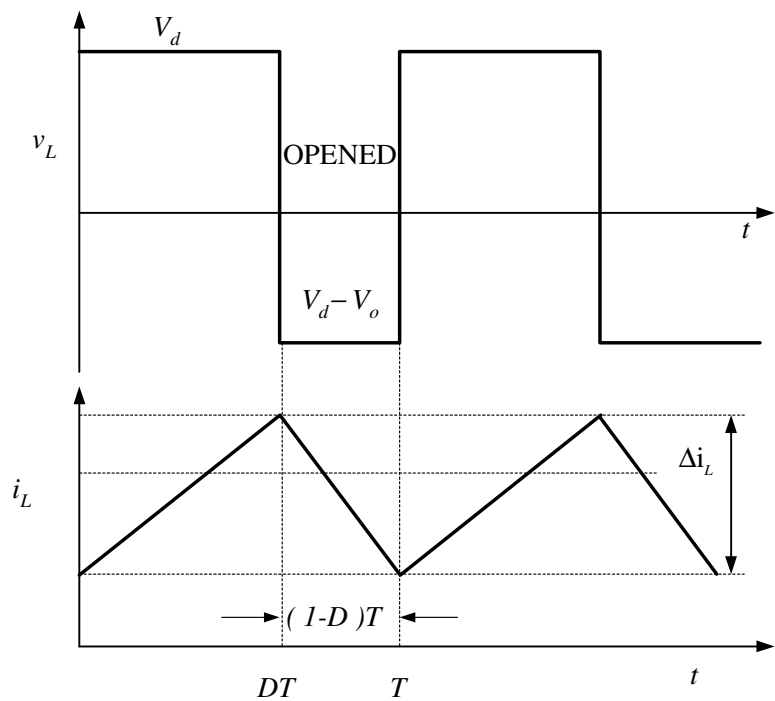
$$v_L = V_d - V_o$$

$$= L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t}$$

$$= \frac{\Delta i_L}{(1-D)T}$$



$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{(V_d - V_o)(1 - DT)}{L}$$

Steady-state operation

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\frac{V_d DT}{L} + \frac{(V_d - V_o)(1-D)T}{L} = 0$$

$$\Rightarrow V_o = \frac{V_d}{1-D}$$

- Boost converter produces output voltage that is greater or equal to the input voltage.
- Alternative explanation:
 - when switch is closed, diode is reversed. Thus output is isolated. The input supplies energy to inductor.
 - When switch is opened, the output stage receives energy from the input as well as from the inductor. Hence output is large.
 - Output voltage is maintained constant by virtue of large C.

Average, Maximum, Minimum Inductor Current

Input power = Output power

$$V_d I_d = \frac{V_o^2}{R}$$

$$V_d I_L = \frac{\left(\frac{V_d}{(1-D)}\right)^2}{R} = \frac{V_d^2}{(1-D)^2 R}$$

Average inductor current :

$$\Rightarrow I_L = \frac{V_d}{(1-D)^2 R}$$

Maximum inductor current :

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

Minimum inductor current :

