Test #2: Solution

Version: 2/4Jan17

Dummy unit: $\Omega \equiv 1$ $A \equiv 1$ $V \equiv 1$ $W \equiv 1$ $kW \equiv 1 \cdot 10^3 \cdot W$ $Hz \equiv 1$

$Nm \equiv 1$ $rpm \equiv 1$

Q1: DC Machine - speed of the motor under full load condition

Under full load: $P_{Rated} := 10.0kW$ $E_{aRated} := 100V$ $V_t := 100V$ $R_a := 0.1\Omega$

$I_{aFL} := \dfrac{P_{Rated}}{E_{aRated}}$ $I_{aFL} = 100\ A$

$E_{aFL} := V_t - I_{aFL} \cdot R_a$ $E_{aFL} = 90\ V$

Under no load: Given: $E_a = 99V$ at No load $\omega_{NL} := 1000rpm$

Therefore, $E_{aNL} := 99V$

(a) With armature reaction, $\phi_{FL} = 0.925\phi_{NL}$ (7.5% reduction in flux due to armature reaction)

$\dfrac{E_{aFL}}{E_{aNL}} = \dfrac{0.925 \cdot \omega_{FL}}{\omega_{NL}}$ Therefore, $\omega_{FL} := \dfrac{E_{aFL}}{E_{aNL}} \cdot \dfrac{\omega_{NL}}{0.925}$ $\omega_{FL} = 982.801\ rpm$ 3 marks

(b) With armature reaction: $I_{f\_AR} := 0.1A$

$I_f := 0.95A$ $I_{f\_eff} := I_f - I_{f\_AR}$ $I_{f\_eff} = 0.85\ A$

From the graph, when $I_f = 0.85A$ $E_a = 96V$ **(Point D)**

At the **point C,** $E_a = 90V$

Now, we know that:

$E_{aD} := 96V$ $E_{aC} := 90V$ $\omega_D := 1000rpm$

Points C and D are related using:

$\dfrac{\omega_D}{\omega_C} = \dfrac{E_{aD}}{E_{aC}}$ and $\omega_C := \dfrac{E_{aC}}{E_{aD}} \cdot \omega_D$ $\omega_C = 937.5\ rpm$ 3 marks

(c) With armature reaction: $\phi_{FL} = 0.925\phi_{NL}$

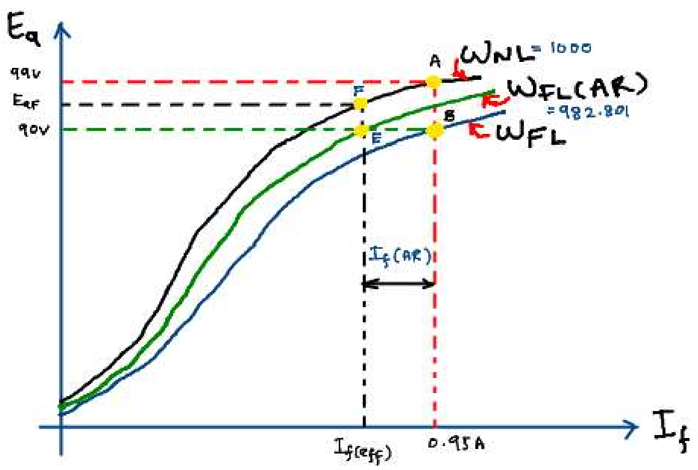
$\omega_F := 1000\text{rpm}$ We need to find E_F

$\omega_E := 982.801\text{rpm}$ From the solution (a) $E_E := 90V$

$$E_F := \frac{\omega_F}{\omega_E} \cdot E_E \qquad E_F = 91.575\,V$$

From the graph, $E_a = 91.575V$ $\quad I_f = 0.72A$ i. e. I_f effective

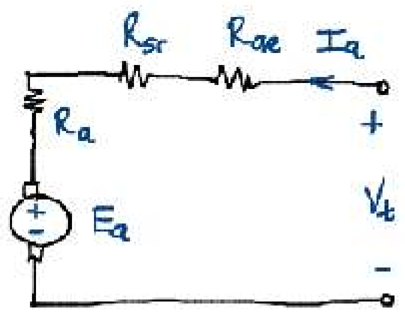
$I_{f\_AR} = I_f - I_{f\_eff} = 0.95 - 0.72 = 0.23A$ 2 marks



Q2: DC Series Motor - Speed control of DC series motor by varying R_a.

