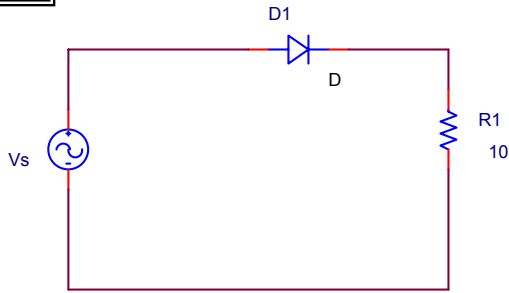
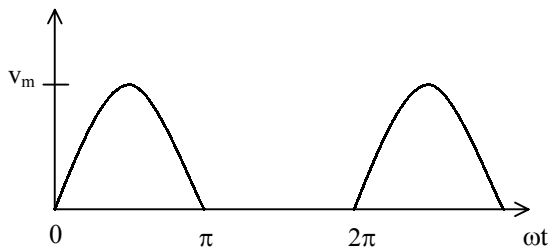


Some Average values of the Output Voltages

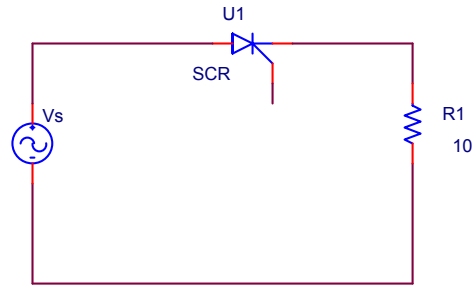
No.1



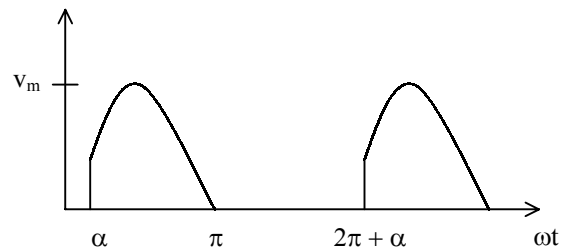
1. Half-Wave Rectifier with R Load



$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } 0 < \omega t < \pi \\ 0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$



1a. Half-Wave Phase-controlled Rectifier with R Load

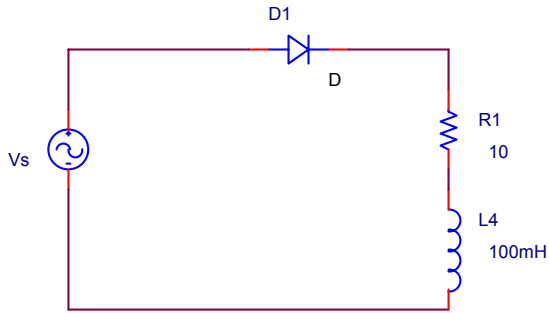


$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } \alpha < \omega t < \pi \\ 0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

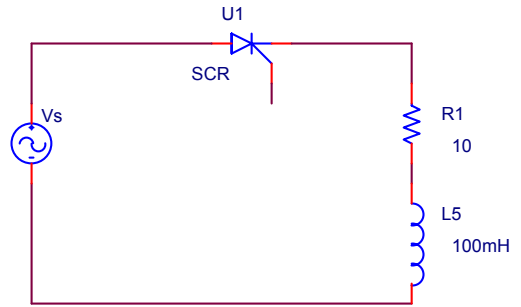
$$\begin{aligned} v_{o,avg} &= \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{2\pi} [-\cos(\omega t)]_{\alpha}^{\pi} \\ &= \frac{V_m}{2\pi} [-\cos(\pi) + \cos(\alpha)] \\ &= \frac{V_m}{2\pi} [1 + \cos(\alpha)] \end{aligned}$$

