Chapter 4
DC to AC Conversion
(INVERTER)

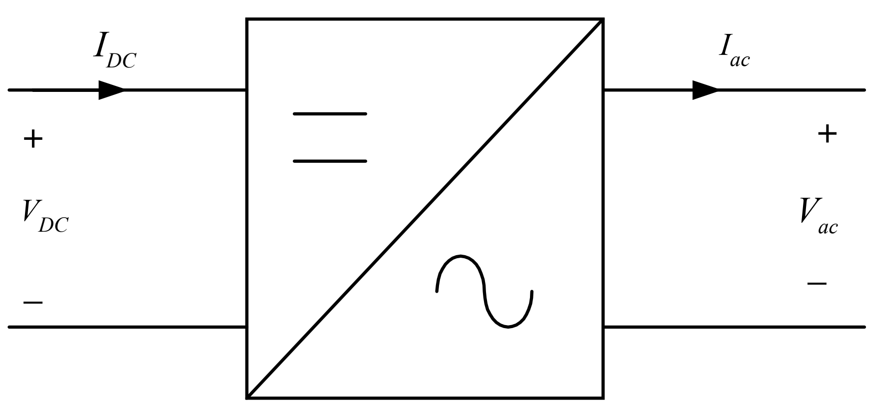
- General concept
- Basic principles/concepts
- Single-phase inverter
  - Square wave
  - Notching
  - PWM
- Harmonics
- Modulation
- Three-phase inverter

DC to AC Converter (Inverter)

- DEFINITION: Converts DC to AC power by switching the *DC input voltage (or current)* in a pre-determined sequence so as to generate *AC voltage (or current) output*.

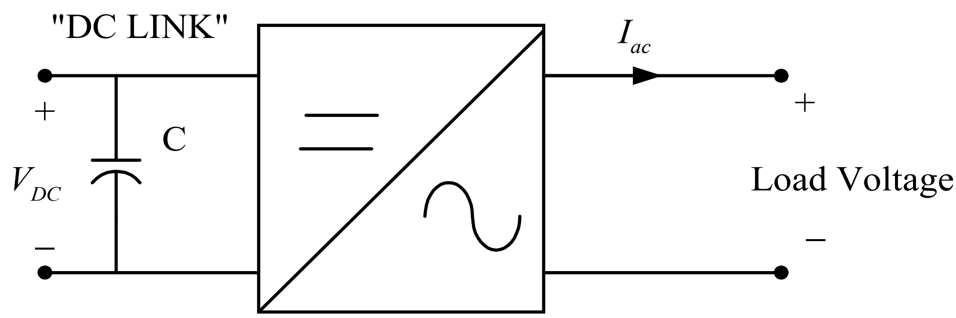
- TYPICAL APPLICATIONS:
  - Un-interruptible power supply (UPS), Industrial (induction motor) drives, Traction, HVDC

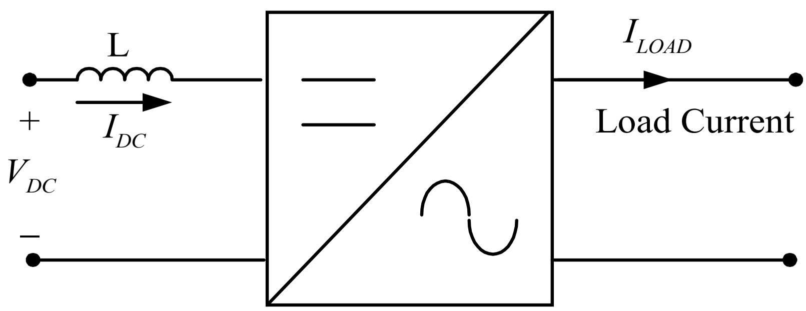
- General block diagram



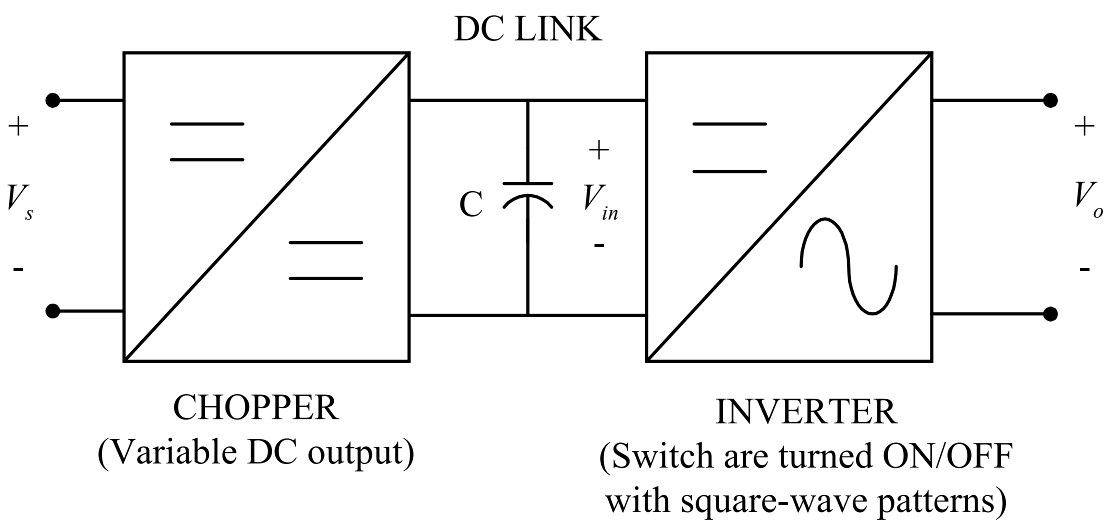
Types of inverter

- Voltage Source Inverter (VSI)
- Current Source Inverter (CSI)





