Chapter 15
Power And Harmonics in Nonsinusoidal Systems

15.1. Average power

in terms of Fourier series

15.2. RMS value of a waveform

15.3. Power factor

THD

Distortion and Displacement factors

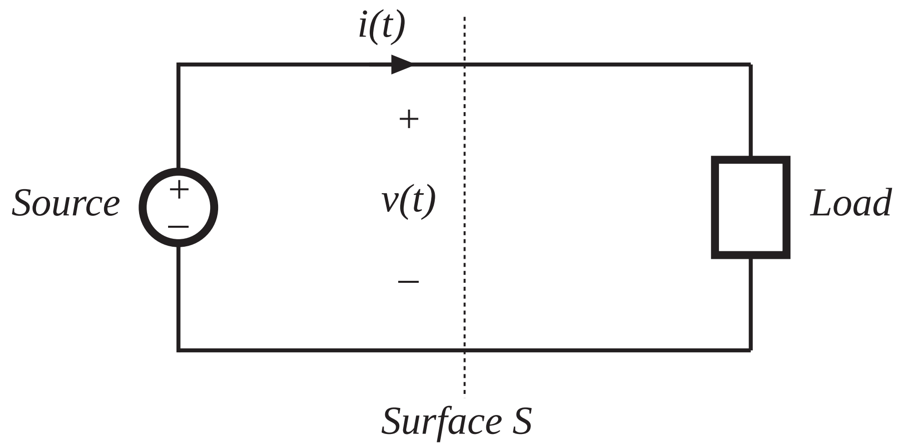
15.4. Power phasors in sinusoidal systems

15.5. Harmonic currents in three-phase systems

15.6. AC line current harmonic standards

15.1. Average power

Observe transmission of energy through surface S



Express voltage and current as Fourier series:

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)$$

relate energy transmission to harmonics

Energy transmittted to load, per cycle

$$W_{cycle} = \int_0^T v(t)i(t)dt$$

This is related to average power as follows:

$$P_{av} = \frac{W_{cycle}}{T} = \frac{1}{T}\int_0^T v(t)i(t)dt$$

Investigate influence of harmonics on average power: substitute Fourier series

$$P_{av} = \frac{1}{T}\int_0^T \left(V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)\right)dt$$

Evaluation of integral

Orthogonality of harmonics: Integrals of cross-product terms are zero

$$\int_0^T \left(V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_m \cos\left(m\omega t - \theta_m\right)\right)dt = \begin{cases} 0 & \text{if } n \neq m \\ \dfrac{V_n I_n}{2}\cos\left(\varphi_n - \theta_n\right) & \text{if } n = m \end{cases}$$

Expression for average power becomes

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos\left(\varphi_n - \theta_n\right)$$

So net energy is transmitted to the load only when the Fourier series of $v(t)$ and $i(t)$ contain terms at the same frequency. For example, if the voltage and current both contain third harmonic, then they lead to the average power

