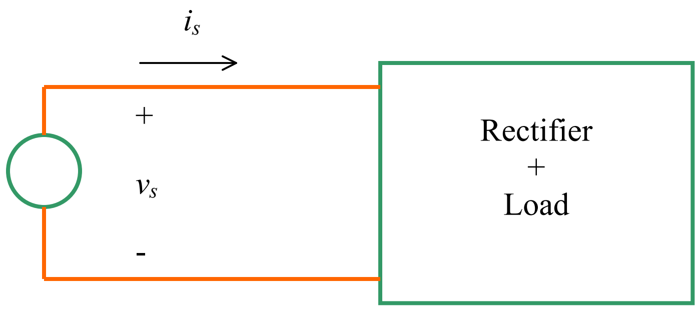
Power Factor: Summary



There are three cases to be considered:

<u>Case I:</u>

v_s – Nonsinusoidal
i_s – Nonsinusoidal

Voltage and current can be expressed in Fourier series as:

$$v_s = V_o + \sum_{n=1}^{\infty} V_n \sin(n\omega t - \varphi_n)$$

$$i_s = I_o + \sum_{n=1}^{\infty} I_n \sin(n\omega t - \theta_n)$$

Power factor, $$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

Where $$V_{rms} = \sqrt{V_o^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}$$

and $$P_{avg} = V_o I_o + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\varphi_n - \theta_n)$$

Case II:

v_s – Sinusoidal
i_s – Nonsinusoidal

Voltage and current can be expressed in Fourier series as:

$$v_s = V_1 \sin(\omega t - \varphi_1)$$

$$i_s = I_o + \sum_{n=1}^{\infty} I_n \sin(n\omega t - \theta_n)$$

Power factor, $$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

Where $$V_{rms} = \frac{V_1}{\sqrt{2}}$$

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}$$

and $$P_{avg} = \frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_1)$$

$$\therefore pf = \frac{\frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_a)}{\frac{V_1}{\sqrt{2}} \sqrt{I_o^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}}$$

$$pf = \frac{\frac{I_1}{\sqrt{2}} \cos(\varphi_1 - \theta_1)}{\sqrt{I_o^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}}$$

$$pf = \frac{I_{1,rms}}{I_{rms}} \cos(\varphi_1 - \theta_1)$$

= (distortion factor)(displacement factor)

Case III:

v_s – Sinusoidal
i_s – Sinusoidal

Voltage and current can be expressed in Fourier series as:

$$v_s = V_1 \sin(\omega t - \varphi_1)$$
$$i_s = I_1 \sin(\omega t - \theta_1)$$

Power factor, $$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

Where $$V_{rms} = \frac{V_1}{\sqrt{2}}$$

$$I_{rms} = \frac{I_1}{\sqrt{2}}$$

and $$P_{avg} = \frac{V_1 I_1}{2}\cos(\varphi_1 - \theta_1)$$

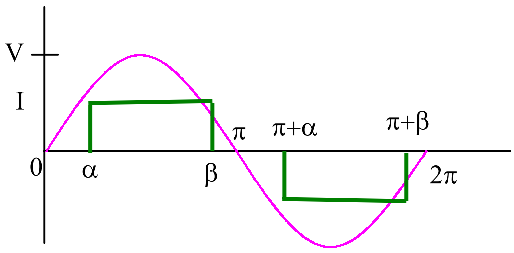
$$\therefore pf = \frac{\frac{V_1 I_1}{2}\cos(\varphi_1 - \theta_1)}{\frac{V_1}{\sqrt{2}}\frac{I_1}{\sqrt{2}}}$$

$$pf = \cos(\varphi_1 - \theta_1)$$

= (displacement factor)

Power Factor Examples

Example 1.0 (Basic Principles)



$$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \;\; ; \; I_{rms} = I\sqrt{\frac{\beta - \alpha}{\pi}}$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{\pi} v(\omega t) i(\omega t) d(\omega t)$$

$$= \frac{1}{\pi}\int_{\alpha}^{\beta} V \sin(\omega t) I d(\omega t)$$

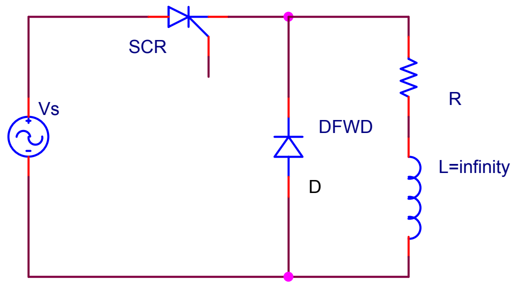
$$= \frac{VI}{\pi}\int_{\alpha}^{\beta} \sin(\omega t) d(\omega t) = \frac{VI}{\pi}\left[-\cos(\omega t)\right]_{\alpha}^{\beta}$$

$$= \frac{VI}{\pi}\left[\cos(\alpha) - \cos(\beta)\right]$$

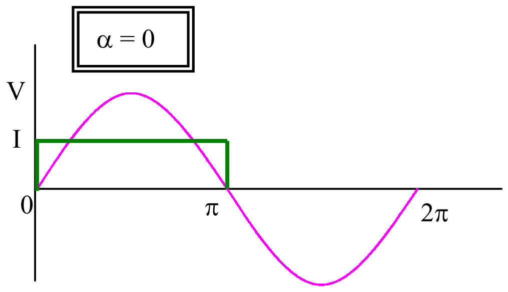
$$\therefore pf = \frac{P_{avg}}{V_{rms} I_{rms}} = \frac{\frac{VI}{\pi}\left[\cos\alpha - \cos\beta\right]}{\frac{VI}{\sqrt{2}}\sqrt{\frac{\beta - \alpha}{\pi}}}$$