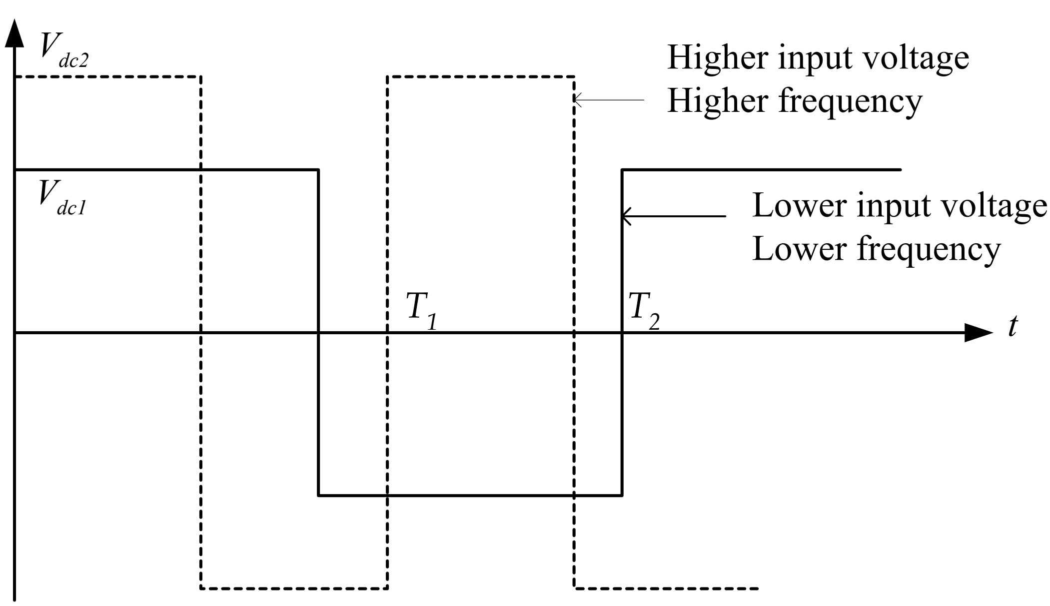
Voltage source inverter (VSI) with variable DC link



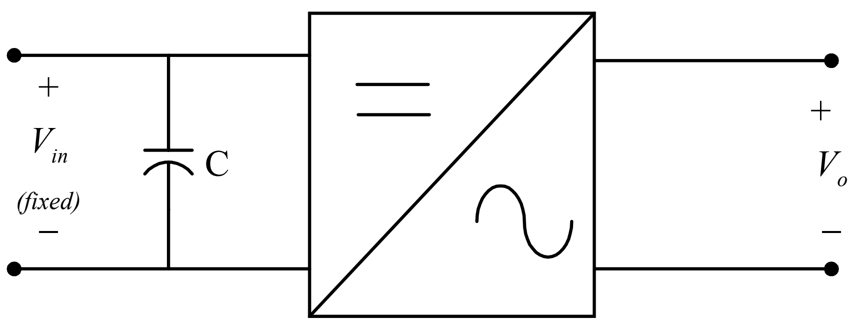
- DC link voltage is varied by a DC-to DC converter or controlled rectifier.
- Generate "square wave" output voltage.
- Output voltage amplitude is varied as DC link is varied.
- Frequency of output voltage is varied by changing the frequency of the square wave pulses.

Variable DC link inverter (2)

- Advantages:
  - simple waveform generation
  - Reliable
- Disadvantages:
  - Extra conversion stage
  - Poor harmonics