When $\alpha = 0$, $V_{o,avg} = \frac{V_m}{2\pi}$

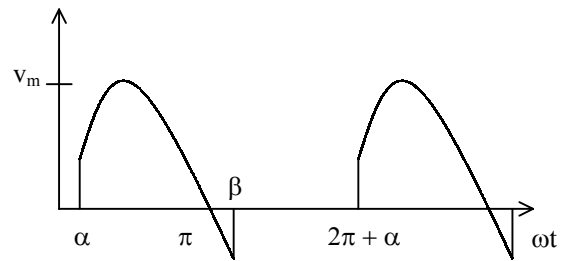
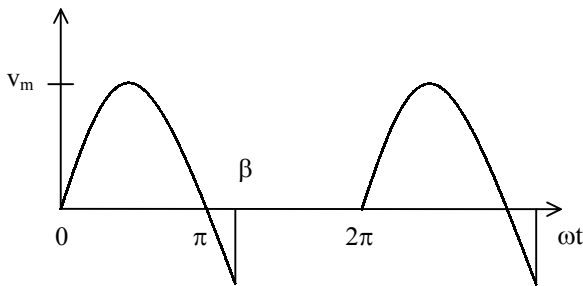
No.2



2. Half-Wave Rectifier with RL Load



2a. Half-Wave Phase-controlled Rectifier with RL Load



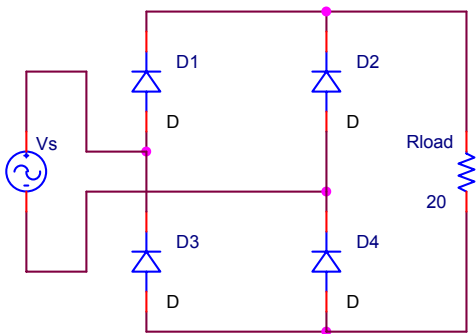
$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } 0 < \omega t < \beta \\ 0 & \text{for } \beta < \omega t < 2\pi \end{cases}$$

$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } \alpha < \omega t < \beta \\ 0 & \text{for } \beta < \omega t < 2\pi + \alpha \end{cases}$$

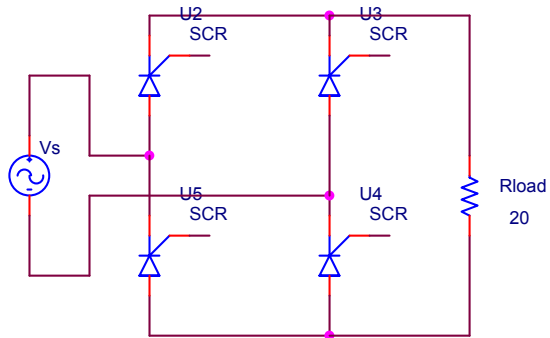
$$\begin{aligned} v_{o,avg} &= \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{2\pi} [-\cos(\omega t)]_{\alpha}^{\beta} \\ &= \frac{V_m}{2\pi} [-\cos(\beta) + \cos(\alpha)] \\ &= \frac{V_m}{2\pi} [\cos(\alpha) - \cos(\beta)] \end{aligned}$$

