

**Tutorial 1**  
**Electromechanical Energy Conversion**

27/9/2017 pin1

Q1. The  $\lambda$ - $i$  relationship for an electromagnetic system is given by

$$i = 8x^2 + 2\lambda^2 e^{2x} + 3x$$

which is valid for the limits  $0 < i < 4A$  and  $3 < x < 10\text{cm}$ . For current  $i = 4 A$  and air gap length  $x = 5 \text{ mm}$ , find the mechanical force on the moveable part, using both energy and co-energy of the field.

Q2. For the Figure Q2(a) and (b), the mechanical system moves from  $x_1 = 3 \text{ mm}$  to  $x_2 = 4 \text{ mm}$  along the  $\lambda i$  path 1 to 2. Determine the following: (i)  $\Delta W_f$ , (ii)  $\Delta W_f'$ , (iii)  $\Delta W_e$ , (iv)  $\Delta W_m$ , (v)  $f_m$ .

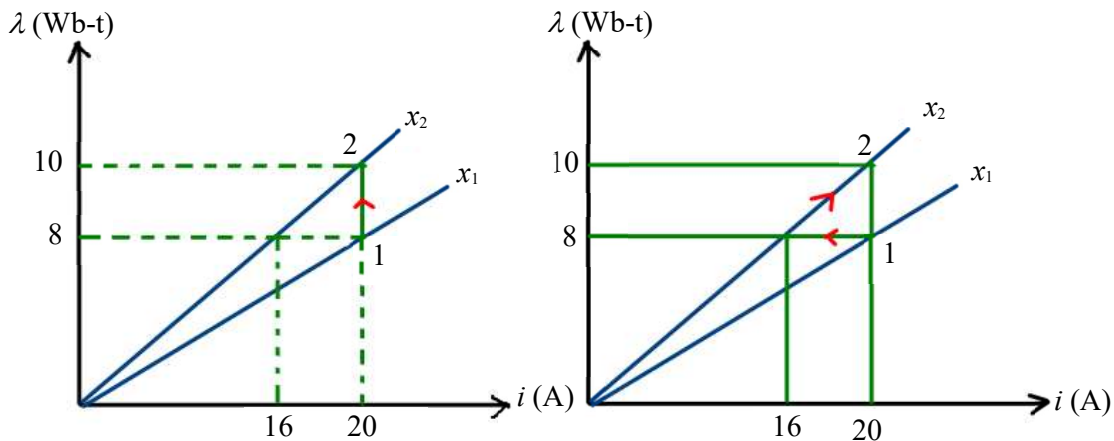


Figure Q2

Q3. The magnetic circuit shown in **Figure Q3** is made of high permeability material so that its reluctance can be negligible. The dimensions are as follows:  $N=1000$  turns,  $g = 4 \text{ mm}$ ,  $a = 10 \text{ cm}$ ,  $b = 20 \text{ cm}$ , and  $i = 10 \text{ Adc}$ .

- (i) Write an expression for the area normal to the flux,  $A_g$ , and the volume of the air gap,  $V_g$ .
- (ii) Obtain an expression for the energy,  $W_f$ , as a function of magnetic flux density,  $B$ , magnetic circuit dimensions, and magnetic field quantities. Then, obtain  $W_f$  as a function of  $\lambda$  and  $g$ .
- (iii) Obtain an expression for the coenergy,  $W_f'$ , as a function of magnetic field strength,  $H$ , magnetic circuit dimensions, and other magnetic field quantities. Then, obtain  $W_f'$  as a function of  $i$  and  $g$ .
- (iv) Obtain an expression for the inductance,  $L$  as a function of  $g$ .
- (v) Find an expression for the force,  $f_m$ , on the moveable part using (ii), (iii) and (iv). Calculate the force,  $f_m$  and show that the obtained results are the same.
- (vi) If the maximum flux density,  $B_{\text{max}}$ , is to be limited to  $1.0 \text{ T}$ , determine  $f_{m(\text{max})}$ .