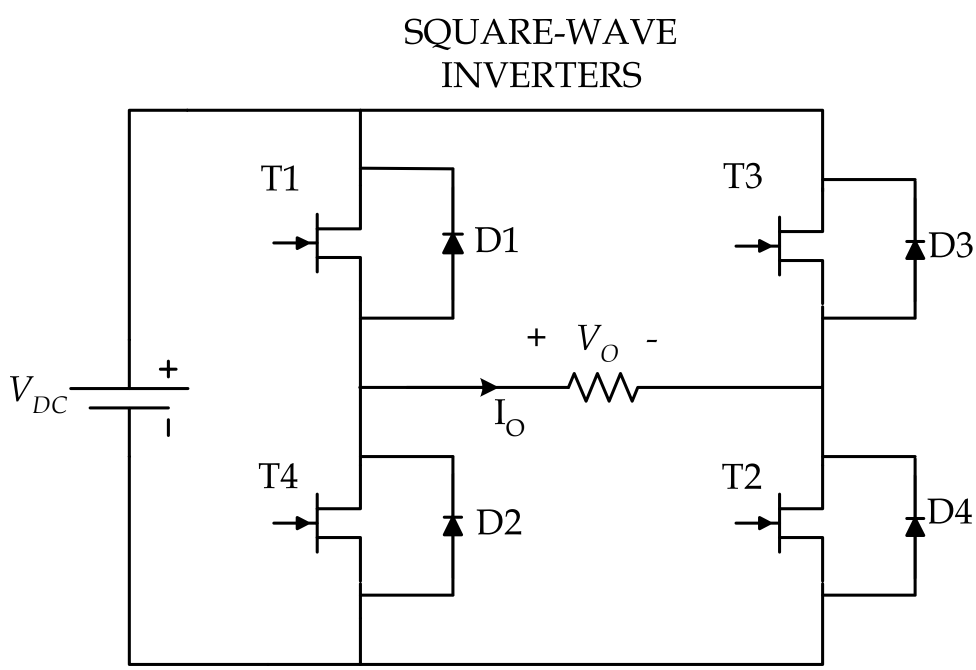
VSI with fixed DC link

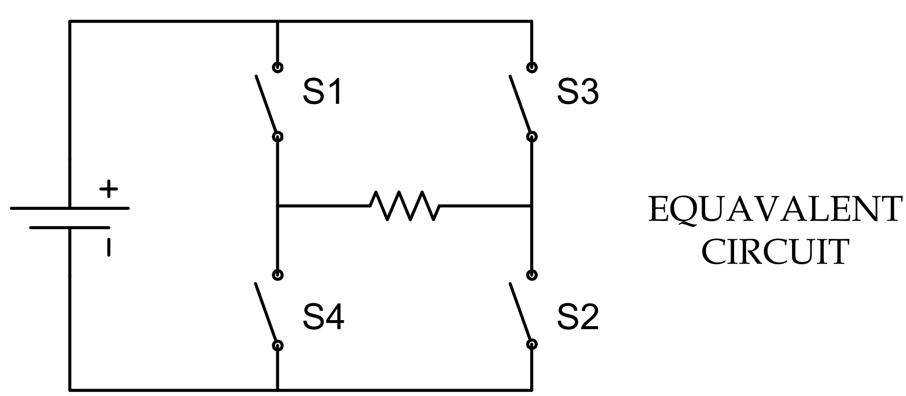


- DC voltage is held constant.
- Output voltage amplitude and frequency are varied simultaneously using PWM technique.
- Good harmonic control, but at the expense of complex waveform generation

Operation of simple square-wave inverter (1)

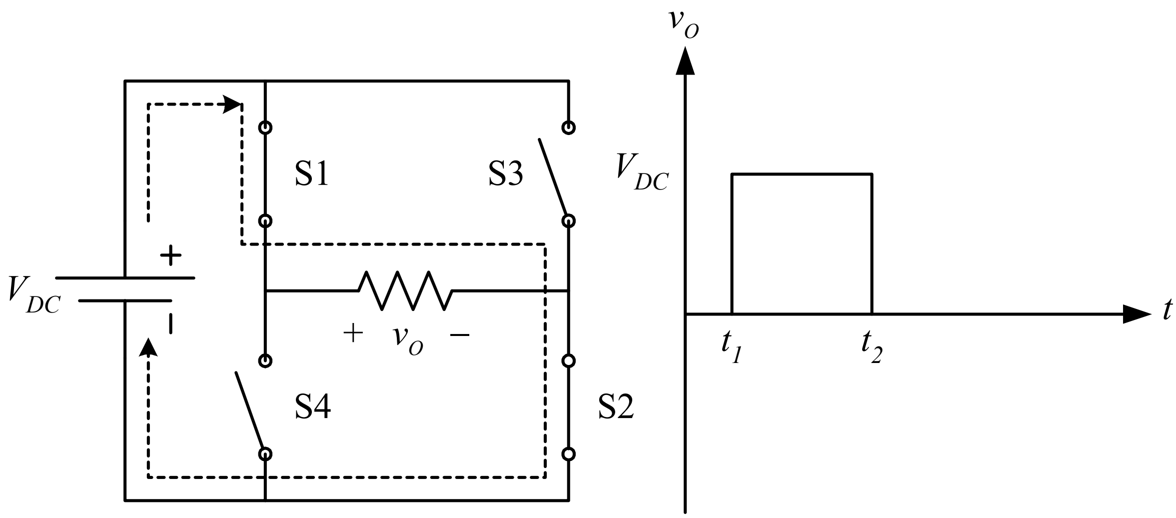
- To illustrate the concept of AC waveform generation



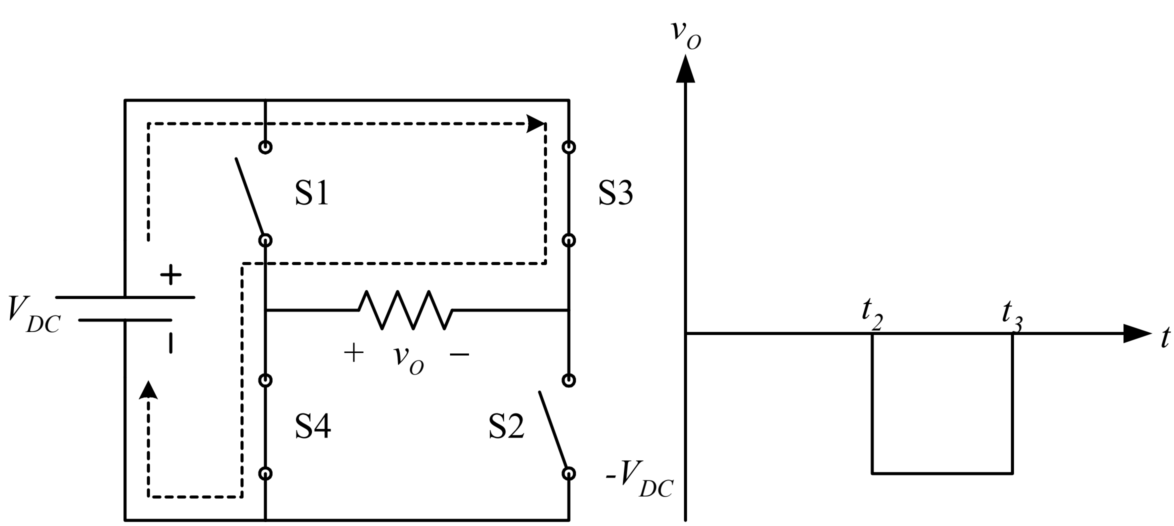


Operation of simple square-wave inverter (2)