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

L and C values

For CCM,

$$I_{\min} \geq 0$$

$$\frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L} \geq 0$$

$$L_{\min} = \frac{D(1-D)^2 TR}{2}$$

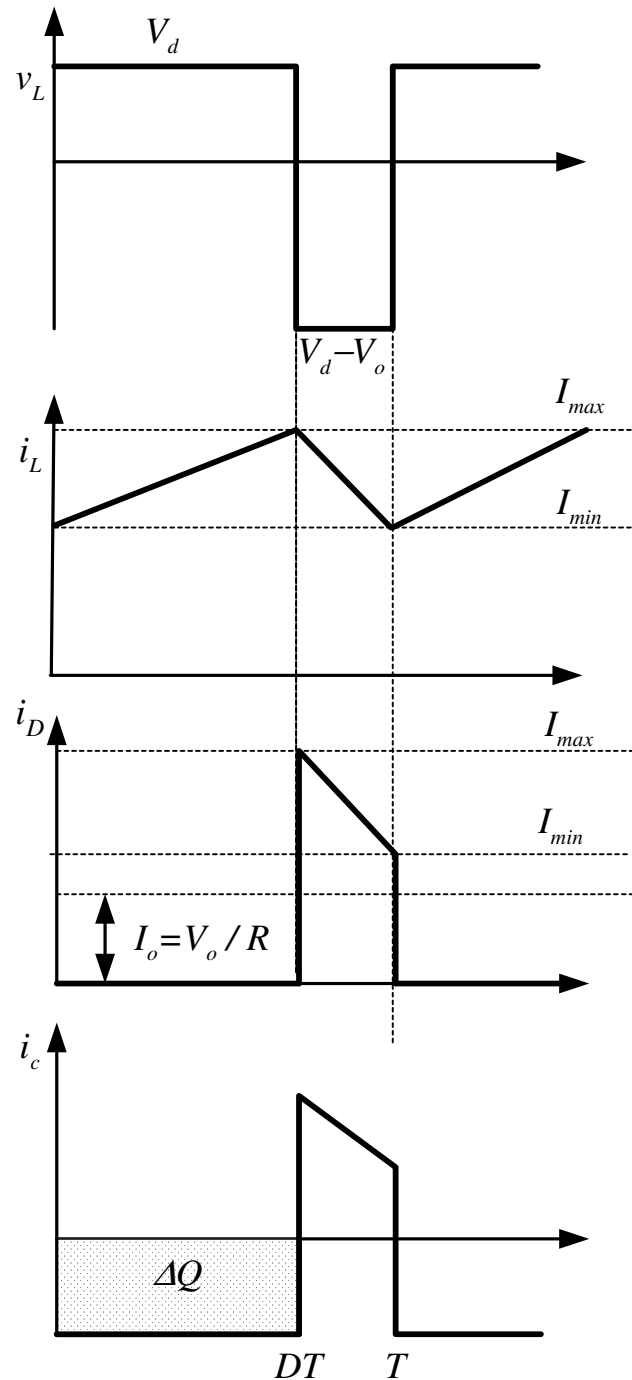
$$= \frac{D(1-D)^2 R}{2f}$$

Ripple factor

$$|\Delta Q| = \left(\frac{V_o}{R} \right) DT = C \Delta V_o$$

$$\Delta V_o = \frac{V_o DT}{RCf} = \frac{V_o D}{RCf}$$

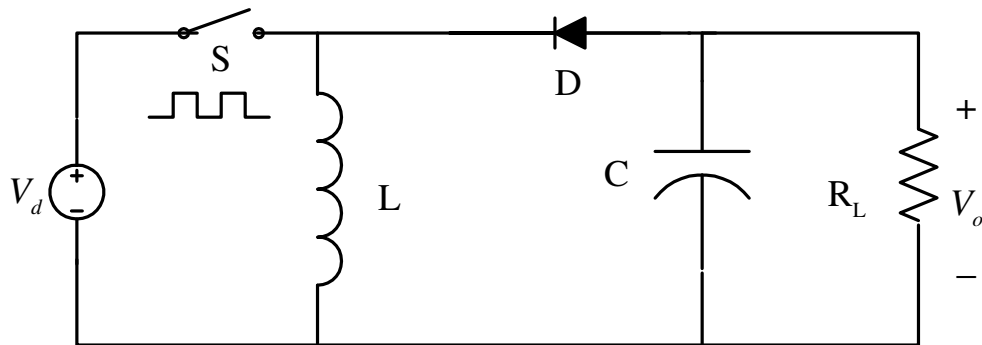
$$r = \frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$



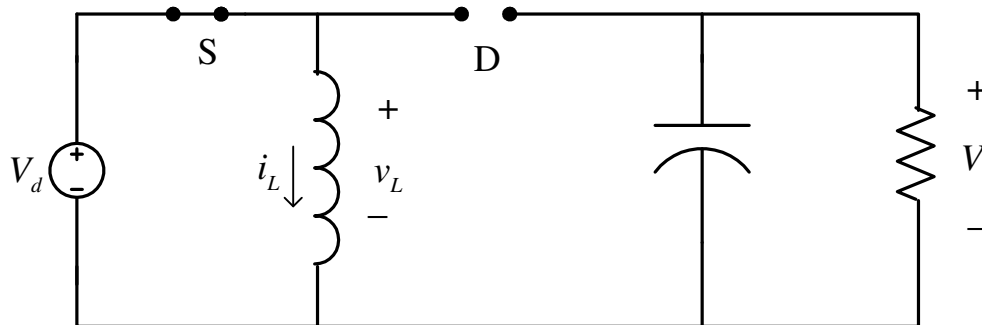
Examples

- The boost converter has the following parameters: $V_d=20V$, $D=0.6$, $R=12.5\text{ohm}$, $L=65\mu\text{H}$, $C=200\mu\text{F}$, $f_s=40\text{KHz}$. Determine (a) output voltage, (b) average, maximum and minimum inductor current, (c) output voltage ripple.
- Design a boost converter to provide an output voltage of 36V from a 24V source. The load is 50W. The voltage ripple factor must be less than 0.5%. Specify the duty cycle ratio, switching frequency, inductor and capacitor size, and power device.