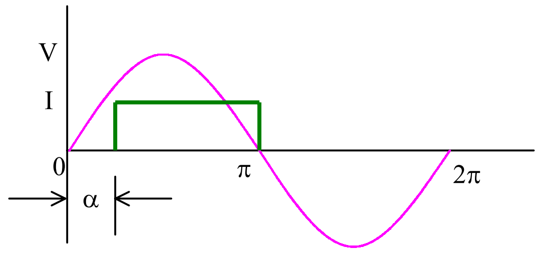
$$= \sqrt{\frac{2}{\pi(\beta - \alpha)}}\left[\cos\alpha - \cos\beta\right]$$

Example 2.0



Half-Wave Phase-controlled
Rectifier with RL Load and FWD





$$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \;;\; I_{rms} = I\sqrt{\frac{\pi}{2\pi}} = \frac{I}{\sqrt{2}}$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t) d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{\pi} V \sin(\omega t) I d(\omega t)$$

$$= \frac{VI}{2\pi}\int_0^{\pi} \sin(\omega t) d(\omega t)$$

$$= \frac{VI}{\pi}$$

$$\therefore pf = \frac{P_{avg}}{V_{rms} I_{rms}} = \frac{VI\sqrt{2}\sqrt{2}}{\pi VI}$$

$$= \frac{2}{\pi}$$

$$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \;;\; I_{rms} = I\sqrt{\frac{\pi - \alpha}{2\pi}}$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t) d(\omega t)$$

$$= \frac{1}{2\pi}\int_\alpha^{\pi} V \sin(\omega t) I d(\omega t)$$

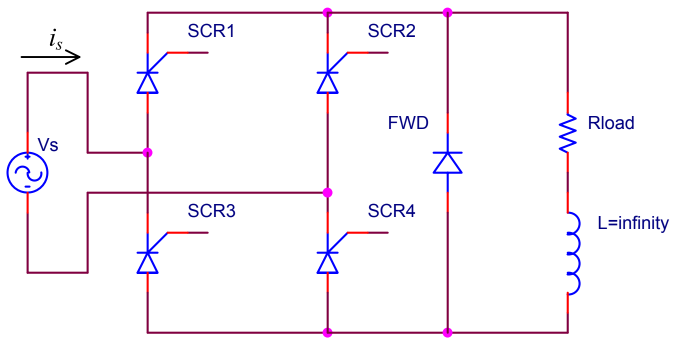
$$= \frac{VI}{2\pi}\int_\alpha^{\pi} \sin(\omega t) d(\omega t)$$

$$= \frac{VI}{2\pi}[1 + \cos\alpha]$$

$$\therefore pf = \frac{P_{avg}}{V_{rms} I_{rms}} = \frac{\frac{VI}{2\pi}[1+\cos\alpha]}{\frac{V}{\sqrt{2}} I\sqrt{\frac{\pi-\alpha}{2\pi}}}$$

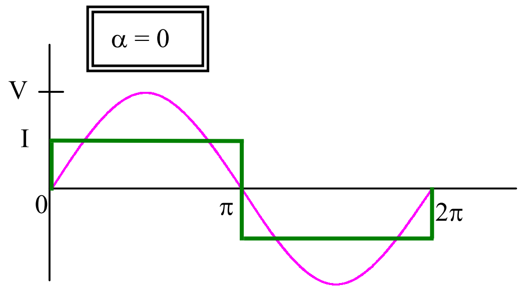
$$= \frac{[1+\cos\alpha]}{\sqrt{\pi(\pi - \alpha)}}$$

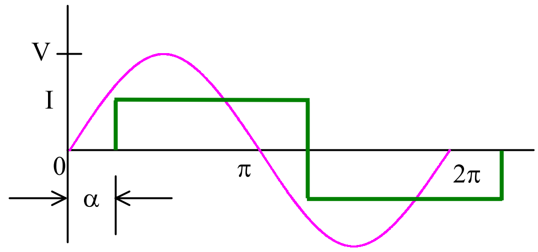
Example 3.0



Full-Wave Phase-controlled Rectifier with RL Load

$$i_s = \begin{cases} i_{SCR1} & when\ SCR1\ on \\ -i_{SCR3} & when\ SCR3\ on \end{cases}$$





$$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}}\ ;\ I_{rms} = I$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t) d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} V \sin(\omega t) I d(\omega t)$$