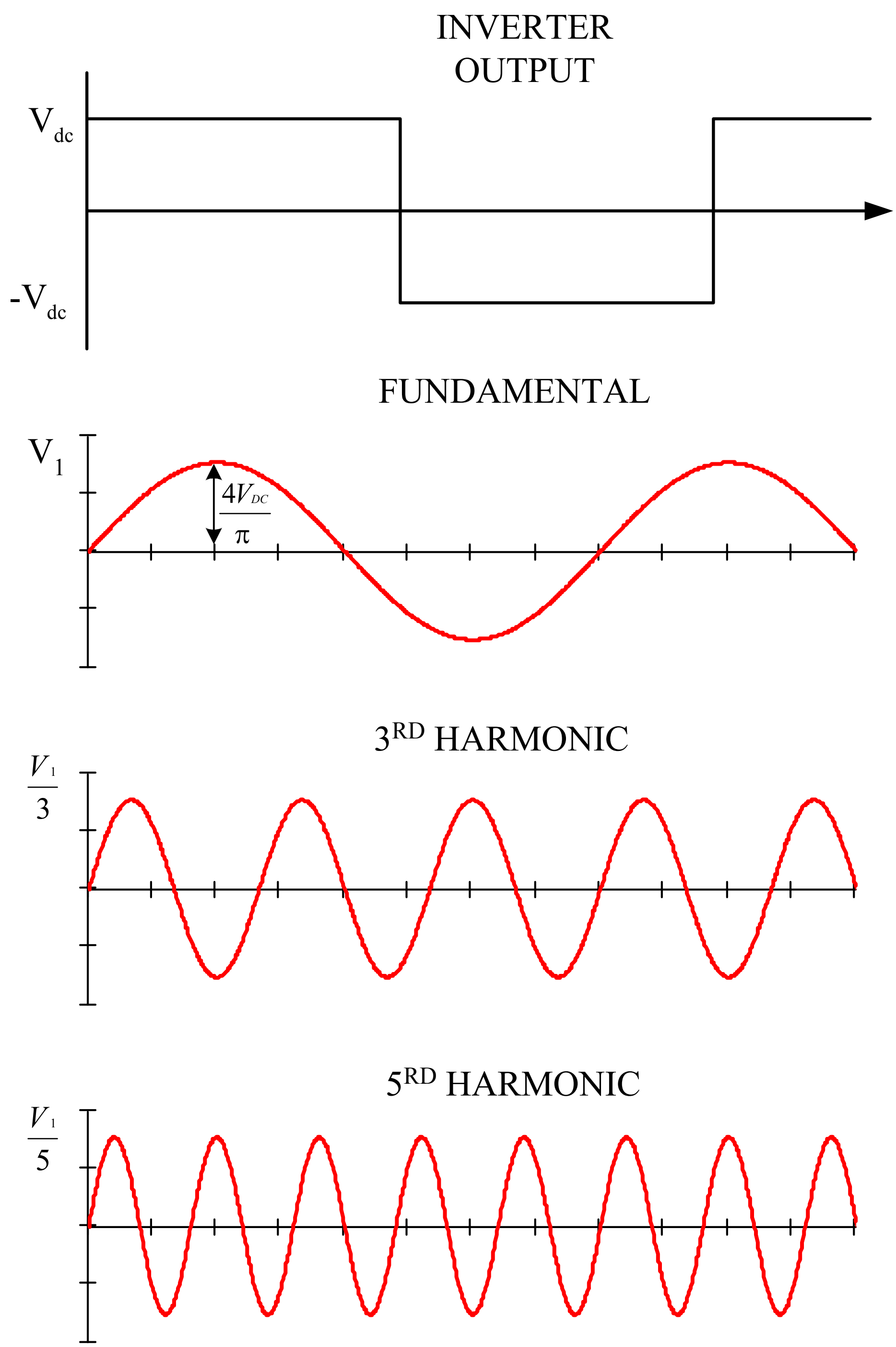
S1,S2 ON; S3,S4 OFF for $t_1 < t < t_2$



S3,S4 ON ; S1,S2 OFF for $t_2 < t < t_3$



Waveforms and harmonics of square-wave inverter

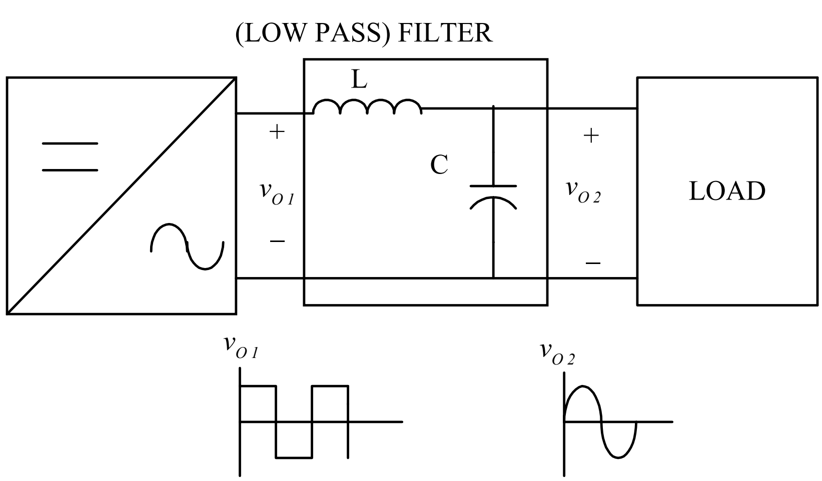


Filtering

- Output of the inverter is "chopped AC voltage with zero DC component".In some applications such as UPS, "*high purity*" sine wave output is required.

- An LC section low-pass filter is normally fitted at the inverter output to reduce the high frequency harmonics.

