

$$v_0 := 40$$

```
>> cd c:\
```

```
>> global v0
```

```
>> v0 = 40
```

$$y'(t) := v_0 - \frac{9.8}{10} \cdot t$$

```
[File] [New] [M-File]
```

```
function yp = yprime(t)
global v0
```

```
yp = v0 - 9.8*t;
```

```
[File] [Save] yprime.m
```

$$y'(0) = 40$$

```
>> yprime(0)
```

$$y_0 := 0$$

```
>> global y0
```

```
>> y0 = 0
```

$$y(t) := y_0 + \int_0^t y'(t) dt$$

```
[File] [New] [M-File]
```

```
function yval = y(t)
global y0
```

```
for i=1:length(t)
```

```
    yval(i) = y0 + ...
                quad8('yprime',0,t(i));
```

```
end
```

```
[File] [Save] y.m
```

$$y(1) = 35.1$$

```
>> y(1)
```

$t := 0, 0.1 .. 10$

```
>> t = [0 : 0.1 : 10];
```

$y(t)$

$y'(t)$



t, t

```
>> plot(t, y(t), 'r-')
```

```
>> hold on
```

```
>> plot(t, yprime(t), 'k-')
```

```
>> hold off
```

$t := 10$

<-- Initial guess for t

```
>> t = 10;
```

Given

$$y(t) = 0$$

$t := \text{Find}(t)$

```
>> t = fzero('y', t)
```

$t = 8.163$

$$y(t) = 5.354 \cdot 10^{-5}$$

```
>> y(t)
```

$$\text{newt}(f, f', t, \text{tol}) := \begin{array}{l} \text{while } |f(t)| > \text{tol} \\ \quad t \leftarrow t - \frac{f(t)}{f'(t)} \\ t \end{array}$$

[File] [New] [M-File]

```
function root = ...
    newt(f,fprime,t, tol)

    while abs( feval(f,t) ) > tol
        t = t - feval(f,t) / ...
            feval(fprime,t);
    end

    root = t;
```

[File] [Save] newt.m

$$t := 10$$

```
>> t = 10;
```

$$t := \text{newt}(y, y', t, 10^{-5})$$

```
>> t = newt('y','yprime',t,1e-5)
```

$$t = 8.163$$

$$y(t) = -5.479 \cdot 10^{-10}$$

```
>> y(t)
```

$$y(t) := y_0 + \int_0^t y'(t) dt \rightarrow 40 \cdot t - \frac{49}{10} \cdot t^2$$

$$y(1) \rightarrow \frac{351}{10}$$

Given

$$y(t) = 0$$

**Symbolic solutions in
Mathcad and MATLAB
via built-in Maple kernel**

$$y(1) = 35.1$$

$$t := \text{Find}(t) \rightarrow \begin{bmatrix} 0 & \frac{400}{49} \end{bmatrix}$$

$$t^T = \begin{bmatrix} 0 \\ 8.163 \end{bmatrix}$$

$$y'(t, y) := v0 - \frac{98}{10} \cdot t$$

[File] [New] [M-File]

```
function yp = yprime2(t,y)
global v0
```

```
yp = v0 - 9.8*t;
```

[File] [Save] yprime2.m

☞ **Reference:** D:\DATA\mathcad\saw\system.mcd(R)