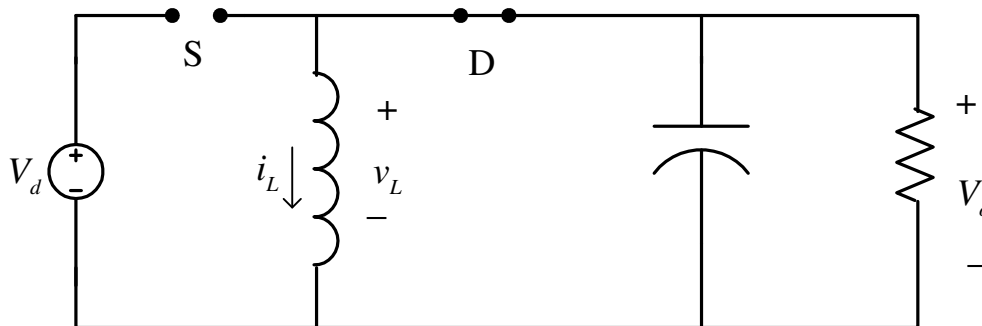
Buck-Boost converter



CIRCUIT OF BUCK-BOOST CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

Buck-boost analysis

Switch closed

$$v_L = V_d = L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_d}{L}$$

$$\Rightarrow (\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

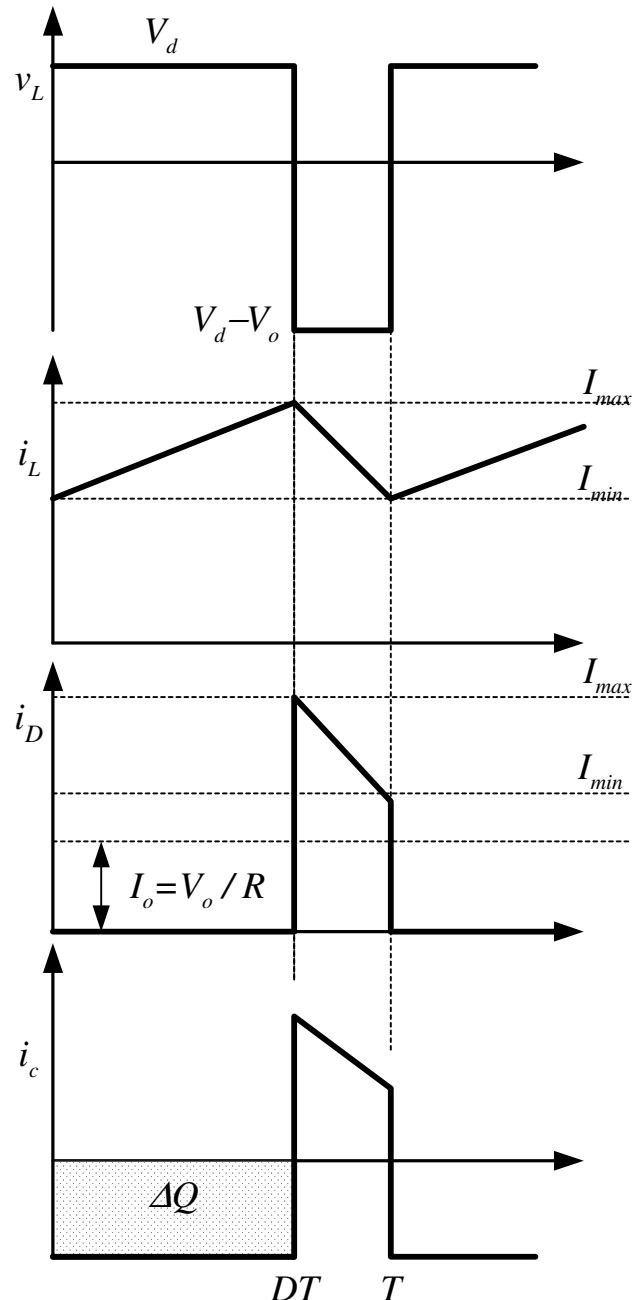
Switch opened

$$v_L = V_o = L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_o}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{V_o (1-D)T}{L}$$



Output voltage

Steady state operation :

$$\Delta_{iL(\text{closed})} + \Delta_{iL(\text{opened})} = 0$$

$$\Rightarrow \frac{V_d DT}{L} + \frac{V_o(1-D)T}{L} = 0$$

Output voltage :

$$\Rightarrow V_o = -V_s \left(\frac{D}{1-D} \right)$$

- NOTE: Output of a buck-boost converter either be higher or lower than input.
 - If $D > 0.5$, output is higher than input
 - If $D < 0.5$, output is lower input
- Output voltage is always negative.
- Note that output is never directly connected to load.
- Energy is stored in inductor when switch is closed and transferred to load when switch is opened.

Average inductor current

Assuming no power loss in the converter, power absorbed by the load must equal power supplied the by source, i.e.

$$P_o = P_s$$

$$\frac{V_o^2}{R} = V_d I_s$$

But average source current is related to average inductor current as :

$$I_s = I_L D$$

$$\Rightarrow \frac{V_o^2}{R} = V_d I_L D$$

Substituting for V_o ,

$$\Rightarrow I_L = \frac{V_o^2}{V_d R D} = \frac{P_o}{V_d D} = \frac{V_d D}{R(1-D)^2}$$

L and C values

Max and min inductor current,

$$\Rightarrow I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d D}{R(1-D)^2} + \frac{V_d DT}{2L}$$

$$\Rightarrow I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d D}{R(1-D)^2} - \frac{V_d DT}{2L}$$

For CCM

$$\frac{V_d D}{R(1-D)^2} + \frac{V_d DT}{2L} = 0$$

$$\Rightarrow L_{\min} = \frac{(1-D)^2 R}{2f}$$

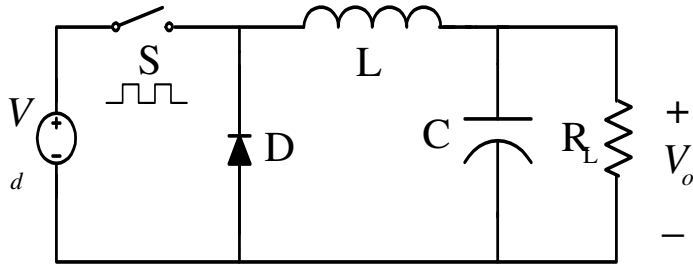
Output voltage ripple,

$$|\Delta Q| = \left(\frac{V_o}{R} \right) DT = C \Delta V_o$$

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$r = \frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

