SEE 3433 (version: 4Mei)

Solution for Final Examination

Semester: 2010/11-02

Question 1: Electromechanical Energy Conversion

(a) Production of torque in a singly excited and doubly excited system — 6 marks

In a singly excited system,

$$T = \frac{1}{2} i^2 \cdot \frac{d}{d\theta} L(\theta)$$

The torque is produced because of variation of self-inductance with rotor position. This is also called reluctance torque.

In a doubly excited system,

$$T = \frac{1}{2} \cdot i_1{}^2 \cdot \frac{d}{d\theta} \cdot L_{11}(\theta) + \frac{1}{2} \cdot i_2{}^2 \cdot \frac{d}{d\theta} \cdot L_{22}(\theta) + i_1 \cdot i_2 \cdot \frac{d}{d\theta} \cdot L_{12}(\theta)$$

The first two terms in the equation represent torques produced because of self-inductance with the rotor position. These torques are the same torque produced in a singly excited system. The third term represents torque produced by the variation of the mutual inductance between the stator and the rotor winding. This torque uniquely belongs to a doubly excited system.

(b) $i = k \cdot \lambda^2 \cdot e^{2x}$ — 6 marks

$$W_f = \int i \, d\lambda = \int 6 \cdot \lambda^2 \cdot e^{2 \cdot x} \, d\lambda = 6 \cdot \frac{\lambda^3}{3} \cdot e^{2 \cdot x} = 2 \cdot \lambda^3 \cdot e^{2 \cdot x}$$

$$f_m = \frac{-\delta}{\delta x} W_f(\lambda, x) \qquad f_m = 4\lambda^3 \cdot e^{2x}$$

$$\lambda := 2 \qquad x := 1$$

$$f_m := 4\lambda^3 \cdot e^{2x} \qquad f_m = 236.45$$

(c) Calculation

(i) $T = i_1 i_2 \cdot \frac{d}{d\theta} L_{12}(\theta)$ — 2 marks

(ii) $i_1 = 4 \qquad i_2 = 8 \sin(10t) \qquad L_{12} = 0.5 \cdot \cos(\theta)$ — 5 marks

$$T = 4 \cdot 8 \cdot \sin(10t) \cdot \frac{d}{d\theta} 0.5 \cos(\theta)$$

$$T = -16 \cdot \sin(10 \cdot t) \cdot \sin(\theta)$$

calculator:

$$\sin(60deg) \cdot 16 = 13.856$$

$$T = -13.856 \cdot \sin(10 \cdot t)$$

Because of T consists only sine function, the average torque is zero.

(iii) $T = 4 \cdot 8 \cdot \sin(10t) \cdot \frac{d}{d\theta} 0.5\cos\left(\omega_m \cdot t + \delta\right)$ 6 marks

$$T = 4 \cdot 8 \cdot \sin(10 \cdot t) \cdot \frac{d}{d\theta} \cdot 0.5 \cdot \cos(10 \cdot t + 60)$$

$$T = -16 \cdot \sin(10 \cdot t) \cdot \sin(10 \cdot t + 60)$$

$$T = 8 \cdot \cos[10 \cdot t - (10 \cdot t + 60)] - 8\cos[10 \cdot t + (10 \cdot t + 60)]$$

$$T = 8 \cdot \cos(-60) - 8\cos(20 \cdot t + 60)$$

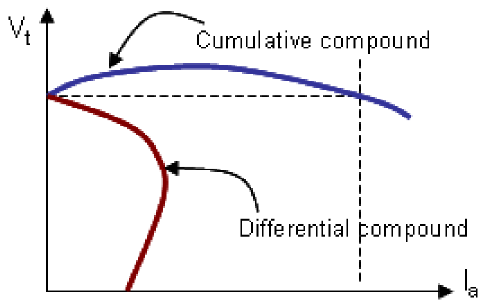
$$T_{ave} := 8 \cdot \cos(-60deg) \qquad T_{ave} = 4\,Nm$$

Question 2: DC Machine (generator)

(a) V_t - I_a characteristics of DC generator 6 marks

In cumulative compound generator, the field of the shunt and series winding add each other. With increasing armature current the field due to series winding increases. This may compensate the drop in terminal voltage due to I_aR_a drop and armature reaction. Then, the terminal voltage may rise.

In differential compound generator, the field of the shunt and series winding oppose each other. With increasing armature current the field due to series winding decreases. Therefore, the terminal voltage drops very quickly with incraesing armature current.