Q4. The magnetic circuit shown in **Figure Q4** is made of high permeability material so that its reluctance can be negligible. The dimensions are as follows:  $N=1000$  turns,  $a = 8 \text{ mm}$ ,  $b = 6 \text{ mm}$ ,  $c = 20 \text{ cm}$ ,  $d = 20 \text{ cm}$ ,  $e = 15 \text{ cm}$  and  $i = 10 \text{ Adc}$ .

- (i) Write an expression for the area normal to the flux,  $A_g$ , and the volume of the air gap,  $V_g$ .
- (ii) Obtain an expression for the energy,  $W_f$ , as a function of magnetic flux density,  $B$ , magnetic circuit dimensions, and magnetic field quantities. Then, obtain  $W_f$  as a function of  $\lambda$  and  $x$ .
- (iii) Obtain an expression for the coenergy,  $W_f'$ , as a function of magnetic field strength,  $H$ , magnetic circuit dimensions, and other magnetic field quantities. Then, obtain  $W_f'$  as a function of  $i$  and  $x$ .

- (iv) Obtain an expression for the inductance,  $L$  as a function of  $x$ .
- (v) Find an expression for the force,  $f_m$ , on the moveable part using (ii), (iii) and (iv). Calculate the force,  $f_m$  at  $x = 2$  mm and show that the obtained results are the same.
- (vi) If the maximum flux density,  $B_{\max}$ , is to be limited to 1.5 T, determine  $f_{m(\max)}$ .

Q5. The magnetic circuit shown in **Figure Q5** is made of high permeability steel so that its reluctance can be negligible. The rotor is free to turn about a vertical axis. The dimensions are as follows:  $t = 20$  cm,  $s = 4$  mm,  $u = 10$  cm,  $N = 1000$  turns, and  $i = 10$  A.

- (i) Write an expression for the effective area normal to the flux,  $A_g$ , and the volume of the air gap,  $V_g$ .
- (ii) Obtain an expression for the energy,  $W_f$ , as a function of magnetic flux density,  $B$ , magnetic circuit dimensions, and magnetic field quantities. Then, obtain  $W_f$  as a function of  $\lambda$  and  $\theta$ .
- (iii) Obtain an expression for the coenergy,  $W_f'$ , as a function of magnetic field strength,  $H$ , magnetic circuit dimensions, and other magnetic field quantities. Then, obtain  $W_f$  as a function of  $i$  and  $\theta$ .
- (iv) Obtain an expression for the inductance,  $L$  as a function of  $\theta$ .
- (v) Find an expression for the force,  $f_m$ , on the moveable part using (ii), (iii) and (iv). Calculate the force,  $f_m$  at  $\theta = 30^\circ$  and show that the obtained results are the same.
- (vi) If the maximum flux density,  $B_{\max}$ , is to be limited to 1.5 T, determine  $f_{m(\max)}$ .

Q6. The magnetic circuit shown in **Figure Q6** is made of high permeability material so that its reluctance can be negligible. The dimensions are as follows:  $N = 1000$  turns,  $a = 40$  cm,  $b = 30$  cm,  $c = 20$  cm,  $x = 5$  mm, and  $i = 10$  Adc.

- (i) Write an expression for the area normal to the flux,  $A_g$ , and the volume of the air gap,  $V_g$ .
- (ii) Obtain an expression for the energy,  $W_f$ , as a function of magnetic flux density,  $B$ , magnetic circuit dimensions, and magnetic field quantities. Then, obtain  $W_f$  as a function of  $\lambda$  and  $x$ .
- (iii) Obtain an expression for the coenergy,  $W_f'$ , as a function of magnetic field strength,  $H$ , magnetic circuit dimensions, and other magnetic field quantities. Then, obtain  $W_f$  as a function of  $i$  and  $x$ .
- (iv) Obtain an expression for the inductance,  $L$  as a function of  $x$ .
- (v) Find an expression for the force,  $f_m$ , on the moveable part using (ii), (iii) and (iv). Calculate the force,  $f_m$  at  $x = 2$  mm and show that the obtained results are the same.
- (vi) If the maximum flux density,  $B_{\max}$ , is to be limited to 1.0 T, determine  $f_{m(\max)}$ .