$$\text{When } \alpha = 0, V_{o,avg} = \frac{V_m}{2\pi} [1 - \cos(\beta)]$$

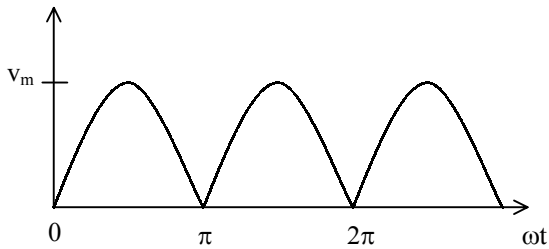
No.3



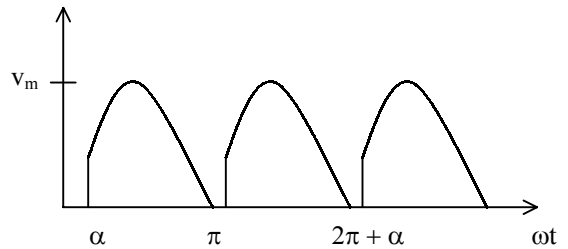
3. Full-Wave Rectifier with R Load



3a. Full-Wave Phase-controlled Rectifier with R Load



$$v_o = V_m \sin(\omega t) \quad \text{for } 0 \leq \omega t < \pi$$

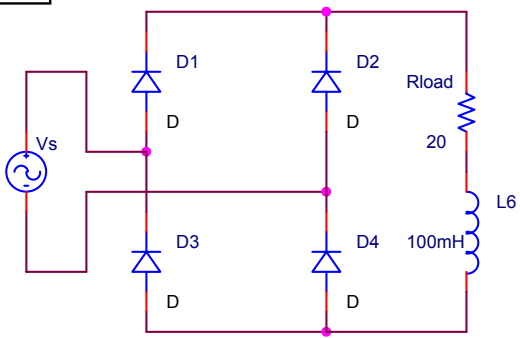


$$v_o = V_m \sin(\omega t) \quad \text{for } \alpha \leq \omega t < \pi$$

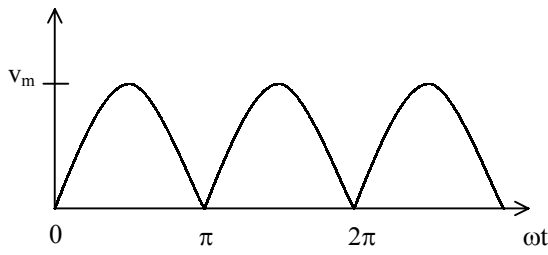
$$\begin{aligned} v_{o,avg} &= \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{\pi} [-\cos(\omega t)]_{\alpha}^{\pi} \\ &= \frac{V_m}{\pi} [-\cos(\pi) + \cos(\alpha)] \\ &= \frac{V_m}{\pi} [1 + \cos(\alpha)] \end{aligned}$$

$$\text{When } \alpha = 0, V_{o,avg} = \frac{V_m}{\pi}$$

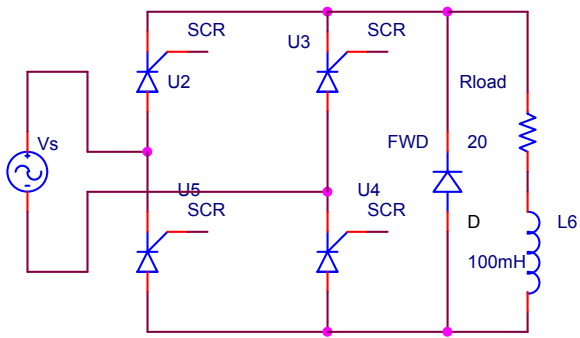
No.4



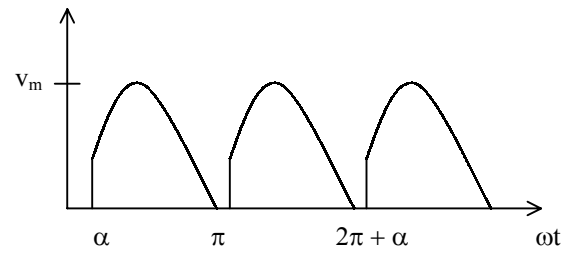
4. Full-Wave Rectifier with RL Load



$$v_o = V_m \sin(\omega t) \quad \text{for } 0 \leq \omega t < \pi$$



4a. Full-Wave Phase-controlled Rectifier with RL Load and FWD



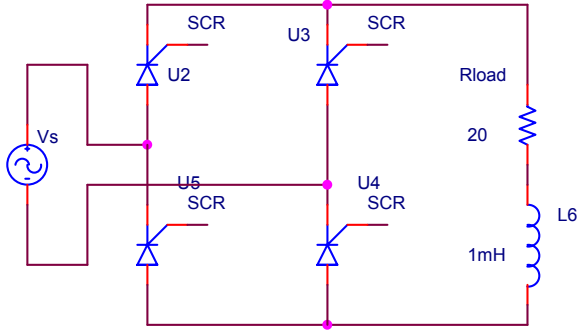
$$v_o = V_m \sin(\omega t) \quad \text{for } \alpha \leq \omega t < \pi$$

$$\begin{aligned} v_{o,avg} &= \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{\pi} [-\cos(\omega t)]_{\alpha}^{\pi} \\ &= \frac{V_m}{\pi} [-\cos(\pi) + \cos(\alpha)] \\ &= \frac{V_m}{\pi} [1 + \cos(\alpha)] \end{aligned}$$