$$= \frac{VI}{\pi}\int_0^{\pi} \sin(\omega t) d(\omega t)$$

$$= \frac{2VI}{\pi}$$

$$\therefore pf = \frac{P_{avg}}{V_{rms} I_{rms}} = \frac{\frac{2VI}{\pi}}{\frac{VI}{\sqrt{2}}}$$

$$= \frac{2\sqrt{2}}{\pi}$$

$$pf = \frac{P_{avg}}{V_{rms} I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}}\ ;\ I_{rms} = I$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t) d(\omega t)$$

$$= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V \sin(\omega t) I d(\omega t)$$

$$= \frac{VI}{\pi}\int_{\alpha}^{\pi+\alpha} \sin(\omega t) d(\omega t)$$

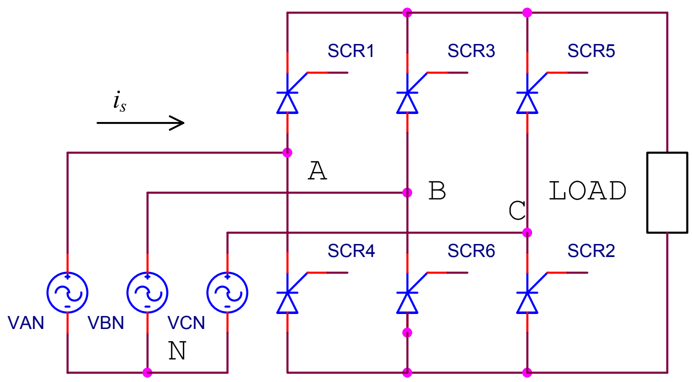
$$= \frac{2VI}{\pi}\cos\alpha$$

$$\therefore pf = \frac{P_{avg}}{V_{rms} I_{rms}} = \frac{\frac{2VI}{\pi}\cos\alpha}{\frac{VI}{\sqrt{2}}}$$

$$= \frac{2\sqrt{2}}{\pi}\cos\alpha$$

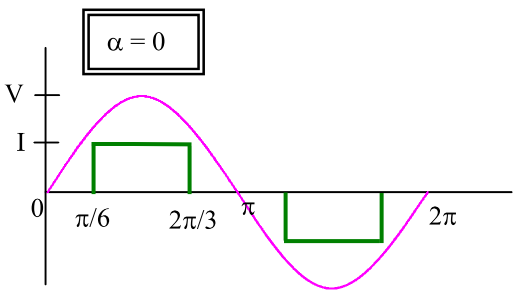
Note: The term cos (α) is actually a term for displacement factor.

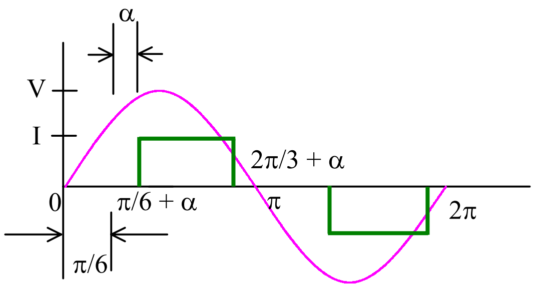
Example 4.0



Three Phase Full-Wave
Phase-controlled Rectifier

$$i_s = \begin{cases} i_{SCR1} & when\ SCR1\ on \\ -i_{SCR4} & when\ SCR4\ on \end{cases}$$





$$pf = \frac{P_{avg}}{V_{rms}I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \ ; \ I_{rms} = I\sqrt{\frac{2}{3}}$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t)i(\omega t)d(\omega t)$$

$$= \frac{1}{\pi}\int_{\pi/6}^{2\pi/3} V\sin(\omega t)Id(\omega t)$$

$$= \frac{VI}{\pi}\int_{\pi/6}^{2\pi/3} \sin(\omega t)d(\omega t)$$

$$= \frac{\sqrt{3}VI}{\pi}$$

$$\therefore pf = \frac{P_{avg}}{V_{rms}I_{rms}} = \frac{\dfrac{\sqrt{3}VI}{\pi}}{\dfrac{V}{\sqrt{2}}\dfrac{I\sqrt{2}}{\sqrt{3}}}$$

$$= \frac{3}{\pi}$$

$$pf = \frac{P_{avg}}{V_{rms}I_{rms}}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \ ; \ I_{rms} = I\sqrt{\frac{2}{3}}$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t)i(\omega t)d(\omega t)$$

$$= \frac{1}{\pi}\int_{\pi/6+\alpha}^{2\pi/3+\alpha} V\sin(\omega t)Id(\omega t)$$

$$= \frac{VI}{\pi}\int_{\pi/6+\alpha}^{2\pi/3+\alpha} \sin(\omega t)d(\omega t)$$

$$= \frac{\sqrt{3}VI}{\pi}\cos\alpha$$

$$\therefore pf = \frac{P_{avg}}{V_{rms}I_{rms}} = \frac{\dfrac{\sqrt{3}VI}{\pi}\cos\alpha}{\dfrac{V}{\sqrt{2}}\dfrac{I\sqrt{2}}{\sqrt{3}}}$$

$$= \frac{3}{\pi}\cos\alpha$$

Note: The term cos (α) is actually a term for displacement factor.

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