## Solution For Test #2

## Question 2: Induction Machine   Exact Approach

Dummy unit: $\Omega \equiv 1$   $rpm \equiv 1$   $V \equiv 1$   $A \equiv 1$   $Hz \equiv 1$   $kW \equiv 10^3$   $Nm \equiv 1$   $W \equiv 1$   $k \equiv 10^3$

Y-connected Induction Machine:

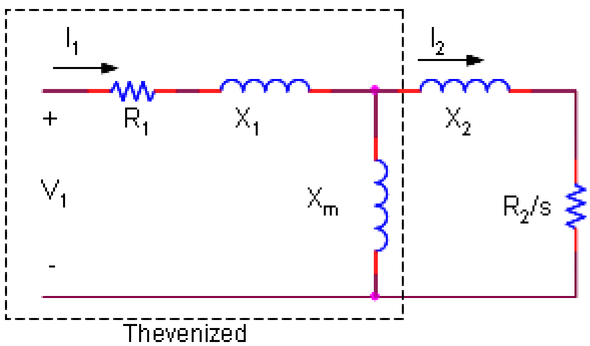
$V_{3\phi} := 415V$   $N_r := 1400rpm$   $f := 50Hz$

$R_1 := 0.25\Omega$   $R_2 := 0.20\Omega$   $p := 4$

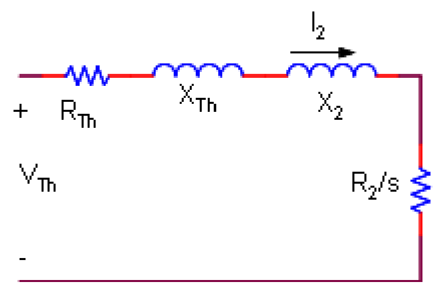
$X_1 := 0.55$   $X_2 := 0.45$   $X_m := 50\Omega$

$V_1 := \frac{V_{3\phi}}{\sqrt{3}}$   $V_1 = 239.6\,V$   $N_s := \frac{120 \cdot f}{p}$   $N_s = 1.5 \times 10^3\,rpm$

We use the exact equivalent circuit first, followed by the aproximate equivalent circuit for comparison.




Equivalent circuit of induction machine (based on IEEE)




Thevenin's equivalent circuit

$$V_{Th} := \frac{X_m}{\left[R_1^{\,2} + (X_1 + X_m)^2\right]^{\frac{1}{2}}} \cdot V_1 \qquad V_{Th} = 236.991\,V$$

$$Z_{Th} := \frac{j \cdot X_m \cdot (R_1 + j \cdot X_1)}{R_1 + j \cdot (X_1 + X_m)} \qquad Z_{Th} = 0.245 + 0.545i \qquad |Z_{Th}| = 0.598 \qquad \arg(Z_{Th}) = 1.149$$

$R_{Th} := Re(Z_{Th})$   $R_{Th} = 0.245\,\Omega$   $X_{Th} := Im(Z_{Th})$   $X_{Th} = 0.545\,\Omega$

We know that Thevenin's theorem modifies the circuit. In this case, $I_1$ is missing in Thevenin's equivalent. Our approach (as in the textbook) is that we use Thevenin's equivalent circuit to find $I_{2prime}$; But, to find $I_1$, we will use the original circuit.
In Mathcad it is hard to write $I_2$ prime; hence it is written without prime for the sake of convenience.

(a) **Exact approach**

(i) The line current at the point A   5 marks

Point A is   at start   $s := 1$   $R_2 \cdot \frac{(1 - s)}{s} = 0$

$$Z_{total} := R_1 + j \cdot X_1 + \frac{j \cdot X_m \cdot \left(\frac{R_2}{s} + j \cdot X_2\right)}{\frac{R_2}{s} + j \cdot (X_2 + X_m)}$$

The impedance seen by the voltage source $V_1$.