Converters in CCM: Summary

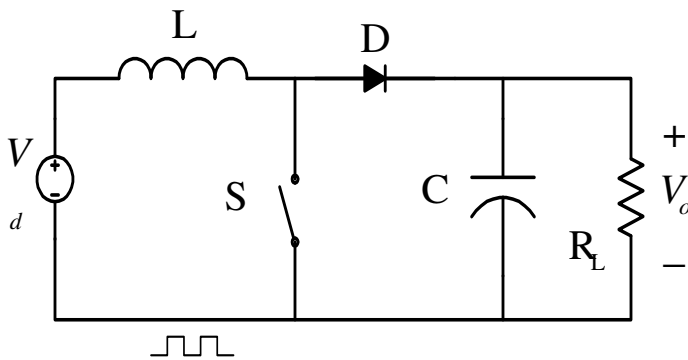


Buck

$$\frac{V_o}{V_d} = D$$

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}$$

$$L_{\min} = \frac{(1-D)R}{2f}$$

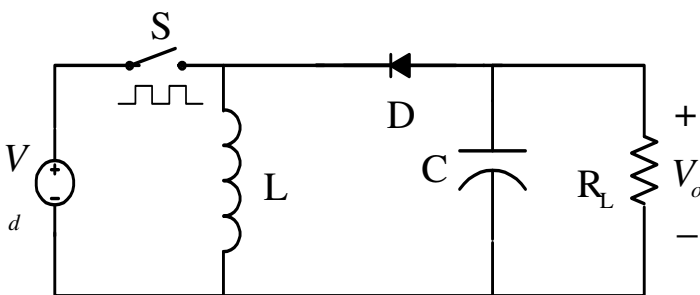


Boost

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

$$L_{\min} = \frac{D(1-D)^2 R}{2f}$$



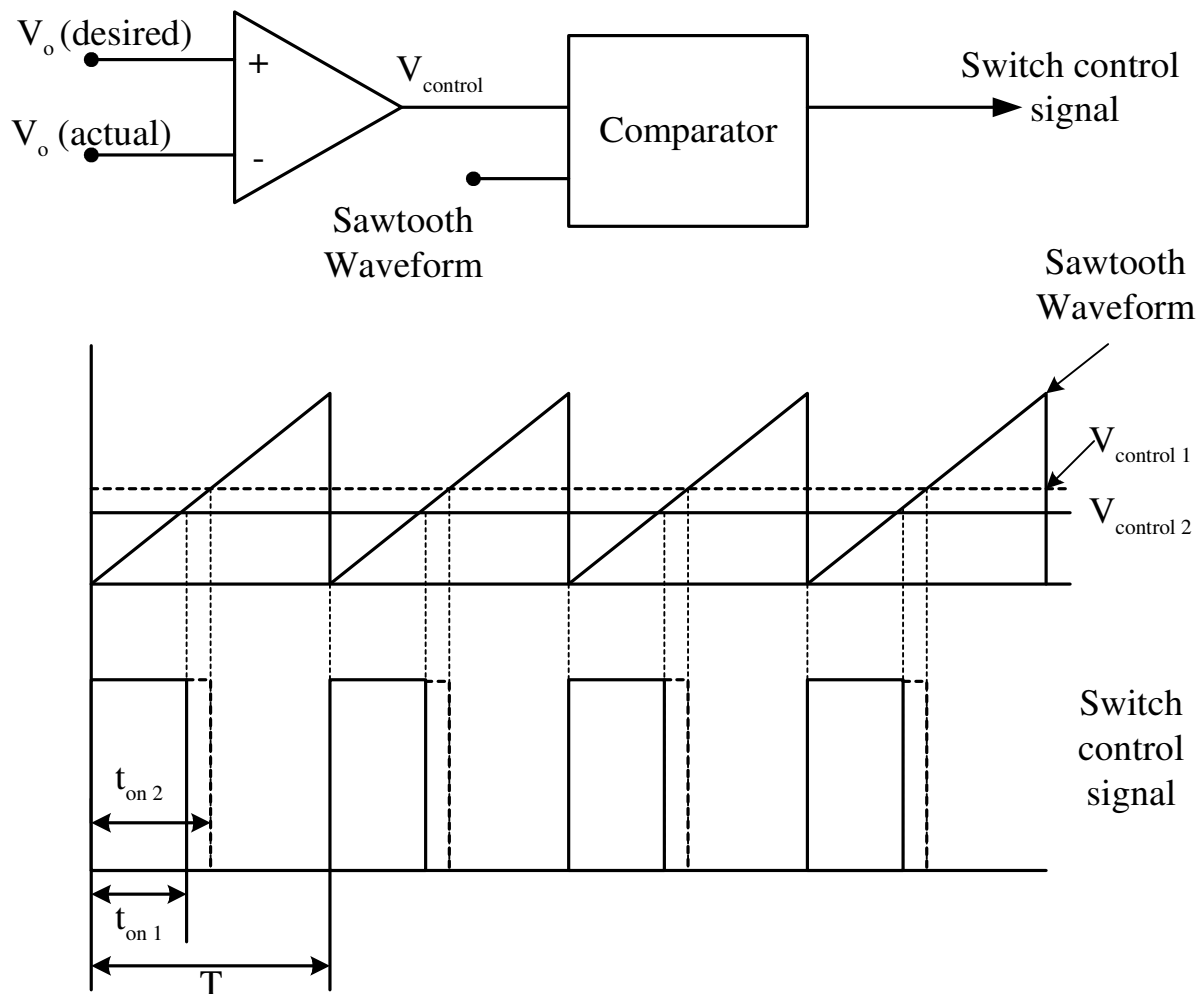
Buck – Boost

$$\frac{V_o}{V_d} = -\frac{D}{1-D}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

$$L_{\min} = \frac{(1-D)^2 R}{2f}$$

Control of DC-DC converter: pulse width modulation (PWM)