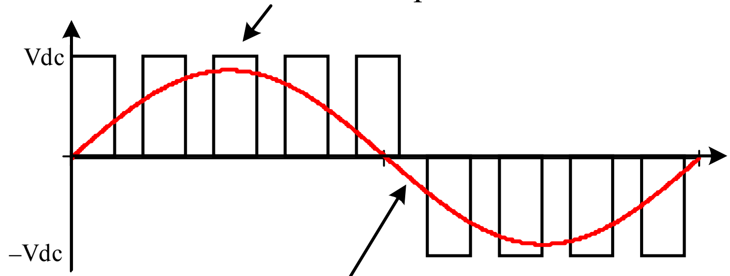
- In some applications such as AC motor drive, filtering is not required.

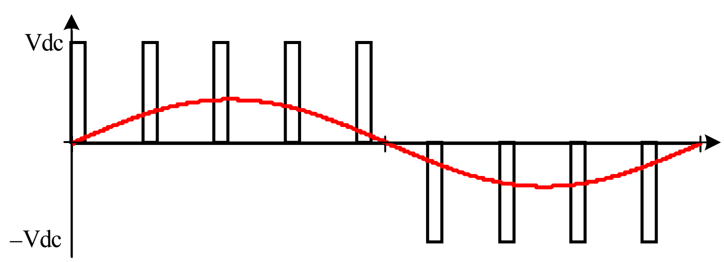


Notes on low-pass filters

- In square wave inverters, maximum output voltage is achievable. However there in NO control in harmonics and output voltage magnitude.
- The harmonics are always at three, five, seven etc times the fundamental frequency.
- Hence the cut-off frequency of the low pass filter is somewhat fixed. The filter size is dictated by the VA ratings of the inverter.
- To reduce filter size, the *PWM switching scheme* can be utilised.
- In this technique, the harmonics are “pushed” to higher frequencies. Thus the cut-off frequency of the filter is increased. Hence the filter components (I.e. L and C) sizes are reduced.
- The trade off for this flexibility is complexity in the switching waveforms.

“Notching”of square wave

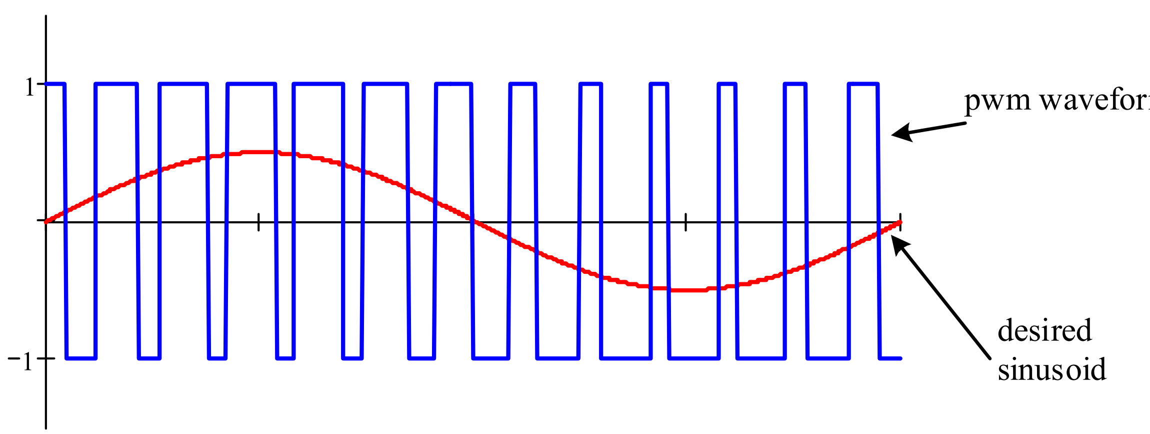




- Notching results in controllable output voltage magnitude (compare Figures above).
- Limited degree of harmonics control is possible

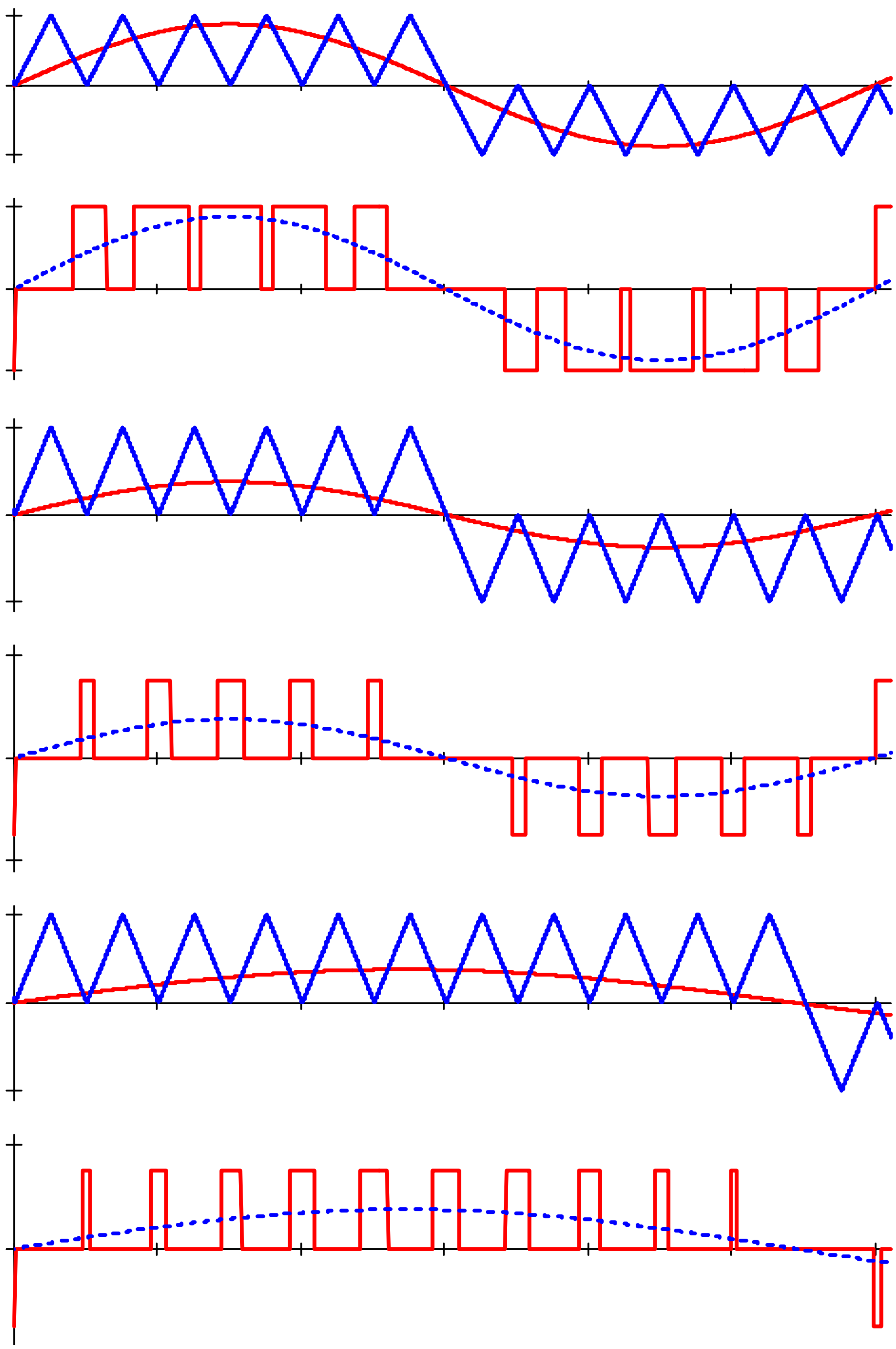
Pulse-width modulation (PWM)