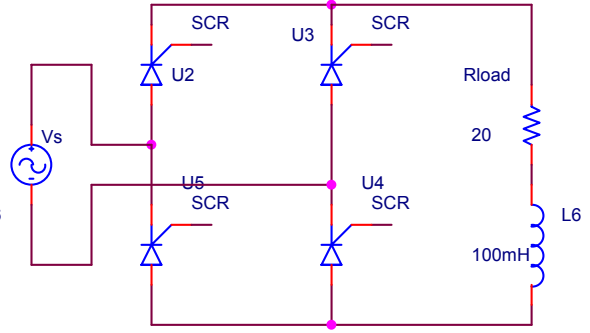
$$\text{When } \alpha = 0, V_{o,avg} = \frac{V_m}{\pi}$$

same as in No. 3

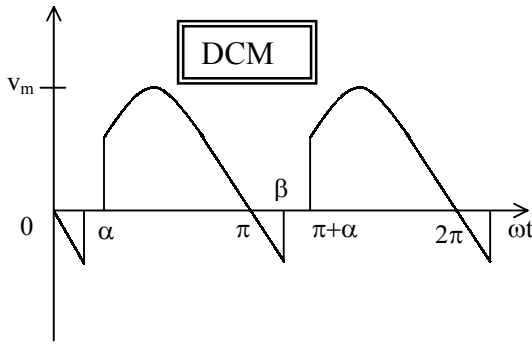
No.5



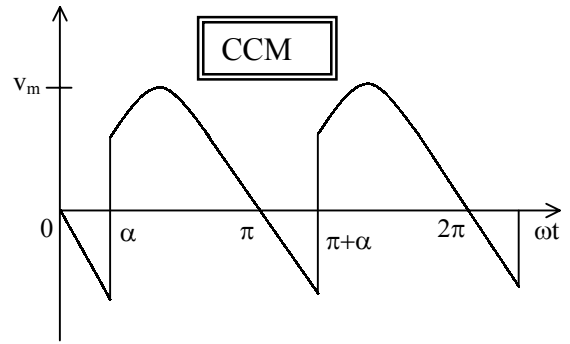
5. Full-Wave Phase-controlled Rectifier with RL Load (Small L)



5a. Full-Wave Phase-controlled Rectifier with RL Load (Large L)



$$v_o = V_m \sin(\omega t) \quad \text{for } \alpha \leq \omega t < \beta$$



$$v_o = V_m \sin(\omega t) \quad \text{for } \alpha \leq \omega t < \pi + \alpha$$

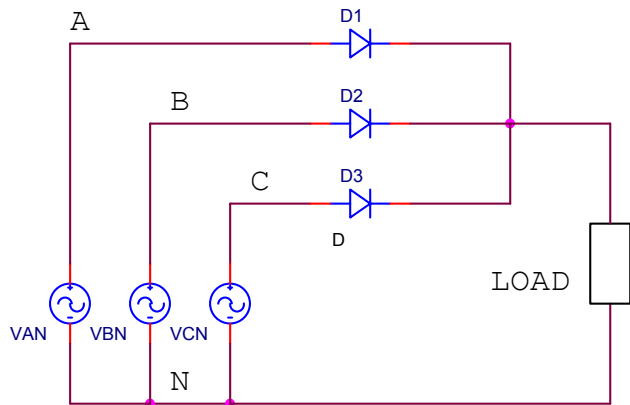
$$\begin{aligned} v_{o,avg} &= \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{\pi} [-\cos(\omega t)]_{\alpha}^{\beta} \\ &= \frac{V_m}{\pi} [-\cos(\beta) + \cos(\alpha)] \\ &= \frac{V_m}{\pi} [\cos(\alpha) - \cos(\beta)] \end{aligned}$$

$$\begin{aligned} v_{o,avg} &= \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{\pi} [-\cos(\omega t)]_{\alpha}^{\pi+\alpha} \\ &= \frac{V_m}{\pi} [-\cos(\pi + \alpha) + \cos(\alpha)] \\ &= \frac{2V_m}{\pi} \cos(\alpha) \end{aligned}$$