(b) Calculation for generated voltage 5 marks

$p := 6 \qquad N := 300 \qquad a := 2 \qquad \phi := 0.025 \qquad N_m := 1000$

$K_a := \frac{N \cdot p}{\pi \cdot a} \qquad K_a = 286.479 \qquad \omega_m := \frac{N_m}{60} \cdot 2\pi$

(i) $E_a := K_a \cdot \phi \cdot \omega_m \qquad E_a = 750\,V$

(ii) $I_a := 25 \cdot a \qquad I_a = 50\,A \qquad P := E_a \cdot I_a \qquad P = 37.5\,kW$

(c) Calculation for DC generator

(i) Diagram of compound DC machine 3 marks

Insert Diagram of Compound DC Machine here

(ii) Draw a field resistance line that intersects the magnetization curve at 100 V. 4 marks

From the graph: $I_f := 0.8 \cdot A$ $E_a := 100 \cdot V$

At no load, $V_t := E_a$ $R_f := \dfrac{V_t}{I_f}$ $R_f = 125\,\Omega$

(iii) $V_{t\_NL} := 100 \cdot V$ $R_f = 125\,\Omega$ $R_{sr} := 0.04 \cdot \Omega$ $R_a := 0.1\Omega$ $N_f := 1200$

$I_{f\_AR} := 0.05 \cdot A$ 7 marks

At full-load: $V_{t\_FL} := V_{t\_NL}$ $V_t := 100 \cdot V$ $I_a := \dfrac{10 \cdot kW}{100 \cdot V}$ $I_a = 100\,A$

$I_f := 1$ $I_t := 100$ (Guess values)

Given

$I_a = I_f + I_t$ $I_f \cdot R_f = I_t \cdot R_{sr} + V_t$

$\begin{pmatrix} I_f \\ I_t \end{pmatrix} := \text{Find}(I_f, I_t)$ $I_f = 0.832\,A$ $I_t = 99.168\,A$

$E_a := V_t + I_t \cdot R_{sr} + I_a \cdot R_a$ $E_a = 113.967\,V$

From the graph, $I_{f\_eff} := 1.20 \cdot A$

$N_{sr} := 5$ (Guess value) Given

$$I_{f\_eff} = I_f + \frac{N_{sr}}{N_f} \cdot I_t - I_{f\_AR}$$

$N_{sr} := \text{Find}(N_{sr})$ $N_{sr} = 5.061$

Question 3: DC Machine (motor)

(a) The terminal voltage speed control 4 marks

The terminal voltage control is the best method to control speed of DC motors. This is because the maximum torque capability is maintained and smooth variation of speed from zero to base speed is guaranteed. This method, however, is relatively expensive because it requires a variable DC supply.

(b) The effect of armature reaction in DC motor operation 4 marks

The effect of armature reaction is that it reduces the useful flux in the air gap. In DC motor operation, the decreasing field will cause the higher speed. It also shifts the magnetic neutral axis of the machine at which the brushes are normally placed.

(c) Calculation for DC motor

$V_t := 100 \cdot V$ $R_a := 0.1 \cdot \Omega$ $\omega_{NL} := 800 \cdot rpm$

(i) At no load, $I_a := 5 \cdot A$ 3 marks

$E_a := V_t - I_a \cdot R_a$ $E_a = 99.5\,V$ $E_{a\_NL} := E_a$

From the magnetization curve, for $E_a = 99.5V$ $I_f = 0.79A$

$V_t := E_a$ $R_f := \frac{V_t}{I_f}$ $R_f = 119.63\,\Omega$

(ii) At full-load $I_a := \frac{10 \cdot kW}{100V}$ $I_a = 100\,A$ $I_{a\_FL} := I_a$ 4 marks

$E_{a\_FL} := V_t - I_{a\_FL} \cdot R_a$ $E_{a\_FL} = 89.5\,V$

Without armature reaction, $\phi_{NL} = \phi_{FL}$

$$\omega_{FL} := \frac{E_{a\_FL}}{E_{a\_NL}} \cdot \omega_{NL} \qquad \omega_{FL} = 719.598\,rpm$$