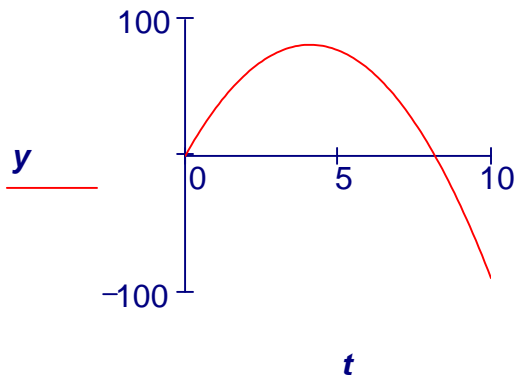
$t := \text{range}(0, 0.1, 10)$

$D(t, y) := \text{system}(t, y, y', 1)$

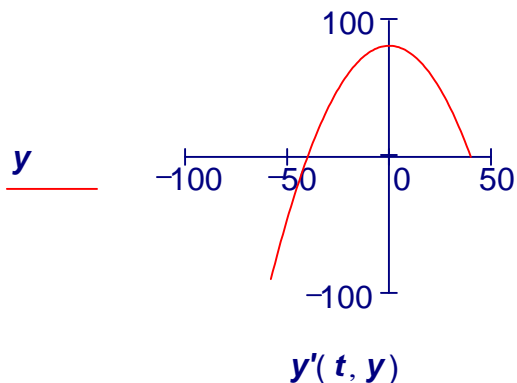
$y := \text{solution}(t, y0, D, 1)$

```
>> t = [0 : 0.1 : 10];
```

```
>> [t,y] = ode45('yprime2',t,y0);
```



```
>> subplot(2,1,1)
>> plot(t, y, 'r-')
```



```
>> subplot(2,1,2)
>> plot(yprime2(t,y), y, 'r-')
```

euler(*t0*, *y0*, *f'*, *tend*, *n*) :=

$$Dt \leftarrow \frac{tend - t0}{n - 1}$$

$$t_1 \leftarrow t0$$

$$y_1 \leftarrow y0$$

for $i \in 2..n$

$$t_i \leftarrow t_{i-1} + Dt$$

$$y_i \leftarrow y_{i-1} + Dt \cdot f'(t_{i-1}, y_{i-1})$$

$$\begin{bmatrix} t \\ y \end{bmatrix}$$

[File] [New] [M-File]

```
function [t,y] = ...
    euler(t0,y0,fprime,tend,n)

delta_t = (tend - t0)/(n - 1);

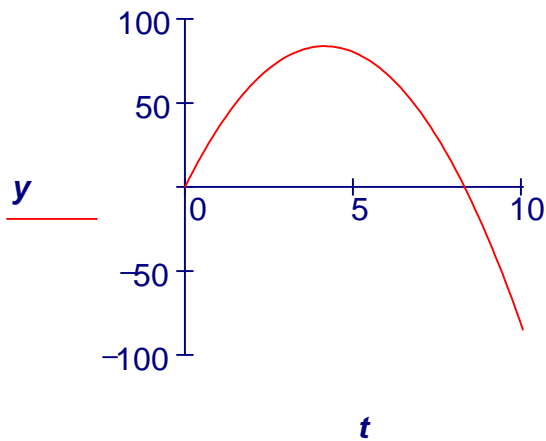
t(1) = t0;
y(1) = y0;

for i=2:n
    t(i) = t(i-1) + delta_t;
    y(i) = y(i-1) + delta_t * ...
        feval(fprime,t(i-1),y(i-1));
end
```

[File] [Save] euler.m

$\begin{bmatrix} t \\ y \end{bmatrix}$:= ***euler***(0, *y0*, *y'*, 10, 100)

[t,y] = ...
euler(0,y0,'yprime2',10,100);



```
>> plot(t, y, 'r-')
```

$$t := \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad v := \begin{bmatrix} 40 \\ 30 \\ 20 \\ 10 \\ 0 \end{bmatrix}$$

```
>> t = [0 1 2 3 4]';
```

```
>> v = [40 30 20 10 0]';
```

$$a := \text{slope}(t, v) \quad a = -10$$

$$b := \text{intercept}(t, v) \quad b = 40$$

```
>> ba = polyfit(t,v,1)
```

```
>> b = ba(1);
```

```
>> a = ba(2);
```

$$\text{ORIGIN} := 1$$

$$M^{<1>} := t^0 \quad M^{<2>} := t^1$$

$$M = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \begin{bmatrix} b \\ a \end{bmatrix} := (M^T \cdot M)^{-1} \cdot M^T \cdot v$$

$$a = -10 \quad b = 40$$

```
>> M = [t.^0 t.^1]
```

```
>> ba = inv(M'*M)*(M'*v)
```

```
>> b = ba(1);
```

```
>> a = ba(2);
```