Q7. In a doubly-excited magnetic system, the developed torque is given by

$$T = \frac{1}{2} i_s^2 \frac{dL_{ss}}{d\theta} + \frac{1}{2} i_r^2 \frac{dL_{rr}}{d\theta} + i_s i_r \frac{dL_{sr}}{d\theta}$$

Determine the expression for the developed torque for the following rotor and stator structures:

- (a) Non-salient stator and salient rotor
- (b) Salient stator and salient rotor
- (c) Salient stator and non-salient rotor
- (d) Non-salient stator and non-salient rotor

*Hints: You just need to rewrite the torque equation by identifying which terms are zeros and which terms are not, and briefly give the reasons.*

Q8. In the system shown in Figure 3.6 (P. C. Sen page 110), the inductances in henrys are given as  $L_{ss} = (2.5 + \cos 2\theta) \times 10^{-3}$ ;  $L_{sr} = 0.37 \cos \theta$ ;  $L_{rr} = 25 + 8 \cos 2\theta$ . Find the torque for current  $i_l = 0.88$  A and  $i_2 = 0.019$  A. Use Microsoft EXCEL or other mathematical software to plot the instantaneous torque and its components.

Q9. The rotating machine of Figure 3.7 (P. C. Sen page 113) has the following parameters:  $L_{ss} = 0.18$  H,  $L_{rr} = 0.056$  H, and  $L_{sr} = 0.079 \cos \theta$  H. The system has two current sources available for its windings. The windings are supplied with either of two sources: source A = 2.45 Adc; source B = 8.65 Arms,  $f = 50$  Hz.

The rotor is locked at  $\theta = 45^\circ$ . Determine the torque developed for the following source connections:

- Source A to both windings
- Source B to both windings
- Source A to the rotor and source B to the stator.

Q10. The rotating machine of Figure Q10 has the following parameters:  $L_{ss} = 0.18$  H,  $L_{rr} = 0.06$  H, and  $L_{sr} = 0.079 \cos \theta$  H. The system has two current sources available for its windings. The windings are supplied with either of two sources: source A = 2.5 Adc; source B = 8.5 Arms,  $f = 50$  Hz.

The rotor is free to turn about its axis and the position of the rotor at any instant is  $\theta = \omega_m t + \delta$ , where  $\omega_m$  is angular velocity of the rotor and  $\delta$  is the rotor position at  $t = 0$ . Determine the torque developed at  $\omega_m = 100\pi$  rad/s and for the following source connections:

- Source A to both windings
- Source B to both windings
- Source A to the rotor and source B to the stator.

Q11. An elementary two-pole cylindrical rotating machine with a uniform air gap is shown in Figure Q11. The rotor is free to rotate about its axis and the position of the rotor at any instant is given as

$$\theta = \omega_m t + \delta,$$

where  $\omega_m$  is angular velocity of the rotor and  $\delta$  is the rotor position at  $t = 0$ . The mutual inductance between the rotor and the stator is given by

$$L_{12} = 7 \cos \theta \text{ H}$$

An AC current source  $i_1 = 8 \cos 50t$  A is applied to the stator winding and another AC current source  $i_2 = 9 \sin 40t$  A is applied to the rotor winding.

- Derive the instantaneous developed torque,  $T$ .
- Let  $\theta = 45^\circ$  at  $t = 0$ , evaluate the average torque developed by the machine at  $\omega_m = 10, 50$  and  $90$  rad/s.
- If the rotor is locked at  $\theta = 45^\circ$ , determine the average torque developed by the machine.