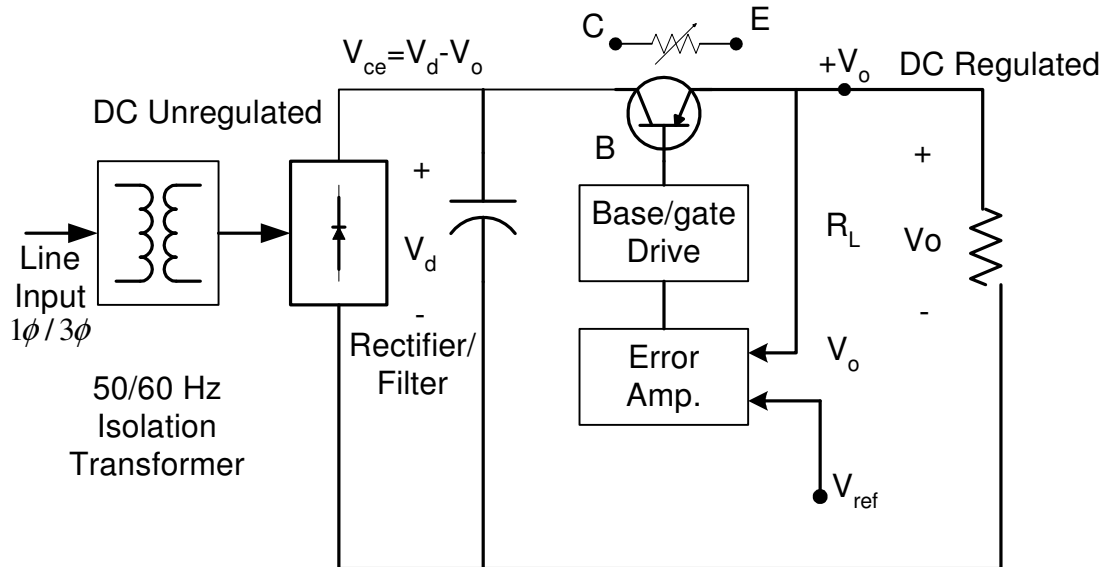


Isolated DC-DC Converter

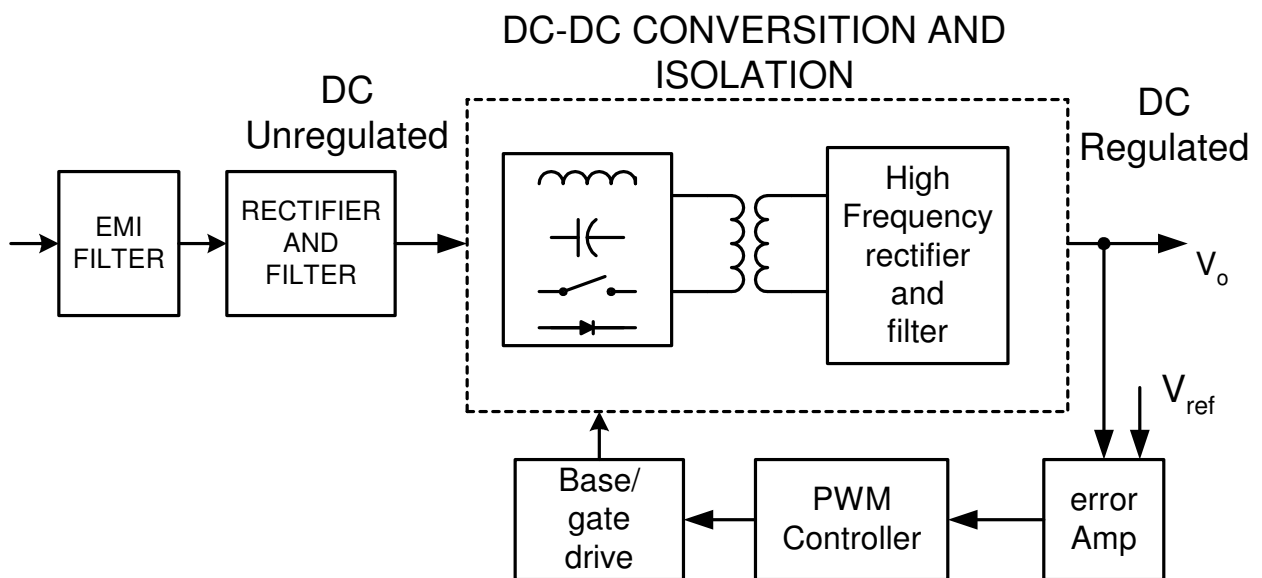
- Isolated DC-DC requires isolation transformer
- Two types: Linear and Switched-mode
- Advantages of switched mode over linear power supply
 - Efficient (70-95%)
 - Weight and size reduction
- Disadvantages
 - Complex design
 - EMI problems
- However above certain ratings, SMPS is the only feasible choice
- Types of SMPS
 - Flyback
 - forward
 - Push-pull
 - Bridge (half and full)

Linear and SMPS block diagram

Basic Block diagram of linear power supply



Basic Block diagram of SMPS



High frequency transformer

Basic function :

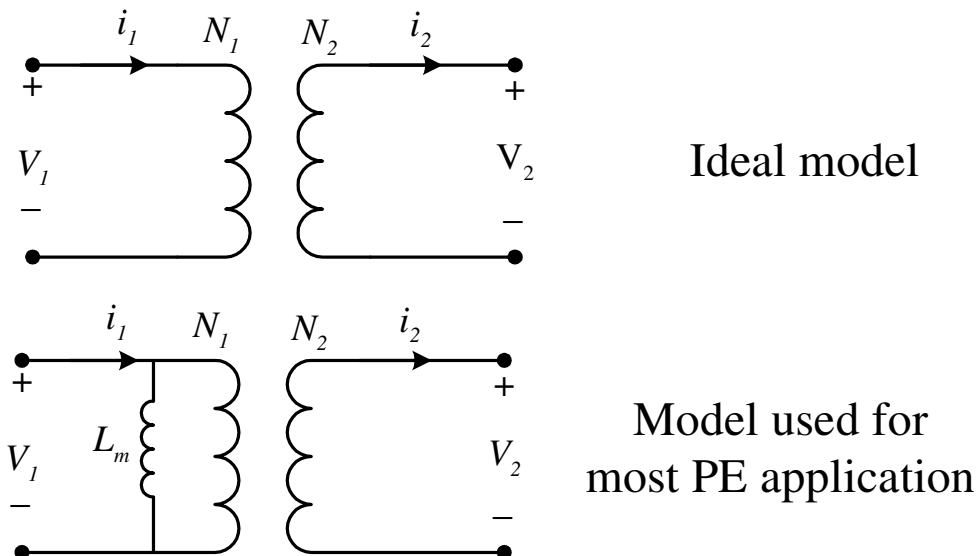
- 1) Input - output electrical isolation
- 2) step up/down time - varying voltage

