TEST#1 SOLUTION

Using Mathcad

Nov-2016

Units: $mm \equiv 10^{-3}$ $cm \equiv 10^{-2}$ $A \equiv 1$ $J \equiv 1$ $Nm \equiv 1$ $T \equiv 1$ $N \equiv 1$

$kW \equiv 10^{3}$ $V \equiv 1$ $turns \equiv 1$ $rpm \equiv 1$ $weber \equiv 1$ $\Omega \equiv 1$

$mWb \equiv 10^{-3}$ $kV \equiv 10^{3}$

Q1)

$$i = 4x\lambda^2 + 5\lambda\cdot(4-2x)^2 + \frac{\lambda}{\sqrt{x}}$$

$$W_f = \int i\,d\lambda = \int \left[4\cdot x\cdot\lambda^2 + 5\cdot\lambda\cdot(4-2\cdot x)^2 + \frac{\lambda}{\sqrt{x}}\right] d\lambda$$

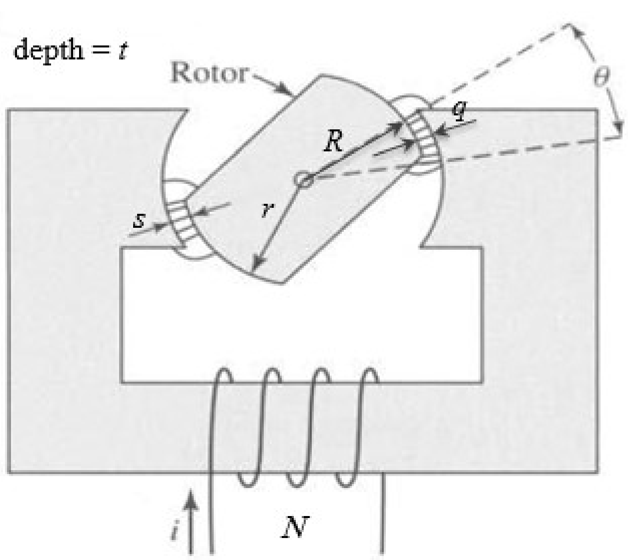
$$W_f = \frac{4}{3}\cdot x\cdot\lambda^3 + \frac{5}{2}\cdot\lambda^2\cdot(4-2\cdot x)^2 + \frac{1}{2}\cdot\frac{\lambda^2}{x^{\left(\frac{1}{2}\right)}}$$

$$f_m = \frac{-\delta}{\delta x} W_f(\lambda, x) \qquad f_m = -\left[\frac{4}{3}\cdot\lambda^3 - 10\cdot\lambda^2\cdot(4-2\cdot x) - \frac{1}{4}\cdot\frac{\lambda^2}{x^{\left(\frac{3}{2}\right)}}\right]$$

$\lambda := 2 \qquad x := 1$

$$f_m := -\left[\frac{4}{3}\cdot\lambda^3 - 10\cdot\lambda^2\cdot(4-2\cdot x) - \frac{1}{4}\cdot\frac{\lambda^2}{x^{\left(\frac{3}{2}\right)}}\right] \qquad f_m = 70.333\,N$$

Q2)



(a) Effective area normal to the flux,

$$A_1 = (R + 0.5 \cdot q) \cdot \theta \cdot t \qquad A_2 = (r + 0.5 \cdot s) \cdot \theta \cdot t$$

$$R_1 = \frac{q}{\mu_o \cdot A_1} \qquad R_2 = \frac{s}{\mu_o \cdot A_2} \qquad R_T = R_1 + R_2 = \frac{1}{\mu_o} \cdot \left(\frac{q}{A_1} + \frac{s}{A_2}\right)$$

$$R_T = \frac{1}{\mu_o \cdot \theta \cdot t}\left(\frac{q}{R + 0.5q} + \frac{s}{r + 0.5s}\right) = \frac{1}{\mu_o \cdot \theta \cdot t} \frac{[q \cdot (r + 0.5s) + s(R + 0.5q)]}{(R + 0.5q) \cdot (r + 0.5s)}$$

$$L = \frac{N^2}{R_T} = \frac{N^2 \cdot \mu_o \cdot \theta \cdot t \cdot (R + 0.5 \cdot q) \cdot (r + 0.5 \cdot s)}{q \cdot (r + 0.5 \cdot s) + s \cdot (R + 0.5 \cdot q)}$$