$Z_{total} = 0.446 + 0.997i$ $\quad |Z_{total}| = 1.092$ $\quad \arg(Z_{total}) = 65.873\,deg$

$I_1 := \frac{V_1}{Z_{total}}$ $\quad I_1 = 89.674 - 200.213i$ $\quad |I_1| = 219.378$ $\quad \arg(I_1) = -65.873\,deg$

(ii) power factor

$\theta := \arg(I_1)$ $\quad \theta = -65.873\,deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.409$ lagging

(a) ## Approximate approach

(i) The line current at the point A

$V_1 := \frac{V_{3\phi}}{\sqrt{3}}$ $\quad V_1 = 239.6\,V$

$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)}$ $\quad I_2 = 89.663 - 199.252i$ $\quad |I_2| = 218.497\,A$ $\quad \arg(I_2) = -65.772\,deg$

$I_m := \frac{V_1}{j \cdot X_m}$ $\quad I_m = -4.792i$ $\quad |I_m| = 4.792$ $\quad \arg(I_m) = -90\,deg$

$I_1 := I_m + I_2$ $\quad I_1 = 89.663 - 204.044i$ $\quad |I_1| = 222.875$ $\quad \arg(I_1) = -66.278\,deg$

Starting current, $\quad I_{start} := |I_1|$ $\quad I_{start} = 222.875\,A$

(ii) power factor

$\theta := \arg(I_1)$ $\quad \theta = -66.278\,deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.402$ lagging

(b) ## Exact approach

(i) Input power at the point C 5 marks

Point C is full-load. Full load slip $\quad s := \frac{N_s - N_r}{N_s}$ $\quad s = 0.067$

$$Z_{total} := R_1 + j \cdot X_1 + \frac{j \cdot X_m \cdot \left(\frac{R_2}{s} + j \cdot X_2\right)}{\ldots}$$

The impedance seen by the voltage source $V_1$.

$$\frac{R_2}{s} + j \cdot (X_2 + X_m)$$

$Z_{total} = 3.186 + 1.171i$ $\quad |Z_{total}| = 3.395$ $\quad \arg(Z_{total}) = 20.172\,\text{deg}$

$$I_1 := \frac{V_1}{Z_{total}} \qquad I_1 = 66.254 - 24.34i \qquad |I_1| = 70.584 \qquad \arg(I_1) = -20.172\,\text{deg}$$

$\theta := \arg(I_1)$ $\quad \theta = -20.172\,\text{deg}$ $\quad pf := \cos(\theta)$ $\quad pf = 0.939$ $\quad$ lagging

Input power: $\quad P_{input} := 3\,|V_1| \cdot |I_1| \cdot pf \qquad P_{input} = 47.623\,\text{kW}$

$$I_2 := \frac{V_{Th}}{R_{Th} + \frac{R_2}{s} + j \cdot (X_{Th} + X_2)} \qquad I_2 = 66.761 - 20.478i \qquad |I_2| = 69.831\,\text{A} \qquad \arg(I_2) = -17.053\,\text{deg}$$

$P_{rot} := 1500\text{W}$ $\quad$ rotational losses (windage plus friction)

Air gap power: $\quad P_{ag} := \dfrac{3\left(|I_2|\right)^2 \cdot R_2}{s} \qquad P_{ag} = 43.887\,\text{kW}$

Rotor Copper loss: $\quad P_2 := s \cdot P_{ag} \qquad P_2 = 2.926 \times 10^3$

Mechanical power: $\quad P_m := (1 - s) \cdot P_{ag} \qquad P_m = 40.961\,\text{kW}$

Output power: $\quad P_{out} := P_m - P_{rot} \qquad P_{out} = 39.461\,\text{kW}$

Motor efficiency, $\quad Eff_{motor} := \dfrac{P_{out}}{P_{input}} \cdot 100 \qquad Eff_{motor} = 82.861$

## Approximate approach

(i) Input power at the point C

$$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)} \qquad I_2 = 67.347 - 20.722i \qquad |I_2| = 70.463\,\text{A} \qquad \arg(I_2) = -17.103\,\text{deg}$$

$$I_m := \frac{V_1}{j \cdot X_m} \qquad I_m = -4.792i \qquad |I_m| = 4.792 \qquad \arg(I_m) = -90\,\text{deg}$$

$$I_1 := I_m + I_2 \qquad I_1 = 67.347 - 25.514i \qquad |I_1| = 72.018 \qquad \arg(I_1) = -20.749\,\text{deg}$$

$\theta := \arg(I_1)$ $\quad \theta = -20.749\,\text{deg}$ $\quad pf := \cos(\theta)$ $\quad pf = 0.935$ $\quad$ lagging

Input power: $\quad P_{input} := 3\,|V_1| \cdot |I_1| \cdot pf \qquad P_{input} = 48.409\,\text{kW}$