Basic input - output relationship

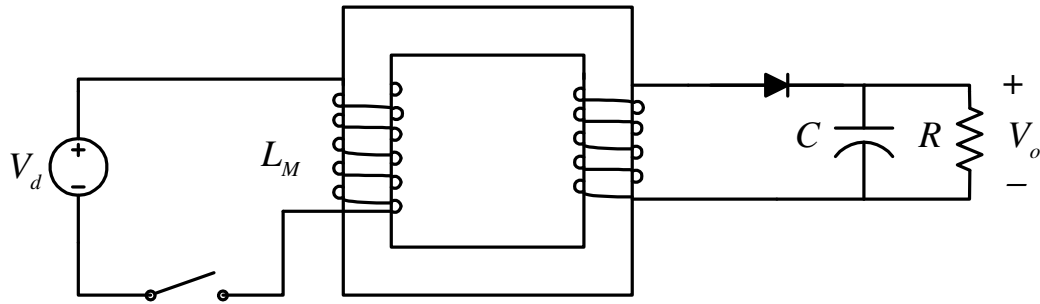
$$\frac{v_1}{v_2} = \frac{N_1}{N_2};$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

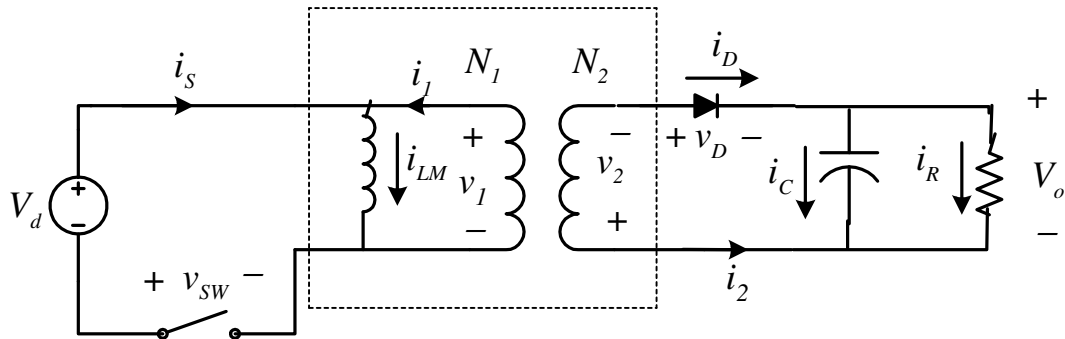
Models :



Flyback Converter

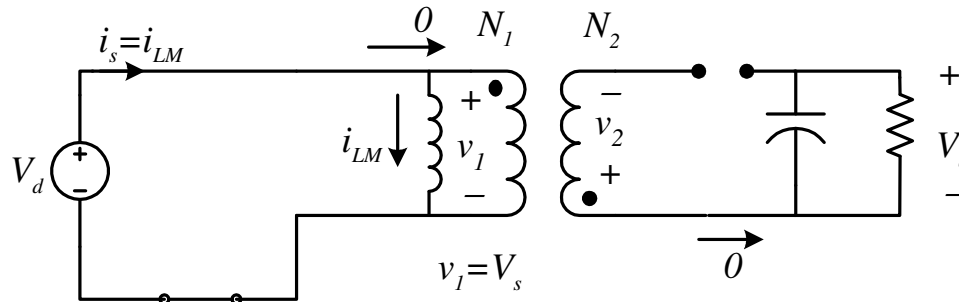


Flyback converter circuit



Model with magnetising inductance

Operation: switch closed



$$v_1 = V_d = L_m \frac{di_{LM}}{dt}$$

$$\frac{di_{LM}}{dt} = \frac{\Delta i_{LM}}{dt} = \frac{\Delta i_{LM}}{DT} = \frac{V_d}{L_m}$$

$$\Rightarrow (\Delta i_{L_m})_{closed} = \frac{V_d DT}{L_m}$$

On the load side of the transformer,

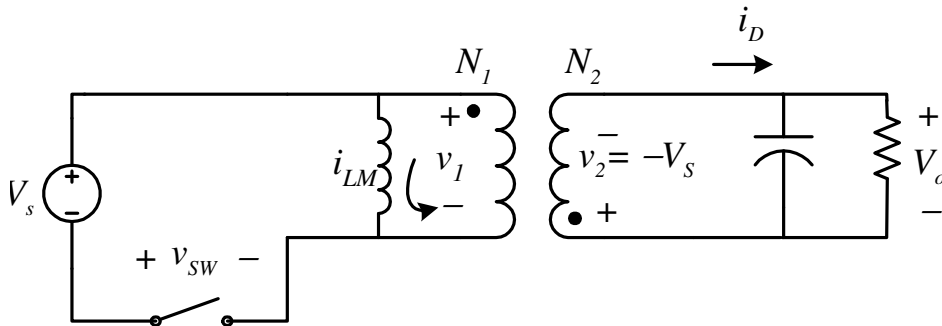
$$v_2 = v_1 \left(\frac{N_2}{N_1} \right) = V_d \left(\frac{N_2}{N_1} \right)$$

$$v_D = -V_o - V_d \left(\frac{N_2}{N_1} \right) < 0, \text{ i.e. diode turned off}$$