- A better square wave notching is shown below - this is known as PWM technique.
- Both amplitude and frequency can be controlled independently. Very flexible.



SINUSOIDAL PULSE-WITDH MODULATED
APPROXIMATION TO SINE WAVE

PWM- output voltage and frequency control



Output voltage harmonics

- Why need to consider harmonics?
  - Waveform quality must match TNB supply. "Power Quality" issue.
  - Harmonics may cause degradation of equipment. Equipment need to be "de-rated".
- Total Harmonic Distortion (THD) is a measure to determine the "quality" of a given waveform.

- DEFINITION of THD (voltage)

$$THDv = \frac{\sqrt{\sum_{n=2}^{\infty}\left(V_{n,RMS}\right)^2}}{V_{1,RMS}} = \frac{\sqrt{\sum_{n=2}^{\infty}\left(V_{RMS}\right)^2 - \left(V_{1,RMS}\right)^2}}{V_{1,RMS}}$$

where n is the harmonics number.
Current THD can be obtained by replacing the harmonic voltage with harmonic current :

$$THDi = \frac{\sqrt{\sum_{n=2}^{\infty}\left(I_{n,RMS}\right)^2}}{I_{1,RMS}}$$

$$I_n = \frac{V_n}{Z_n}$$

Z_n is the impedance at harmonic frequency.

Fourier Series

- Study of harmonics requires understanding of wave shapes. Fourier Series is a tool to analyse wave shapes.

Fourier Series

$$a_o = \frac{1}{\pi}\int_0^{2\pi} f(v)\,d\theta$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\cos(n\theta)\,d\theta$$

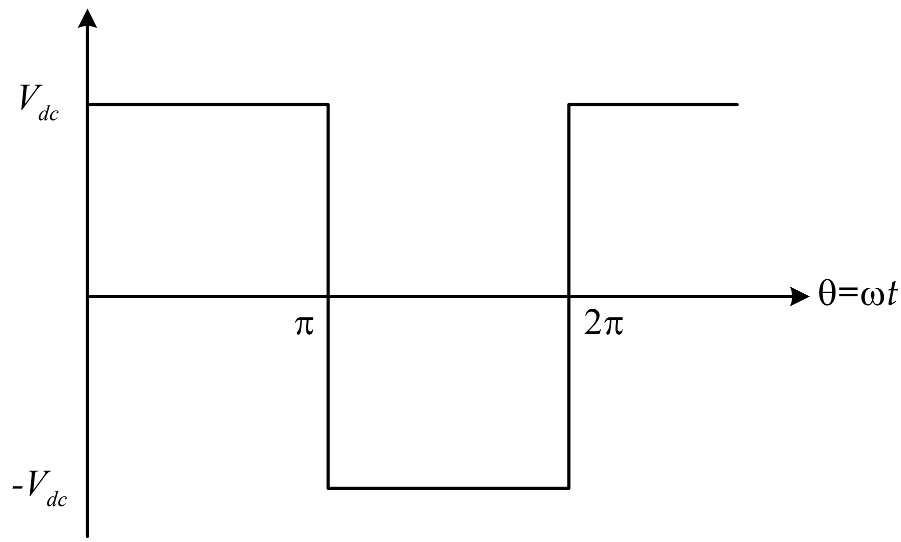
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(v)\sin(n\theta)\,d\theta$$

Inverse Fourier

$$f(v) = \frac{1}{2}a_o + \sum_{n=1}^{\infty}\left(a_n \cos n\theta + b_n \sin n\theta\right)$$

where $\theta = \omega t$

Harmonics of square-wave (1)



$$a_o = \frac{1}{\pi}\left[\int_0^{\pi} V_{dc}\,d\theta + \int_{\pi}^{2\pi} -V_{dc}\,d\theta\right] = 0$$

$$a_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} \cos(n\theta)\,d\theta - \int_{\pi}^{2\pi} \cos(n\theta)\,d\theta\right] = 0$$

$$b_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} \sin(n\theta)\,d\theta - \int_{\pi}^{2\pi} \sin(n\theta)\,d\theta\right]$$