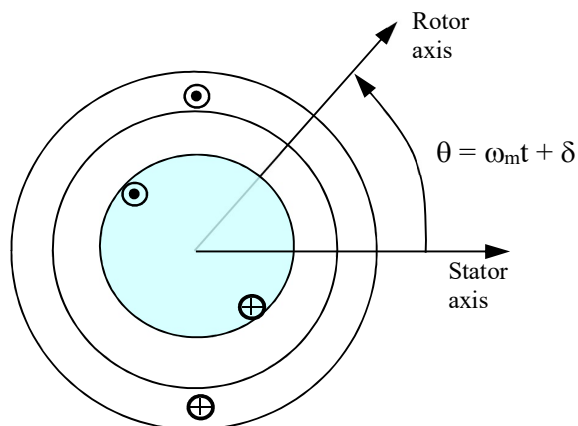


Figure Q10

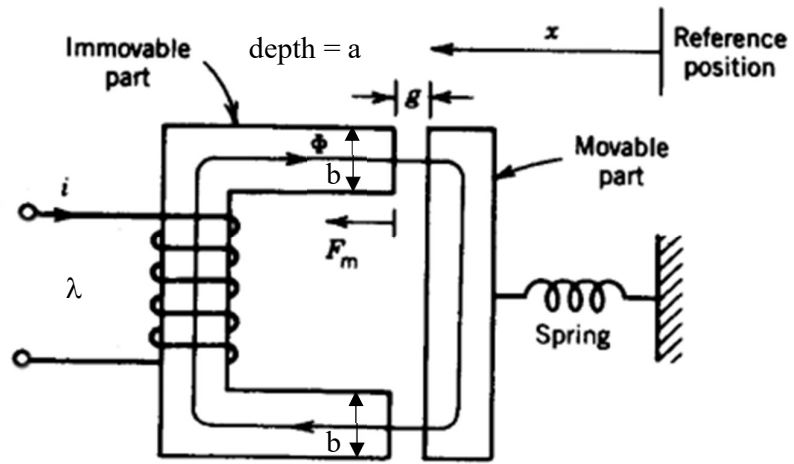


Figure Q3

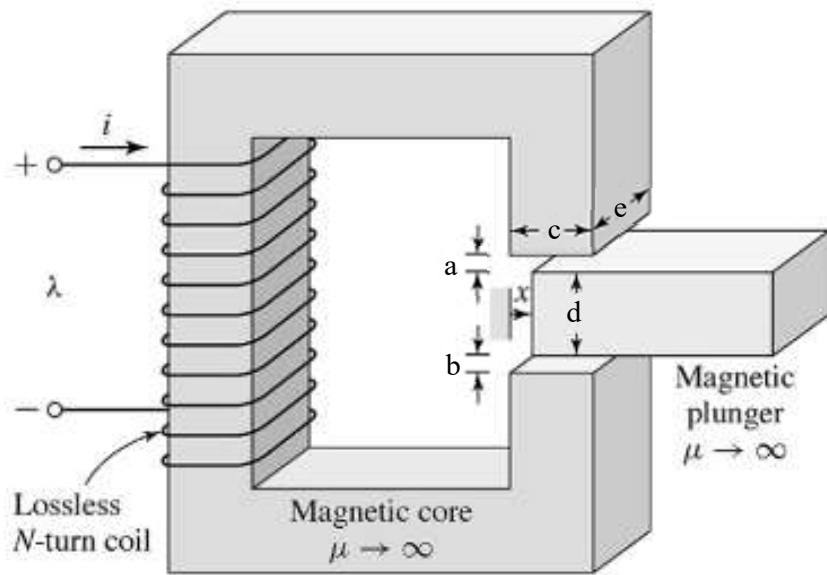


Figure Q4