(b) To find expression for the torque,

$$T = \frac{1}{2} \cdot i^2 \cdot \frac{d}{d\theta} L(\theta) \qquad \text{and} \qquad L(\theta) = \frac{N^2}{R_T}$$

$$T = \frac{1}{2} \cdot i^2 \cdot \frac{d}{d\theta}\left[\frac{N^2 \cdot \mu_o \cdot \theta \cdot t \cdot (R + 0.5 \cdot q) \cdot (r + 0.5 \cdot s)}{q \cdot (r + 0.5 \cdot s) + s \cdot (R + 0.5 \cdot q)}\right]$$

$$T = \frac{1}{2} i^2 \cdot \frac{N^2 \cdot \mu_o \cdot t \cdot (R + 0.5 \cdot q) \cdot (r + 0.5 \cdot s)}{q \cdot (r + 0.5 \cdot s) + s \cdot (R + 0.5 \cdot q)}$$

(c) The torque, T

$q := 3mm \qquad R := 5.8cm \qquad r := 5.9cm \qquad s := 2mm \qquad t := 5cm$

$N := 1000 \qquad i := 10 \qquad \mu_o := 4\pi \cdot 10^{-7}$

$$T := \frac{1}{2} i^2 \frac{N^2 \cdot \mu_o \cdot t \cdot (R + 0.5 \cdot q) \cdot (r + 0.5 \cdot s)}{q \cdot (r + 0.5 \cdot s) + s \cdot (R + 0.5 \cdot q)} \qquad T = 37.51\,Nm$$

Q3) $p := 8 \qquad \omega_{rpm} := 1400rpm \qquad \phi := 30mWb$

$N := 72 \cdot 16 \qquad N = 1.152 \times 10^3$ turns

The number of parallel path, $a := 2$ for wave winding

a) To find the generated voltage, E_a

$$\omega := \omega_{rpm} \cdot \frac{2\pi}{60} \qquad K_a := \frac{N \cdot p}{\pi \cdot a} \qquad K_a = 1.467 \times 10^3$$

$E_a := K_a \cdot \phi \cdot \omega$ $\quad\quad$ $E_a = 6.451\,kV$

$$E_t := \frac{E_a}{\frac{N}{a}} \quad\quad E_t = 11.2\,V$$

b) To find the rated torque

$I_t := 20A$ $\quad\quad$ $I_a := a \cdot I_t$ $\quad\quad$ $I_a = 40\,A$

$T := K_a \cdot \phi \cdot I_a$ $\quad\quad$ $T = 1.76 \times 10^3\,Nm$

c) To find the total armature resistance

Number of turns connected in series,
(Number of turns for each parallel path)

$$N_{series} := \frac{N}{a} \quad\quad N = 1.152 \times 10^3 \quad\quad a = 2$$

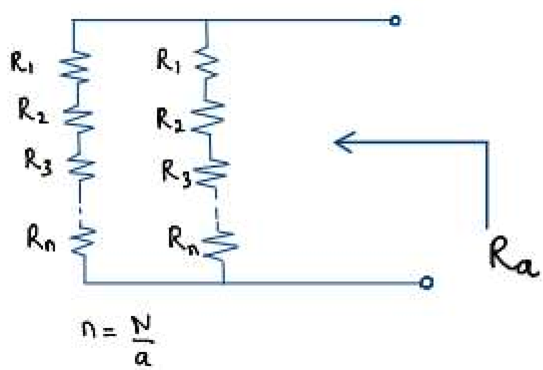
$N_{series} = 576$

$R_{turn} := 0.001 \cdot \Omega$

Therefore, the resistance for each parallel path, $R_{path} := N_{series} \cdot R_{turn}$

$R_{path} = 0.576\,\Omega$

The armature resistance, $R_a := \frac{R_{path}}{a}$ $\quad\quad$ $R_a = 0.288$



Q4) 1. Separately Excited DC generator

$R_a := 0.1\Omega$ $E_a := 120V$ $I_f := 1.15A$

$I_a := \frac{14.4kW}{120}$ $I_a = 120\,A$

(i) Terminal voltage at full-load without armature reaction

$V_t := E_a - I_a \cdot R_a$ $V_t = 108\,V$

(ii) Terminal voltage at full-load with armature reaction

$I_{f\_AR} := 0.05A$

$I_{f\_eff} := I_f - I_{f\_AR}$ $I_{f\_eff} = 1.1$

From the graph, $I_f = 1.1A$ $E_a := 118V$

$V_t := E_a - I_a \cdot R_a$ $V_t = 106\,V$

2. Shunt DC generator

(i) Terminal voltage at full-load without armature reaction

$I_a = 120\,A$ $I_a \cdot R_a = 12\,V$

From the graph, $V_t = 102V$

(ii) Terminal voltage at full-load with armature reaction

$I_a \cdot R_a = 12\,V$ $I_{f\_AR} = 0.05A$

From the graph, $V_t = 93V$