$V_t := 350V \qquad R_a := 0.5\Omega \qquad R_{sr} := 0.3\Omega$

An Equivalent circuit of the series DC motor



(a) To determine key quantities at the operating point 1.

Operation at point 1: $\omega_{m1} := 600 \cdot \text{rpm} \qquad I_{a1} := 30A \qquad R_{ae1} := 0$

Therefore,

$$E_{a1} := V_t - I_{a1} \cdot \left(R_a + R_{sr} + R_{ae1}\right) \qquad E_{a1} = 326\,V$$

$$P_1 := E_{a1} \cdot I_{a1}$$ $P_1 = 9.78\,kW$

$$T_1 := \frac{E_{a1} \cdot I_{a1}}{\frac{\omega_{m1}}{60} \cdot 2\pi}$$ $T_1 = 155.654\,Nm$

3 marks

(b) To determine key quantities at the operating point 2.

Operation at point 2: $\omega_{m2} := 500 \cdot rpm$

Since the load torque is proportional to square root of the motor speed,

$$T_2 := \left(\frac{\omega_{m2}}{\omega_{m1}}\right)^{2.25} \cdot T_1$$ $T_2 = 103.276\,Nm$

To find K_{sr}: $T_1 = K_{sr} \cdot I_a{}^2$ $K_{sr} := \frac{T_1}{I_{a1}{}^2}$ $K_{sr} = 0.173$

$$I_{a2} := \sqrt{\frac{T_2}{K_{sr}}}$$ $I_{a2} = 24.437\,A$

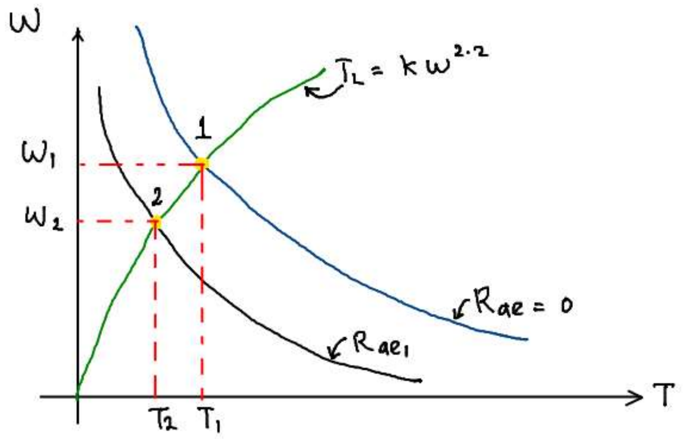
$$E_{a2} := K_{sr} \cdot I_{a2} \cdot \omega_{m2} \cdot \frac{2\pi}{60}$$ $E_{a2} = 221.288\,V$

$$R_{ae2} := \frac{V_t - E_{a2}}{I_{a2}} - (R_a + R_{sr})$$ $R_{ae2} = 4.467\,\Omega$

$$P_2 := E_{a2} \cdot I_{a2}$$ $P_2 = 5.408\,kW$

3 marks

(c) Speed-torque characteristics



2 marks

Q3: Induction Machine - power flow

$V_L := 380V$ $I_L := 96A$ $P_{out} := 47.5kW$ $N_r := 575rpm$ $p := 10$ $f := 50Hz$

Δ-connected $P_{rotat} := 1kW$ $\eta := 0.92$

$N_s := \frac{120 \cdot f}{p}$ $N_s = 600\,rpm$

$s := \frac{N_s - N_r}{N_s}$ $s = 0.042$

i. $P_m := P_{out} + P_{rotat}$ $P_m = 48.5\,kW$ 1 mark

ii. $P_{ag} := \frac{P_m}{(1 - s)}$ $P_{ag} = 50.609\,kW$ 1 mark

iii. $P_{rotor} := s \cdot P_{ag}$ $P_{rotor} = 2.109\,kW$ 1 mark

iv. $P_{in} := \frac{P_{out}}{\eta}$ $P_{in} = 51.63\,kW$ 1 mark

v. $P_{stator} := P_{in} - P_{ag}$ $P_{stator} = 1.022\,kW$ 1 mark

vi. $P_{Losses} := P_{rotat} + P_{rotor} + P_{stator}$ $P_{Losses} = 4.13\,kW$ 1 mark