Harmonics of square wave (2)

Solving,

$$b_n = \frac{V_{dc}}{n\pi}\left[-\cos(n\theta)\Big|_0^{\pi} + \cos(n\theta)\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{V_{dc}}{n\pi}\left[(\cos 0 - \cos n\pi) + (\cos 2n\pi - \cos n\pi)\right]$$

$$= \frac{V_{dc}}{n\pi}\left[(1 - \cos n\pi) + (1 - \cos n\pi)\right]$$

$$= \frac{2V_{dc}}{n\pi}\left[(1 - \cos n\pi)\right]$$

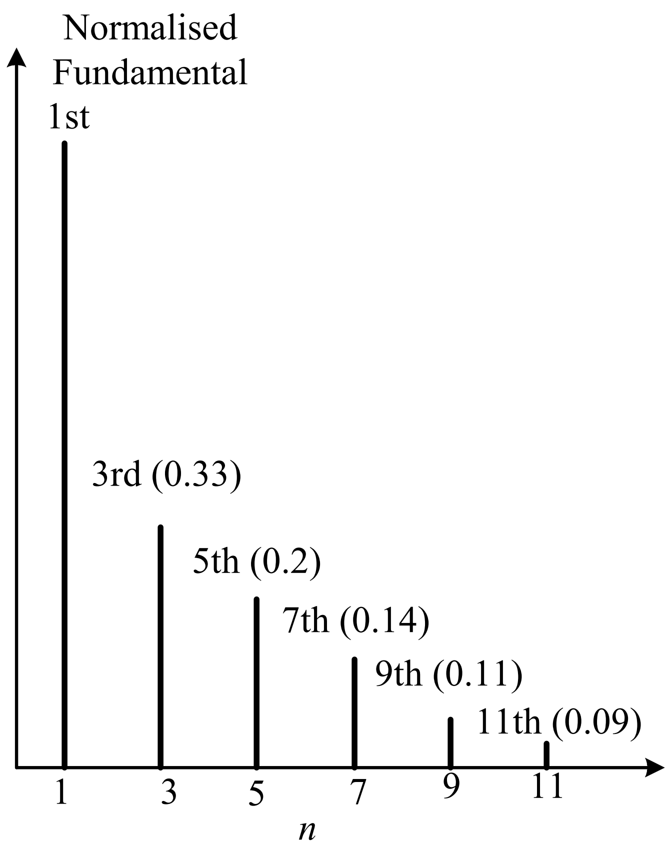
when n is even, $\cos n\pi = 1$

$b_{n=} 0$

when n is odd, $\cos n\pi = -1$

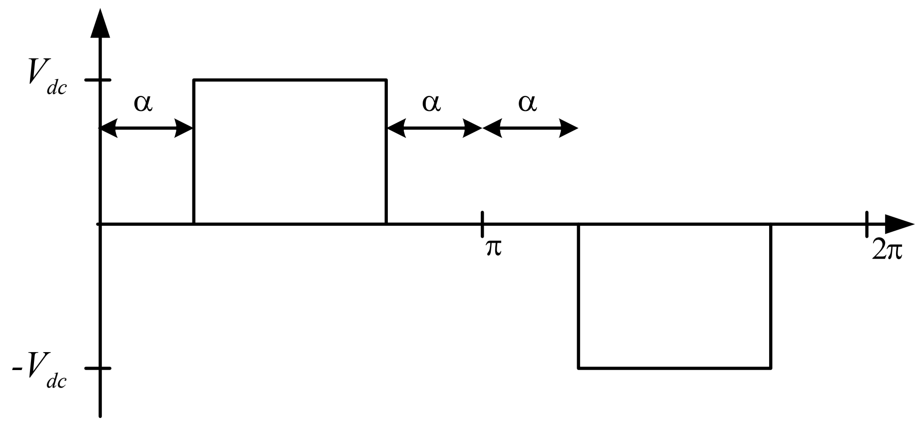
$$b_{n=} \frac{4V_{dc}}{n\pi}$$

Spectra of square wave



- Spectra (harmonics) characteristics:
  - Harmonic decreases as n increases. It decreases with a factor of $(1/n)$.
  - Even harmonics are absent
  - Nearest harmonics is the 3rd. If fundamental is 50Hz, then nearest harmonic is 150Hz.
  - Due to the small separation between the fundamental an harmonics, output low-pass filter design can be quite difficult.

Quasi-square wave (QSW)



Note that $a_n = 0$.

Due to half - wave symmetry,

$$b_n = 2\left[\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}\sin(n\theta)d\theta\right] = \frac{2V_{dc}}{n\pi}\left[-\cos n\theta\Big|_{\alpha}^{\pi-\alpha}\right]$$

$$= \frac{2V_{dc}}{n\pi}\left[\cos(n\alpha) - \cos n(\pi - \alpha)\right]$$

Expanding,

$$\cos n(\pi - \alpha) = \cos(n\pi - n\alpha)$$
$$= \cos n\pi \cos n\alpha + \sin n\pi \sin n\alpha$$
$$= \cos n\pi \cos n\alpha$$