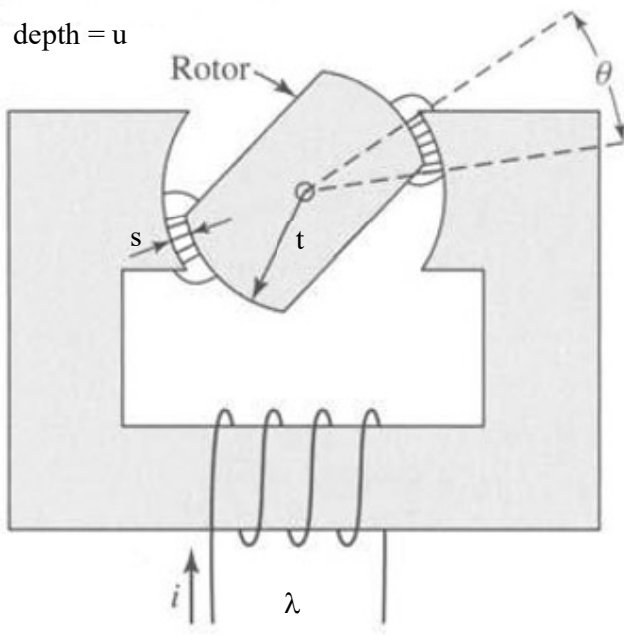


Figure Q5

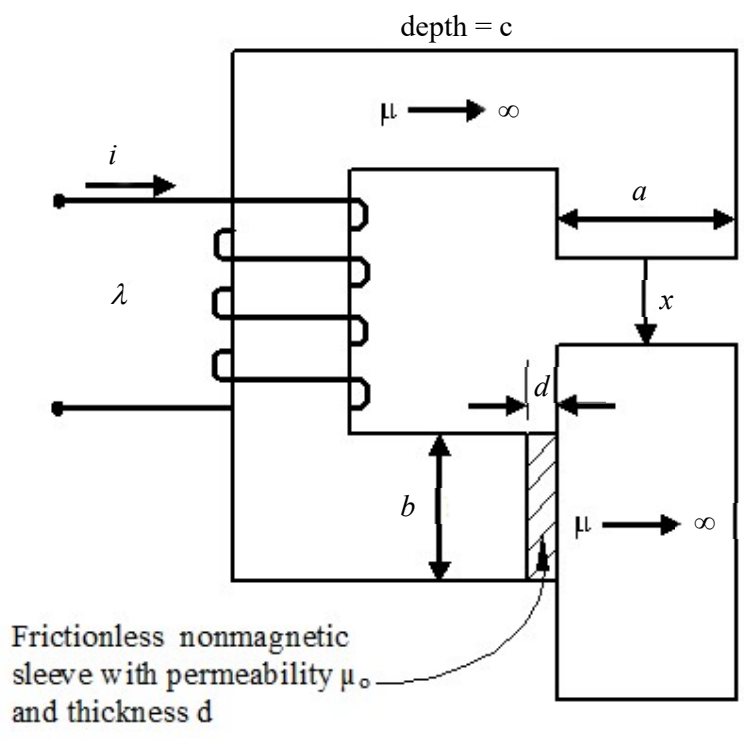


Figure Q6

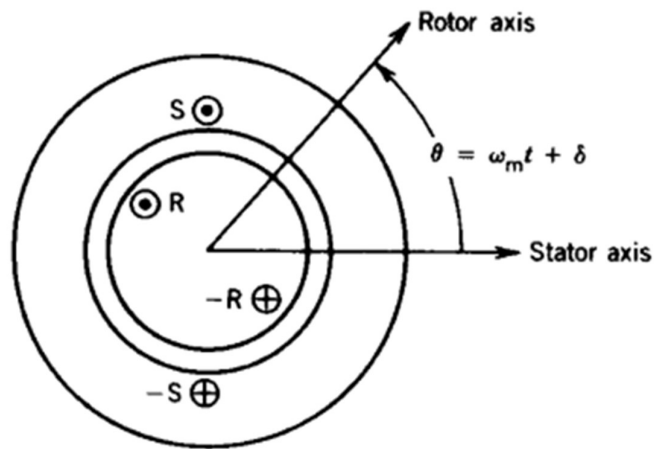


Figure Q10