vii. $P_{losses} := P_{in} - P_{out}$ $P_{losses} = 4.13\,kW$

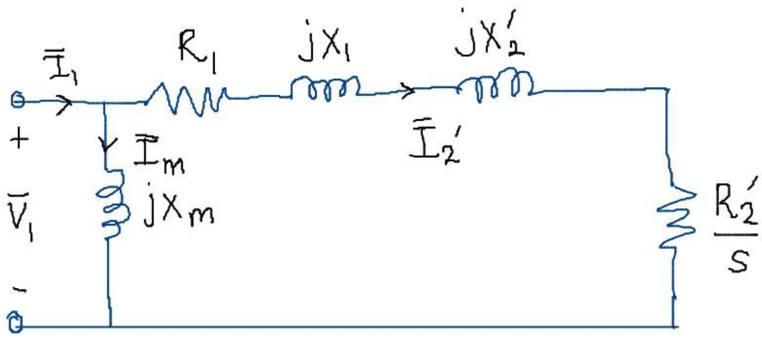
$P_{in} = \sqrt{3} \cdot V_L \cdot I_L \cdot pf$ $pf := \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L}$ $pf = 0.817$ lagging 1 mark

Q4: Induction Machine - Equivalent Circuit Analysis

$R_1 := 1.4\Omega$ $R_2 := 0.6\Omega$ $X_1 := 2\Omega$ $X_2 := 1\Omega$ $X_m := 50\Omega$

$V_L := 415V$ $N_r := 960rpm$ $p := 6$ $f := 50Hz$

We use an approximate equivalent circuit. The calculation uses per phase quantities.



Connection Type: Delta

$V_1 := V_L$ $\quad V_1 = 415\,V$ $\quad N_s := \frac{120f}{p}$ $\quad N_s = 1 \times 10^3\,rpm$

(a) Under starting condition, i.e. when $s := 1$ **; to find line current and power factor**

	mag	**phase**
$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 115.1\,A$	$\arg(I_2) = -56.31\,deg$
$I_m := \frac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8.3\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 122.093\,A$	$\arg(I_1) = -58.471\,deg$

The line current, since the connection is delta, $I_L := \sqrt{3}\,\lvert I_1 \rvert$ $\quad I_L = 211.472\,A$

The power factor, $pf = \cos(\theta)$

$pf := \cos(\arg(I_1))$ $\quad pf = 0.523$ $\quad$ lagging $\quad$ 3 marks

(b) Under maximum torque condition, i.e. when $s = s_{Tmax}$ **; to find the maximum torque**

$$s_{Tmax} := \frac{R_2}{\sqrt{R_1^{\,2} + (X_1 + X_2)^2}} \qquad s_{Tmax} = 0.181$$

$s := s_{Tmax}$

	mag	**phase**
$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 74.309\,A$	$\arg(I_2) = -32.492\,deg$
$I_m := \frac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8.3\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 79.078\,A$	$\arg(I_1) = -37.571\,deg$

$$P_{ag} := (\lvert I_2 \rvert)^2 \frac{R_2}{s} \qquad P_{ag} = 18.281\,kW$$

and $\omega_s := N_s \cdot \frac{2\pi}{60}$

$$T := \frac{P_{ag}}{\omega_s} \qquad T = 174.567\,Nm$$ per phase

3 marks

(c) Under full load, i.e. when $s := \frac{N_s - N_r}{N_s}$; **to find efficiency at the full-load**

$s = 0.04$

	mag	phase
$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$	$\lvert I_2 \rvert = 24.892\,A$	$\arg(I_2) = -10.366\,deg$
$I_m := \frac{V_1}{j \cdot X_m}$	$\lvert I_m \rvert = 8.3\,A$	$\arg(I_m) = -90\,deg$
$I_1 := I_2 + I_m$	$\lvert I_1 \rvert = 27.62\,A$	$\arg(I_1) = -27.56\,deg$

$P_{ag} := (\lvert I_2 \rvert)^2 \cdot \frac{R_2}{s}$ $\qquad P_{ag} = 9.294\,kW$

$P_m := (\lvert I_2 \rvert)^2 \cdot \frac{R_2}{s} \cdot (1 - s)$ $\qquad P_m = 8.922\,kW$

$P_{rotational} := 0.275kW$ $\qquad P_{out} := P_m - \frac{P_{rotational}}{3}$ $\qquad P_{out} = 8.831\,kW$

$P_{in} := \lvert V_1 \rvert \cdot \lvert I_1 \rvert \cdot \cos(\arg(I_1))$ $\qquad P_{in} = 10.161\,kW$

Efficiency, $\eta := \frac{P_{out}}{P_{in}} \cdot 100$ $\qquad \eta = 86.903$

3 marks