(ii) Motor Efficiency

$P_{rot} := 1500W$ rotational losses (windage plus friction)

Air gap power: $P_{ag} := \frac{3\left(|I_2|\right)^2 \cdot R_2}{s}$ $P_{ag} = 44.685\,kW$

Rotor Copper loss: $P_2 := s \cdot P_{ag}$ $P_2 = 2.979 \times 10^3$

Mechanical power: $P_m := (1 - s) \cdot P_{ag}$ $P_m = 41.706\,kW$

Output power: $P_{out} := P_m - P_{rot}$ $P_{out} = 40.206\,kW$

Motor efficiency, $Eff_{motor} := \frac{P_{out}}{P_{input}} \cdot 100$ $Eff_{motor} = 83.055$

## Exact approach

(c) (i) Air gap power and developed torque at the point B 5 marks

(Point B is at which maximum torque is developed)

$s_{Tmax} := \frac{R_2}{\sqrt{R_1{}^2 + (X_1 + X_2)^2}}$ $s_{Tmax} = 0.194$ $s := s_{Tmax}$

$I_2 := \frac{V_{Th}}{R_{Th} + \frac{R_2}{s} + j \cdot (X_{Th} + X_2)}$ $I_2 = 115.493 - 90.125i$ $|I_2| = 146.497\,A$ $\arg(I_2) = -37.967\,deg$

Air gap power, $P_{ag} := \frac{3\left(|I_2|\right)^2 \cdot R_2}{s}$ $P_{ag} = 66.365\,kW$

$\omega_s := \frac{N_s}{60} \cdot 2\pi$ $T := \frac{P_{ag}}{\omega_s}$ $T_{max} := T$ $T_{max} = 422.495\,Nm$

(ii) Total losses of the motor

Stator copper loss: $P_1 := 3 \cdot \left(|I_2|\right)^2 \cdot R_1$ $P_1 = 16.096\,kW$

Rotor copper loss: $P_2 := 3 \cdot \left(|I_2|\right)^2 \cdot R_2$ $P_2 = 12.877\,kW$

Rotational losses: $P_{rot} = 1.5\,kW$
(windage and friction)

$P_{total} := P_1 + P_2 + P_{rot}$ $P_{total} = 30.473\,kW$

## Approximate approach

(i) Air gap power and developed torque at the point B 5 marks

(Point B is at which maximum torque is developed)

$$I_2 := \frac{V_1}{R_1 + \frac{R_2}{s} + j \cdot (X_1 + X_2)} \qquad I_2 = 116.223 - 90.744i \quad |I_2| = 147.453\,A \quad arg(I_2) = -37.982\,deg$$

Air gap power, $P_{ag} := \frac{3(|I_2|)^2 \cdot R_2}{s}$ $\quad P_{ag} = 67.235\,kW$

$$\omega_s := \frac{N_s}{60} \cdot 2\pi \qquad T := \frac{P_{ag}}{\omega_s} \qquad T_{max} := T \qquad T_{max} = 428.029\,Nm$$

(ii) Total losses of the motor

Stator copper loss: $P_1 := 3 \cdot (|I_2|)^2 \cdot R_1$ $\quad P_1 = 16.307\,kW$

Rotor copper loss: $P_2 := 3 \cdot (|I_2|)^2 \cdot R_2$ $\quad P_2 = 13.045\,kW$

Rotational losses: (windage and friction) $P_{rot} = 1.5\,kW$

$$P_{total} := P_1 + P_2 + P_{rot} \qquad P_{total} = 30.852\,kW$$

(d) Speed of the motor (rotor), $N_r$, at the point D is synchronous speed, $N_s$. At this point, slip s = 0, which means that no relative motion between rotating magnetic field of the stator and rotor conductors. As such, no emf would be induced and no current would flow in the rotor conductors. Therefore, no torque would be developed. Naturally, the rotor speed would not achieve synchronous speed; the rotor speed, $N_r$, is always slightly less than synchronous speed.

1.5 marks

(e) $X_m$, in both transformer and induction machine, represents magnetizing inductance. This value reflects the amount of magnetizing current required to establish flux in the magnetic circuit. The major different between magnetic circuit in transformer and induction machine is that induction machine has air gap, while transformer does not have. Due to the air gap, higher value of current is required to establish flux in the induction machine. This corresponds to lower value of $X_m$ in induction machine.

1.5 marks