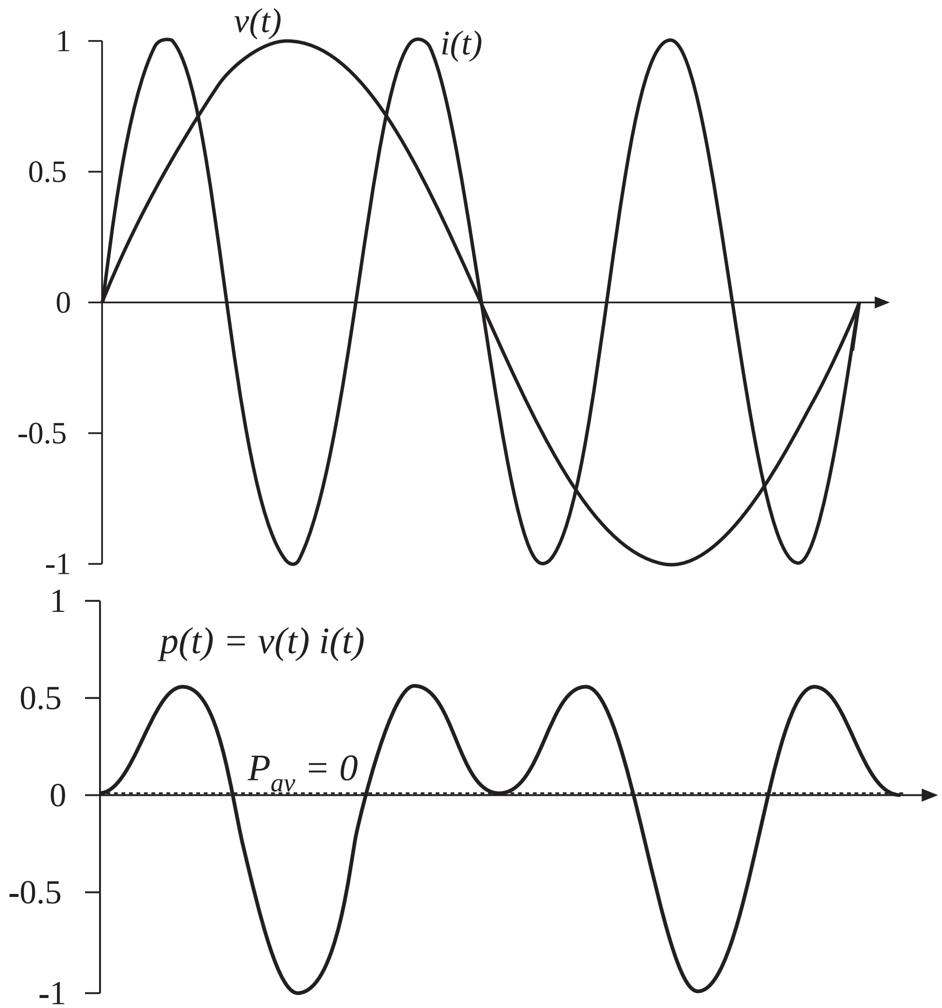
$$\frac{V_3 I_3}{2}\cos\left(\varphi_3 - \theta_3\right)$$