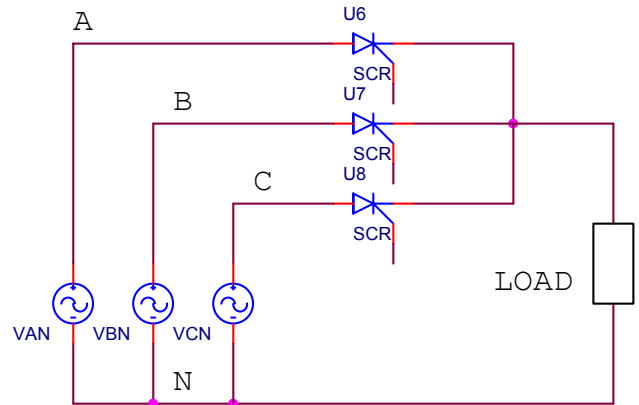
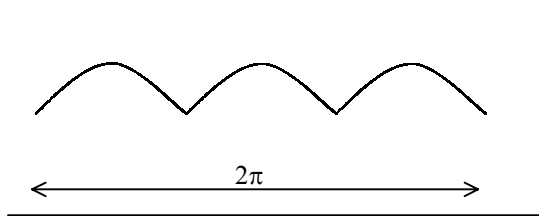
DCM/CCM Boundary:

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{for CCM}$$

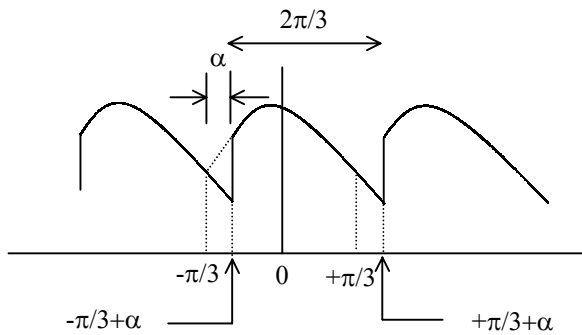
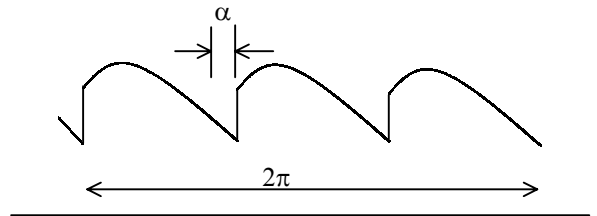
No.6



6. Three Phase Half-Wave Rectifier



6a. Three Phase Half-Wave Phase-controlled Rectifier

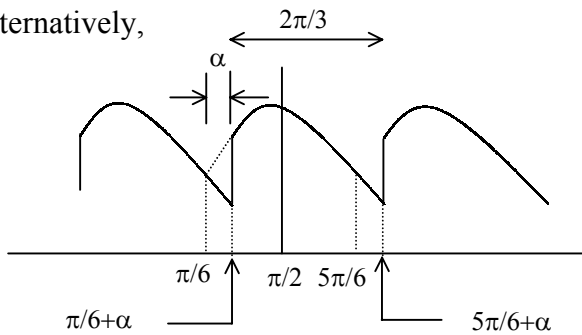


$$v_{o,avg} = \frac{1}{2\pi/3} \int_{-\pi/3+\alpha}^{+\pi/3+\alpha} V_m \cos(\omega t) d(\omega t)$$

$$= \frac{3V_m}{2\pi} \left[\sin(\omega t) \right]_{-\pi/3+\alpha}^{+\pi/3+\alpha}$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)$$

Alternatively,



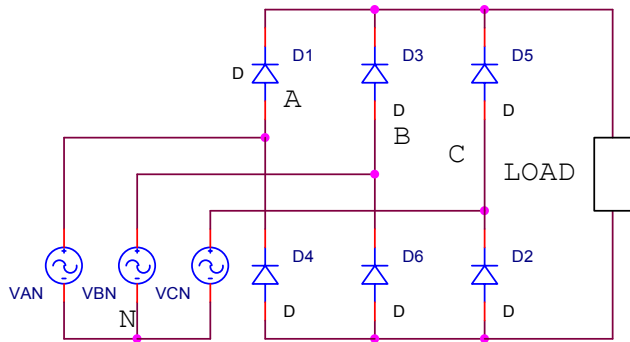
$$v_{o,avg} = \frac{1}{2\pi/3} \int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin(\omega t) d(\omega t)$$

$$= \frac{3V_m}{2\pi} \left[-\cos(\omega t) \right]_{\pi/6+\alpha}^{5\pi/6+\alpha}$$