Therefore,

$$i_2 = 0 \text{ and } i_1 = 0$$

Switch opened



$$v_1 = -V_o \left(\frac{N_1}{N_2} \right)$$

But $v_2 = -V_o$

$$\Rightarrow v_1 = v_2 \left(\frac{N_1}{N_2} \right) = -V_o \left(\frac{N_1}{N_2} \right)$$

$$v_1 = L_m \frac{di_{Lm}}{dt} = -V_o \left(\frac{N_1}{N_2} \right)$$

$$\frac{di_{Lm}}{dt} = \frac{\Delta i_{Lm}}{dt} = \frac{\Delta i_{Lm}}{(1-D)T} = \frac{-V_o}{L_m} \left(\frac{N_1}{N_2} \right)$$

$$\Rightarrow (\Delta i_{Lm})_{open} = -\frac{V_o (1-D)T}{L_m} \left(\frac{N_1}{N_2} \right)$$

Voltage across the switch :

$$v_{SW} = V_d + V_o \left(\frac{N_1}{N_2} \right)$$

Output voltage

For steady - state operation,

$$\left(\Delta i_{L_m}\right)_{closed} + \left(\Delta i_{L_m}\right)_{opened} = 0$$

$$\Rightarrow \frac{V_d DT}{L_m} - \frac{V_o(1-D)T}{L_m} \left(\frac{N_1}{N_2}\right) = 0$$

$$\Rightarrow V_o = V_d \left(\frac{D}{1-D}\right) \left(\frac{N_2}{N_1}\right)$$

- Input output relationship is similar to buck-boost converter.
- Output can be greater or less than input, depending upon D.
- Additional term, i.e. transformer ratio is present.

Flyback waveforms

$$P_s = P_o$$

$$V_d I_s = \frac{V_o^2}{R}$$

I_s is related to I_{L_m} as :

$$I_s = \frac{I_{L_m} DT}{T} = (I_{L_m})D$$

Solving for I_{L_m}

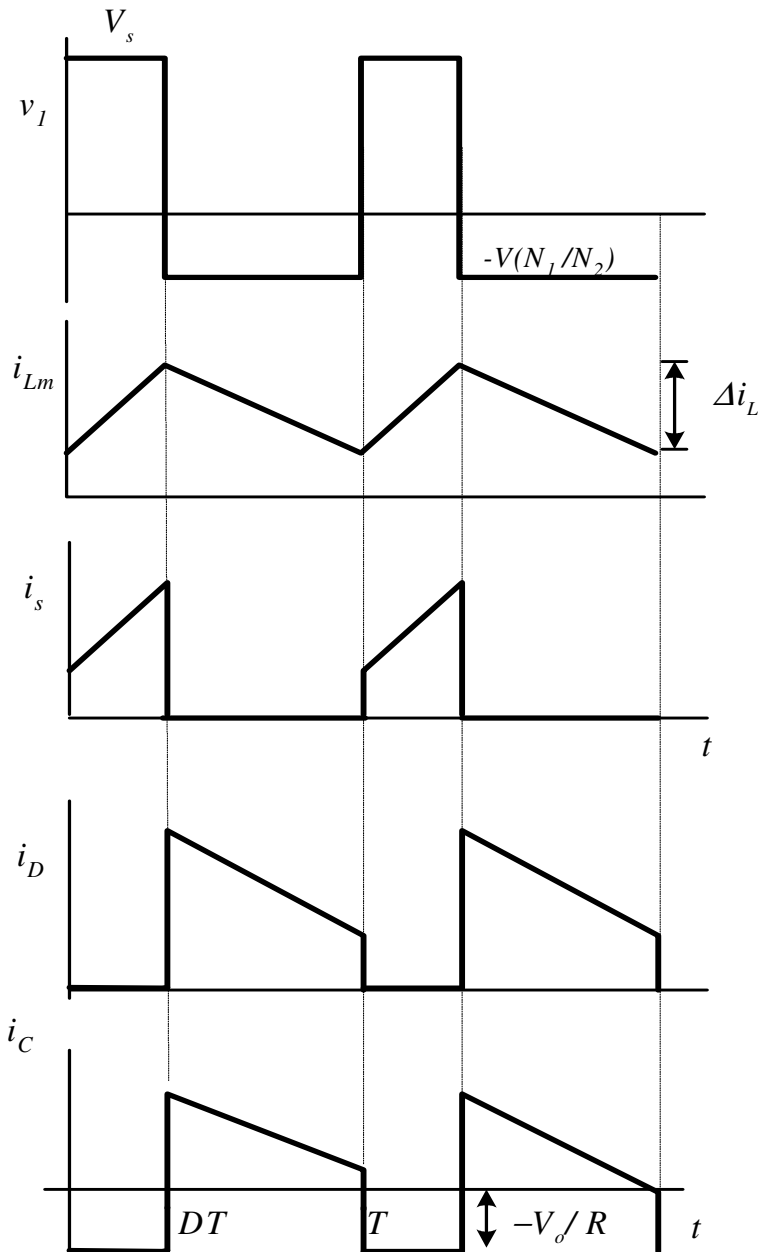
$$V_d (I_{L_m})D = \frac{V_o^2}{R}$$

$$\Rightarrow I_{L_m} = \frac{V_o^2}{V_d DR}$$

I_{L_m} can be written as :

$$I_{L_m} = \frac{V_d D}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 i_c$$

$$= \frac{V_o}{(1-D)R} \left(\frac{N_2}{N_1} \right)$$



Max, Min inductor current

$$I_{L_m, \max} = I_{L_m} + \frac{\Delta i_{L_m}}{2}$$
$$= \frac{V_d D}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 + \frac{V_d D T}{2L_m}$$

$$I_{L_m, \min} = I_{L_m} - \frac{\Delta i_{L_m}}{2}$$
$$= \frac{V_d D}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 - \frac{V_d D T}{2L_m}$$

For CCM, $I_{L_m, \min} = 0$

$$\frac{V_d D}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 = \frac{V_d D T}{2L_m} = \frac{V_d D}{2L_m f}$$

$$\Rightarrow (L_m)_{\min} = \frac{V_d (1-D)^2 R}{2f} \left(\frac{N_1}{N_2} \right)^2$$

Ripple calculation is similar to boost,

$$r = \frac{\Delta V_0}{V_0} = \frac{D}{RCf}$$

Example

The Flyback converter has these specifications:

DC input voltage: 40V

Output voltage: 25V

Duty cycle: 0.5

Rated load: 62.5W

Max peak-peak inductor current ripple:
25% of the average inductor current.