Example 1

Voltage: fundamental only

Current: third harmonic only

Power: zero average

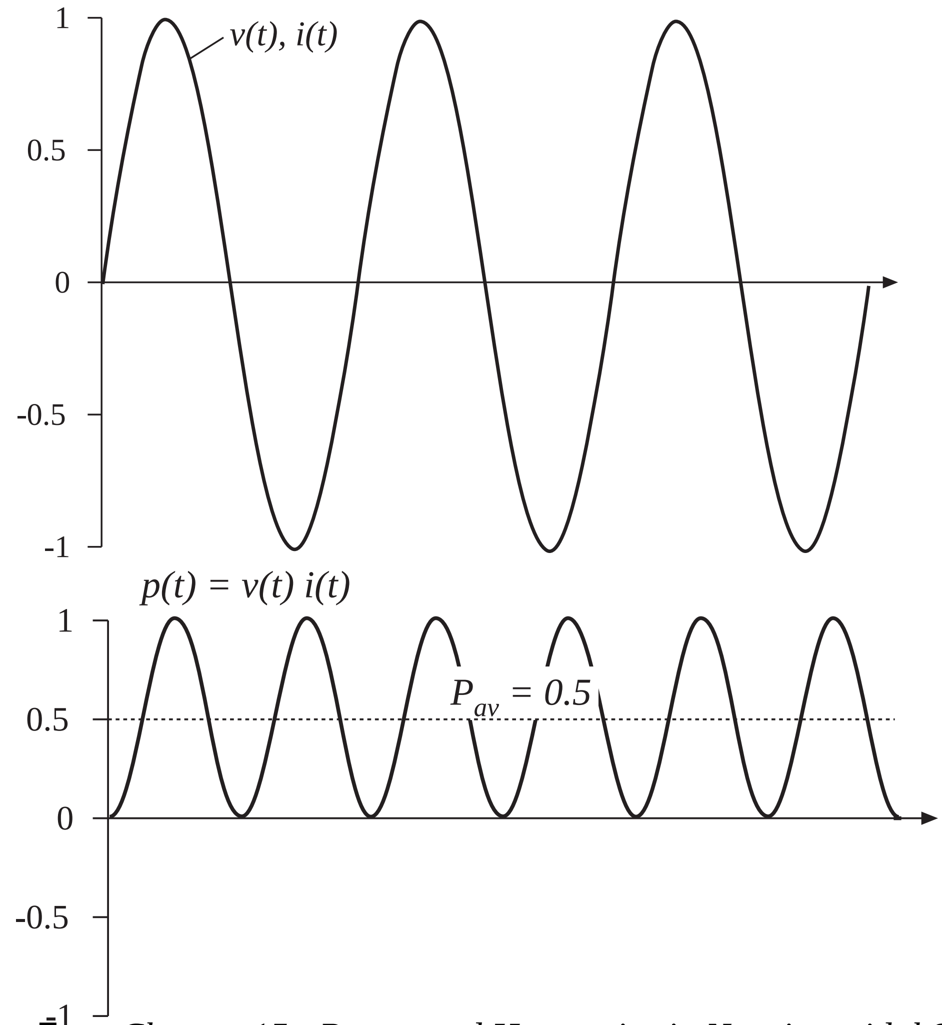


Example 2

Voltage: third harmonic only

Current: third harmonic only, in phase with voltage

Power: nonzero average



Example 3

Fourier series:

$$v(t) = 1.2 \cos(\omega t) + 0.33 \cos(3\omega t) + 0.2 \cos(5\omega t)$$

$$i(t) = 0.6 \cos(\omega t + 30°) + 0.1 \cos(5\omega t + 45°) + 0.1 \cos(7\omega t + 60°)$$

Average power calculation:

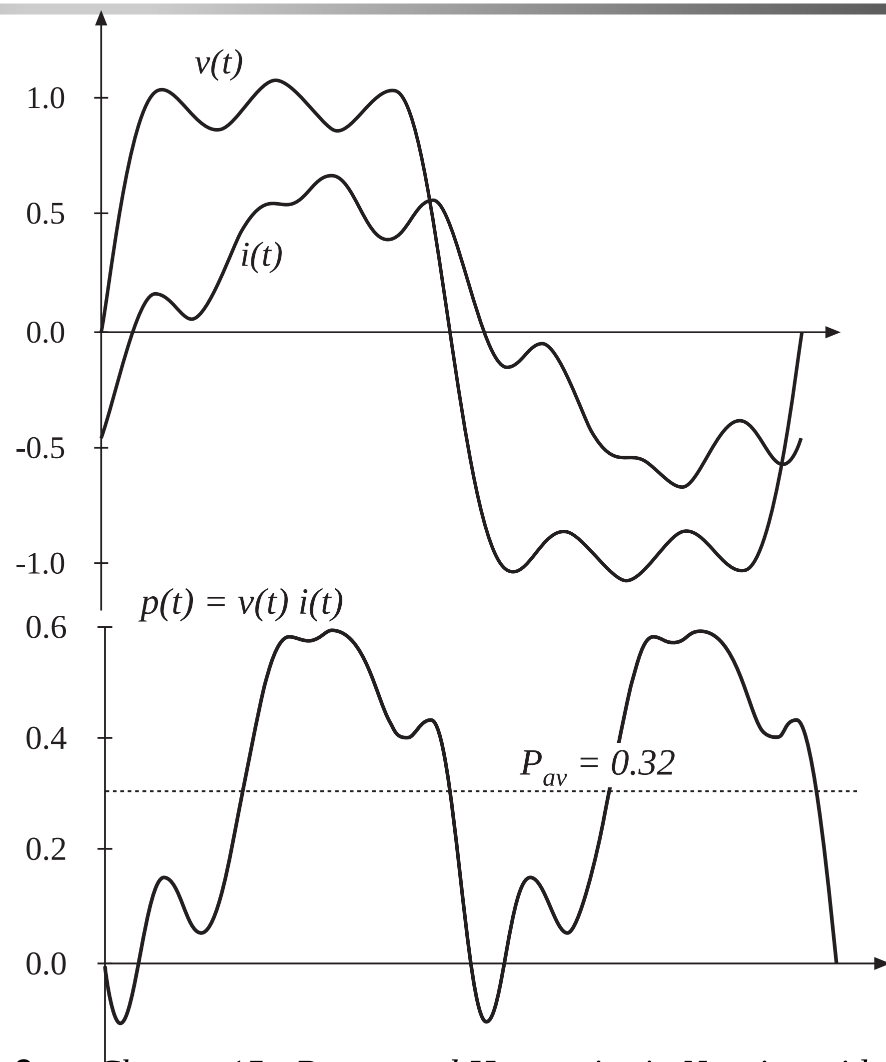
$$P_{av} = \frac{(1.2)(0.6)}{2} \cos(30°) + \frac{(0.2)(0.1)}{2} \cos(45°) = 0.32$$