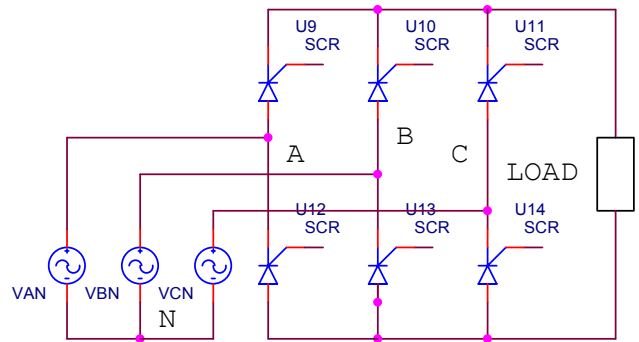
$$= \frac{3\sqrt{3}V_m}{\pi} \cos(\alpha)$$

Note: V_m is the amplitude of line to neutral voltage (phase voltage).

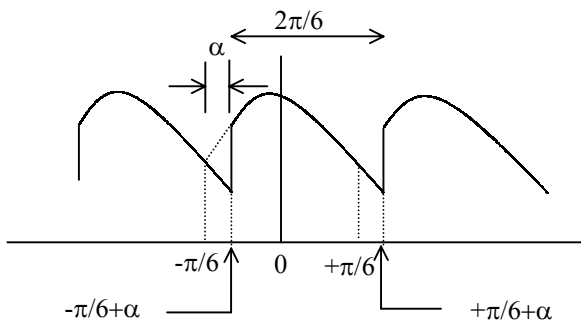
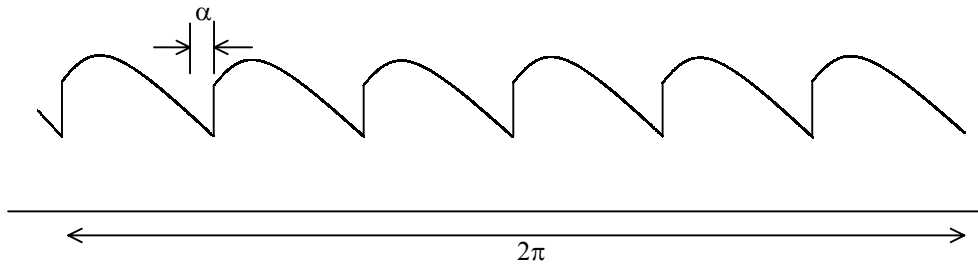
No.7



7. Three Phase Full-Wave Rectifier



7a. Three Phase Full-Wave Phase-controlled Rectifier

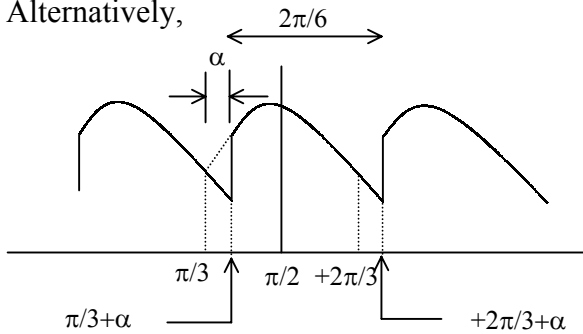


$$v_{o,avg} = \frac{1}{2\pi/6} \int_{-\pi/6+\alpha}^{\pi/6+\alpha} V_m \cos(\omega t) d(\omega t)$$

$$= \frac{3V_m}{\pi} [\sin(\omega t)]_{-\pi/6+\alpha}^{\pi/6+\alpha}$$

$$= \frac{3V_m}{\pi} \cos(\alpha)$$

Alternatively,



$$v_{o,avg} = \frac{1}{2\pi/6} \int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_m \sin(\omega t) d(\omega t)$$

$$= \frac{3V_m}{\pi} [-\cos(\omega t)]_{\pi/3+\alpha}^{2\pi/3+\alpha}$$

$$= \frac{3V_m}{\pi} \cos(\alpha)$$

Note: V_m is the amplitude of line to line voltage (line voltage).