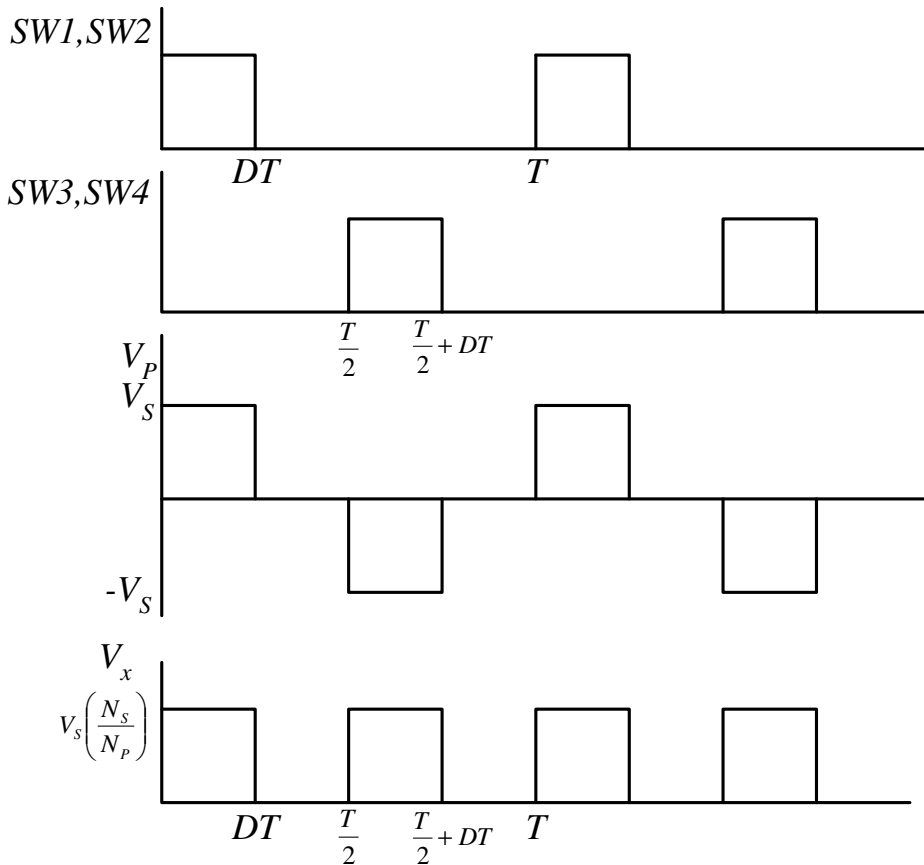
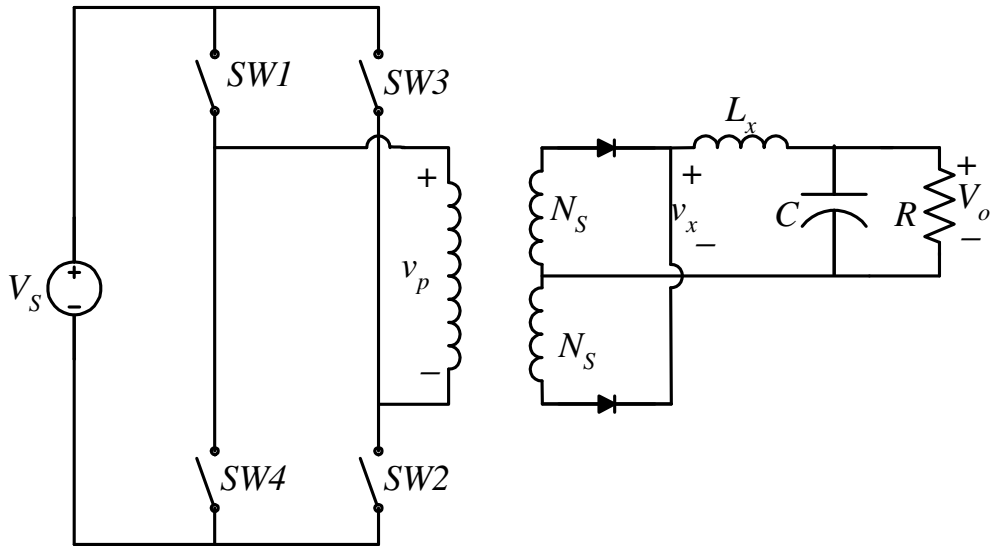
Maximum peak-peak output voltage: 0.1V

Switching frequency: 75kHz

Based on the abovementioned specifications, determine

- a) Transformer turns ratio
- b) Value of magnetizing inductor L_m .
- c) Maximum and minimum inductor current.
- d) Value of capacitor C.

Full-bridge converter



Full bridge: basic operation

- Switch “pair”: [S1 & S2];[S3 & S4].
- Each switch pair turn on at a time as shown. The other pair is off.
- “AC voltage” is developed across the primary. Then transferred to secondary via high frequency transformers.
- On secondary side, diode pair is “high frequency full wave rectification”.
- The choke (L) and (C) acts like the “buck converter” circuit.
- Output Voltage

$$V_o = 2V_s \left(\frac{N_s}{N_p} \right) \cdot D$$