(iii) With armature reaction, $\phi_{FL} = 0.95\phi_{NL}$ 5 marks

$$\frac{E_{a\_FL}}{E_{a\_NL}} = \frac{0.95 \cdot \omega_{FL}}{\omega_{NL}} \qquad \omega_{FL} := \frac{1}{0.95} \cdot \frac{E_{a\_FL}}{E_{a\_NL}} \cdot \omega_{NL} \qquad \omega_{FL} = 757.472\,rpm$$

(iv) $E_a = 89.5V$ at 700 rpm

5 marks

$$E_{a\_800} := \frac{800}{700} \cdot 89.5 \qquad E_{a\_800} = 102.286\,V$$

From magnetization curve 800 rpm: for $E_a = 78.313V$ $I_f = 0.85A$

$I_{f\_eff} := 0.85A$ $I_f := 0.79A$ $I_a = 100$ $I_{f\_AR} := 0.15A$

$N_{sr} := 5$ (guess value) Given $I_{f\_eff} = I_f + \frac{N_{sr}}{N_f} \cdot (I_a + I_f) - I_{f\_AR}$

$N_{sr} := Find(N_{sr})$ $N_{sr} = 2.5$

Question 4: Induction Machine

(a) Terminal voltage speed control is limited in operating range 3 marks

The torque developed by an induction motor is proportional to the square of the applied voltage. By reducing the terminal voltage, the torque will reduce drastically and the capability to drive load will reduce as the same proportion. Therefore the speed of the motor may be controlled over a limited range by varying the line voltage.

(b) Development of torque in IM and IM can not operate at synchronous speed 4 marks

A three-phase voltage is applied to the stator of IM. This three phase voltage produces a rotating magnetic field in the stator, rotating at synchronous speed given by

$$N_s = \frac{120 \cdot f}{p}$$

The rotating magnetic field passes over the rotor conductors and induces the voltage in them. Because the conductors in the rotor are short-circuited, the current flows and causes magnetic fiel in the rotor too. The interaction between stator magnetic field and rotor magnetic field will produce torque.

If the induction motor was running at synchronous speed, then the rotor would be stationary relative to the rotating magnetic field and there would be no induced voltage. Then, no current would flow in the rotor's conductor and no flux would be produced. With no rotor field, the induced torque would be zero.

Q4(c) Calculation for Induction machine

Dummy unit: $V \equiv 1$ $rpm \equiv 1$ $\Omega \equiv 1$ $Hz \equiv 1$ $kW \equiv 10^3$ $A \equiv 1$

$Nm \equiv 1$ $kVAr \equiv 10^3$ $k \equiv 10^3$

$V_{3\phi} := 415$ $N_r := 950$ $f := 50$ $R_1 := 0.25$ $R_2 := 0.2$ $p := 6$ $X_1 := 0.65$ $X_2 := 0.6$ $X_M := 60$

$$V_1 := \frac{V_{3\phi}}{\sqrt{3}} \qquad V_1 = 239.6\,V \qquad N_s := \frac{120 \cdot f}{p} \qquad N_s = 1 \times 10^3$$

$$Z_{Th} := \frac{j \cdot X_M \cdot (R_1 + j \cdot X_1)}{R_1 + j \cdot (X_1 + X_M)} \qquad Z_{Th} = 0.245 + 0.644i \qquad R_{Th} := Re(Z_{Th}) \qquad X_{Th} := Im(Z_{Th})$$

$$V_{Th} := \frac{X_M}{\sqrt{R_1{}^2 + (X_1 + X_M)^2}} \cdot V_1 \qquad V_{Th} = 237.03\,V$$

(i) Line current, power factor, and input power at full load 6 marks

At the full-load, $s := \frac{N_s - N_r}{N_s}$ $s = 0.05$

Based on the original circuit:

$$Z_1 := R_1 + j \cdot X_1 + \frac{j \cdot X_M \cdot \left(\frac{R_2}{s} + j \cdot X_2\right)}{j \cdot X_M + \frac{R_2}{s} + j \cdot X_2} \qquad Z_1 = 4.154 + 1.502i \qquad |Z_1| = 4.417$$

$$\arg(Z_1) = 19.875\,\text{deg}$$

$$pf := \cos(\arg(Z_1)) \qquad pf = 0.94 \quad \text{lagging}$$

$$I_1 := \frac{V_1}{Z_1} \qquad I_1 = 51.011 - 18.441i \qquad |I_1| = 54.241 \qquad \arg(I_1) = -19.875\,\text{deg}$$