Example 3

Voltage: 1st, 3rd, 5th

Current: 1st, 5th, 7th

Power: net energy is transmitted at fundamental and fifth harmonic frequencies



15.2. Root-mean-square (RMS) value of a waveform, in terms of Fourier series

$$\text{(rms value)} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

Insert Fourier series. Again, cross-multiplication terms have zero average. Result is

$$\text{(rms value)} = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

- Similar expression for current
- Harmonics always increase rms value
- Harmonics do not necessarily increase average power
- Increased rms values mean increased losses

15.3. Power factor

For efficient transmission of energy from a source to a load, it is desired to maximize average power, while minimizing rms current and voltage (and hence minimizing losses).

Power factor is a figure of merit that measures how efficiently energy is transmitted. It is defined as

$$\text{power factor} = \frac{\text{(average power)}}{\text{(rms voltage) (rms current)}}$$

Power factor always lies between zero and one.

15.3.1. Linear resistive load, nonsinusoidal voltage

Then current harmonics are in phase with, and proportional to, voltage harmonics. All harmonics result in transmission of energy to load, and unity power factor occurs.

$$I_n = \frac{V_n}{R} \qquad \theta_n = \varphi_n \qquad \text{so} \quad \cos(\theta_n - \varphi_n) = 1$$

$$(\text{rms voltage}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}} = \sqrt{\frac{V_0^2}{R^2} + \sum_{n=1}^{\infty} \frac{V_n^2}{2R^2}}$$

$$= \frac{1}{R}(\text{rms voltage})$$

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\varphi_n - \theta_n)$$

15.3.2. Nonlinear dynamical load, sinusoidal voltage

With a sinusoidal voltage, current harmonics do not lead to average power. However, current harmonics do increase the rms current, and hence they decrease the power factor.

$$P_{av} = \frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_1)$$

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}$$

$$(\text{power factor}) = \left(\frac{\frac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) \left(\cos(\varphi_1 - \theta_1) \right)$$

$$= (\text{distortion factor})\,(\text{displacement factor})$$

Distortion factor

Defined only for sinusoidal voltage.

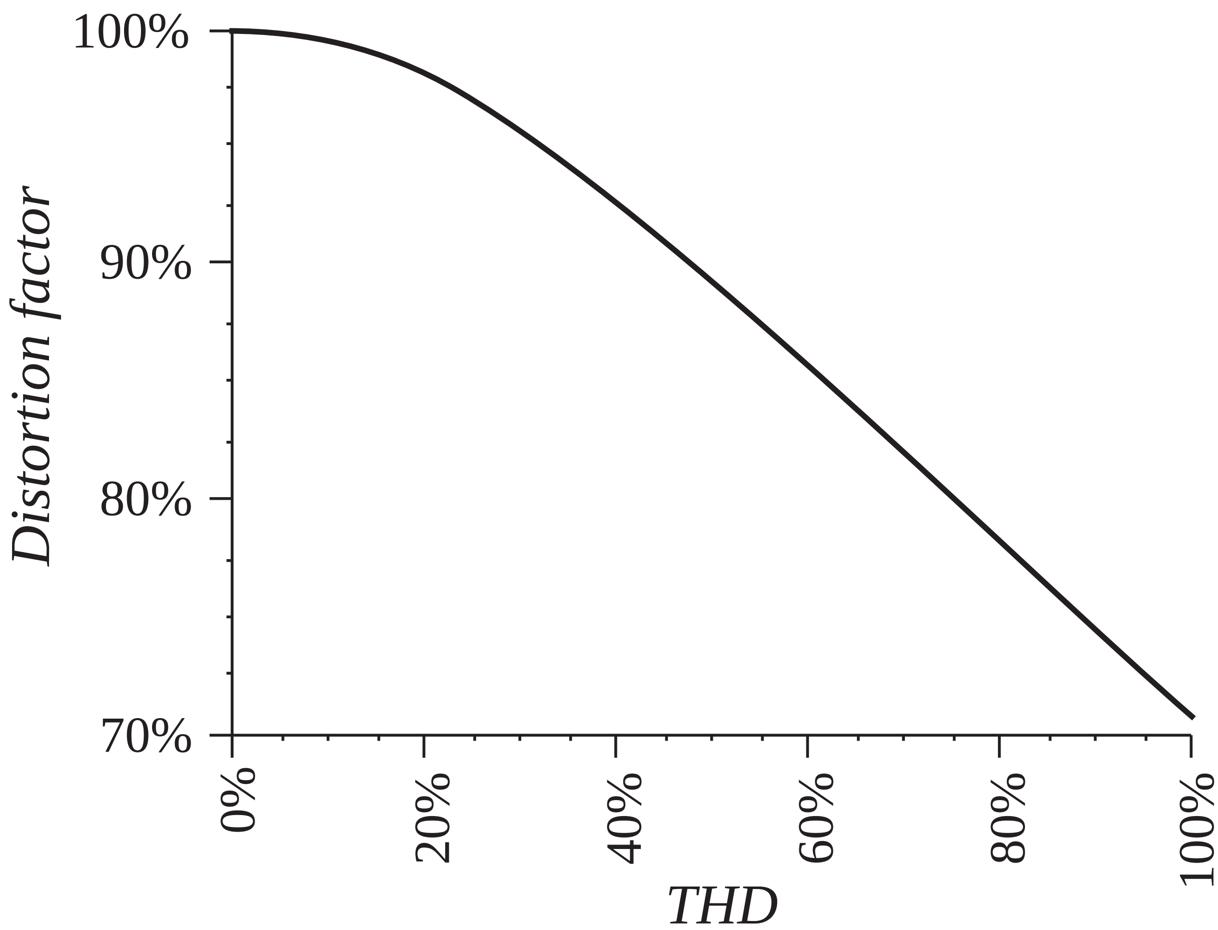
$$(\text{distortion factor}) = \left(\frac{\frac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) = \frac{(\text{rms fundamental current})}{(\text{rms current})}$$

Related to Total Harmonic Distortion (THD):

$$(\text{THD}) = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1}$$

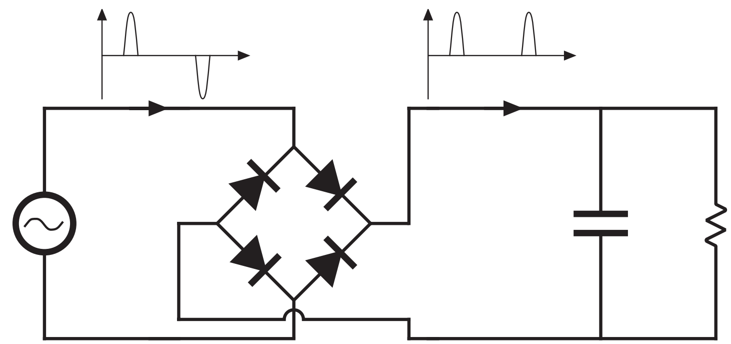
$$(\text{distortion factor}) = \frac{1}{\sqrt{1 + (\text{THD})^2}}$$

Distortion factor vs. THD



Peak detection rectifier example

Conventional single-phase peak detection rectifier



Typical ac line current spectrum