$$P_{in} := 3\,|V_1| \cdot |I_1| \cdot pf \qquad P_{in} = 36.666\,\text{kW}$$

(ii) Current and torque at starting 6 marks

At start, $s := 1$

Input impedance, $Z_{in} := Z_{Th} + \frac{R_2}{s} + j \cdot X_2 \qquad Z_{in} = 0.445 + 1.244i$

$$I_2 := \frac{V_1}{Z_{in}} \qquad I_2 = 61.043 - 170.779i$$

$$P_{ag} := 3\left(|I_2|\right)^2 \cdot \frac{R_2}{s} \qquad P_{ag} = 19.735\,\text{kW}$$

$$\omega_{syn} := \frac{N_s}{60} \cdot 2\pi$$

$$T_{st} := \frac{P_{ag}}{\omega_{syn}} \qquad T_{st} = 188.456\,\text{Nm}$$

$$E_1 := I_2 \cdot \left(\frac{R_2}{s} + j \cdot X_2\right) \qquad E_1 = 114.676 + 2.47i$$

$$I_M := \frac{E_1}{j \cdot X_M} \qquad I_M = 0.041 - 1.911i$$

$$I_1 := I_M + I_2 \qquad I_1 = 61.084 - 172.691i$$

(iii) Efficiency at which the maximum developed torque occurs 6 marks

$$S_{Tmax} := \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}} \qquad P_{rot} := 1200 \quad \text{(Rotational losses)} \qquad S_{Tmax} = 0.158$$

$$s := S_{Tmax} \qquad Z_{in} := Z_{Th} + \frac{R_2}{s} + j \cdot X_2$$

$$I_2 := \frac{V_{Th}}{Z_{in}} \qquad I_2 = 93.476 - 76.882i \qquad |I_2| = 121.031 \qquad \arg(I_2) = -39.437\,\text{deg}$$

$$E_1 := I_2 \cdot \left(\frac{R_2}{s} + j \cdot X_2\right) \qquad E_1 = 164.645 - 41.392i$$

$$I_M := \frac{E_1}{j \cdot X_M} \qquad I_M = -0.69 - 2.744i$$

$$I_1 := I_M + I_2 \qquad I_1 = 92.786 - 79.626i \qquad |I_1| = 122.268 \qquad \arg(I_1) = -40.635\,\text{deg}$$

$$pf := \cos(\arg(I_1)) \qquad pf = 0.759$$

Air gap power: $P_{ag} := \frac{3(|I_2|)^2 \cdot R_2}{s}$ $\quad P_{ag} = 5.572 \times 10^4$ $\quad \omega_{syn} := \frac{N_s}{60} \cdot 2\pi$

Rotor Copper loss: $P_2 := s \cdot P_{ag}$ $\quad P_2 = 8.789 \times 10^3$

Mechanical power: $P_{mech} := (1 - s) \cdot P_{ag}$ $\quad P_{mech} = 4.693 \times 10^4$

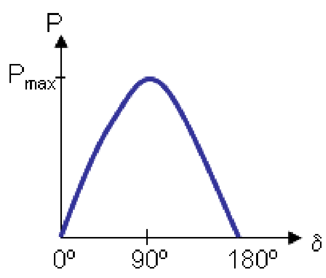
Output power: $P_{out} := P_{mech} - P_{rot}$ $\quad P_{out} = 4.573 \times 10^4$

Input power: $P_{input} := 3\,|V_1| \cdot |I_1| \cdot pf$ $\quad P_{input} = 6.669 \times 10^4$

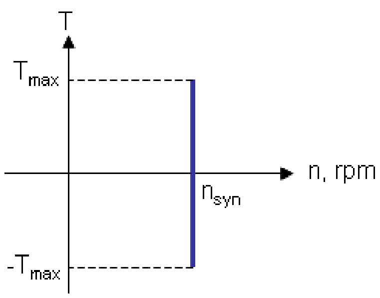
$$\text{Efficiency} := \frac{P_{out}}{P_{input}} \cdot 100 \qquad \text{Efficiency} = 68.564$$

Question 5: Synchronous machine

(a) Power angle characteristics and torque characteristics **4 marks**



Power angle characteristics



Torque-speed characteristics

(b) Power factor of synchronous motor can be controlled by field current **6 marks**

The power transfer equations:

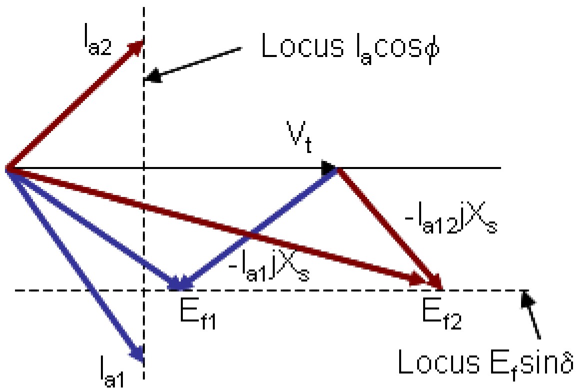
$$P = 3V_t \cdot I_a \cdot \cos\phi$$

and

$$P = \frac{3 \cdot V_t \cdot E_f}{X_s} \cdot \sin\delta$$

The motor connected to infinite bus, hence

V_t = constant



For constant power operation, $I_a \cdot \cos\phi = \text{constant}$

and $E_f \cdot \sin\delta = \text{constant}$

These conditions must satisfy this phasor relationship:

$$E_f = V_t - j \cdot I_a \cdot X_s$$

The excitation voltage E_f changes linearly with the field current I_f. Therefore, as I_f is changed, E_f willl change along the locus of E_f and Ia will change along the locus of I_a, signifying a change in the power factor angle ϕ.

(c) Calculation

(i) Excitation voltage and the power angle 5 marks

$P := 10k \qquad V_{3\phi} := 415 \qquad pf := 0.8 \qquad X_s := 10$

$I_a := \dfrac{P}{\sqrt{3} \cdot V_{3\phi} \cdot pf} \qquad I_a = 17.39\,A \qquad \theta := -acos(pf) \qquad \theta = -36.87\,deg$

$V_t := \dfrac{V_{3\phi}}{\sqrt{3}} \qquad V_t = 239.6\,V \qquad E_f = V_t + I_a \cdot jX_s$

$E_f := V_t + I_a \cdot e^{j\cdot\theta} \cdot j \cdot X_s \qquad E_f = 343.941 + 139.121i$

$|E_f| = 371.012\,V \quad arg(E_f) = 22.023\,deg$

Power angle $\delta := arg(E_f) \qquad \delta = 22.023\,deg$

Excitation voltage $E_f := |E_f| \qquad E_f = 371.012\,V$

Include Phasor diagram

(ii) Reactive kVA supplied by the machine 5 marks

$E_{f1} := 1.15 \cdot E_f$

$$\frac{V_t \cdot E_f}{X_s} \cdot \sin(\delta) = \frac{V_t \cdot E_{f1}}{X_s} \sin(\delta 1)$$

$$E_f \cdot \sin(\delta) = E_{f1} \cdot \sin(\delta 1)$$

$\delta 1 := asin\left(\dfrac{E_f \cdot \sin(\delta)}{E_{f1}}\right) \qquad \delta 1 = 19.03\,deg$

$E_{f1} := 1.15 \cdot E_f \cdot e^{j\cdot\delta 1}$

$I_a := \dfrac{E_{f1} - V_t}{j \cdot X_s} \qquad I_a = 13.912 - 16.374i \qquad |I_a| = 21.486\,A \quad arg(I_a) = -49.648\,deg$

Power factor angle, $\theta := |\arg(I_a)|$ $\theta = 49.648\,deg$

Power factor, $pf := \cos(\theta)$ $pf = 0.647$ lagging

Reactive power, $Q := 3 \cdot |V_t| \cdot |I_a| \cdot \sin(\theta)$

$Q = 11.77\,kVAr$ Include Phasor diagram

(iii) $\delta := 90deg$ $E_f = 371.012\,V$ 3 marks

$$P_{max} := \frac{3 \cdot V_t \cdot E_f}{X_s} \qquad P_{max} = 26.668\,k$$

$$I_a := \frac{j \cdot E_f - V_t}{jX_s} \qquad I_a = 37.101 + 23.96i \qquad |I_a| = 44.165\,A \qquad \arg(I_a) = 32.854\,deg$$

Power factor angle, $\theta := |\arg(I_a)|$ $\theta = 32.854\,deg$

Power factor, $pf := \cos(\theta)$ $pf = 0.84$ leading

Reactive power, $Q := 3 \cdot |V_t| \cdot |I_a| \cdot \sin(\theta)$

$Q = 17.222\,kVAr$

Include Phasor diagram