Harmonics control

$$\Rightarrow b_n = \frac{2V_{dc}}{n\pi}\left[\cos(n\alpha) - \cos n\pi \cos n\alpha\right]$$

$$= \frac{2V_{dc}}{n\pi}\cos(n\alpha)\left[1 - \cos n\pi\right]$$

If n is even, $\Rightarrow b_n = 0,$

If n is odd, $\Rightarrow b_n = \frac{4V_{dc}}{n\pi}\cos(n\alpha)$

In particular, amplitude of the fundamental is :

$$b_1 = \frac{4V_{dc}}{\pi}\cos(\alpha)$$

The fundamental , b_1, is controlled by varying α

Harmonics can also be controlled by adjusting α,
For example if $\alpha = 30^o$, then $b_3 = 0$, or the third harmonic is eliminated from the waveform. In general, harmonic n will be eliminated if :

$$\alpha = \frac{90^o}{n}$$

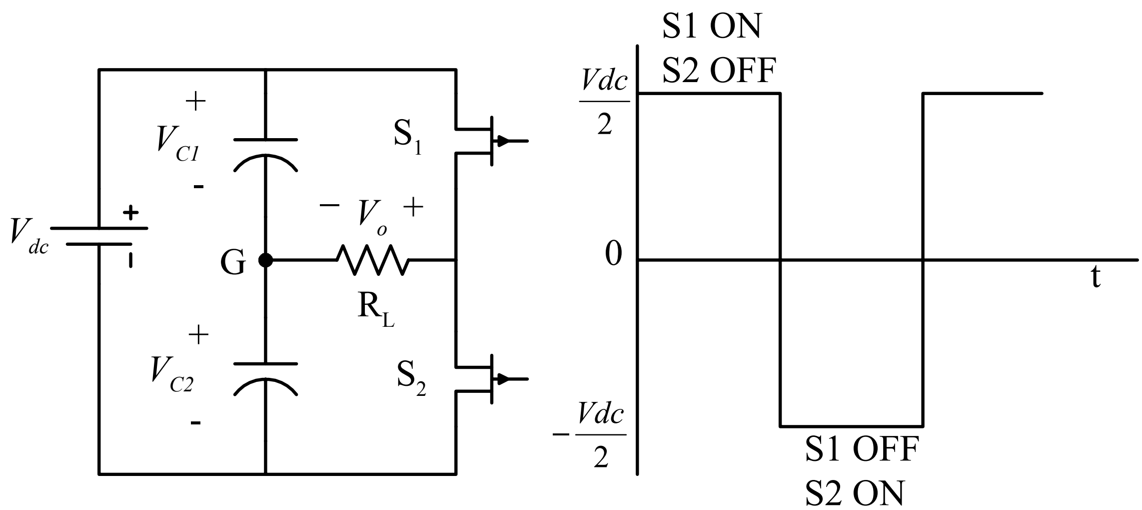
Example

A full - bridge single phase inverter is fed by square wave signals. The DC link voltage is 100V. The load is $R = 10R$ and $L = 10mH$ in series. Calculate:

a) the THDv using the "exact" formula.
b) the THDv by using the first three non - zero harmonics
c) the THDi by using the first three non - zero harmonics

Repeat (b) and (c) for quasi - square wave case with $\alpha = 30$ degrees

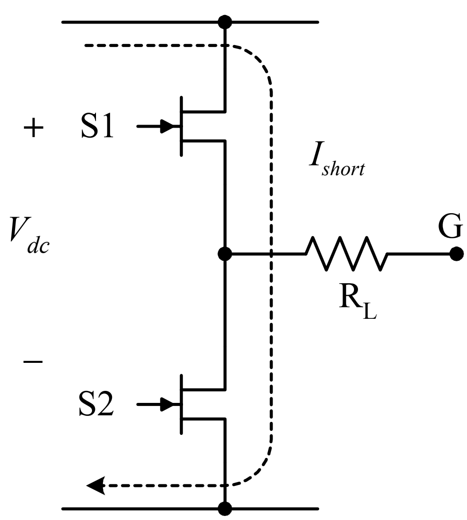
Half-bridge inverter (1)



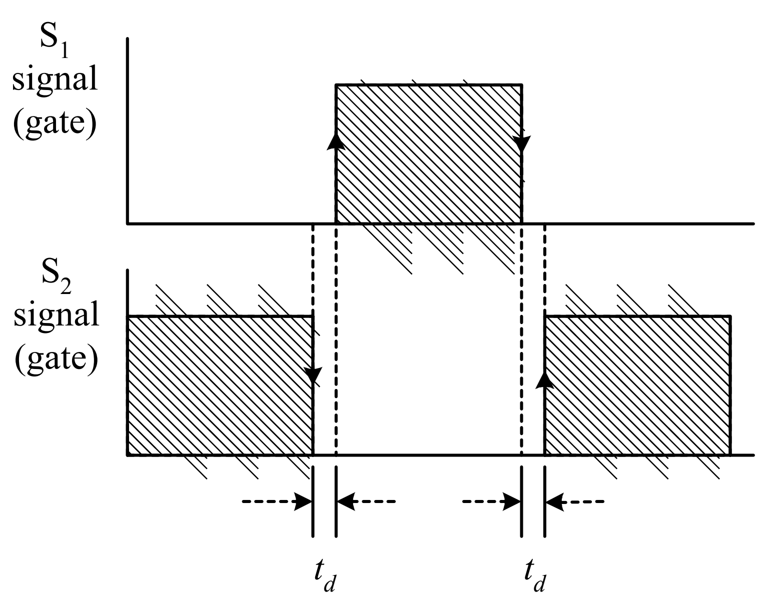
- Also known as the “inverter leg”.
- Basic building block for full bridge, three phase and higher order inverters.
- G is the “centre point”.
- Both capacitors have the same value. Thus the DC link is equally “spilt”into two.

Half-bridge inverter (2)

- The top and bottom switch has to be "complementary", i.e. If the top switch is closed (on), the bottom must be off, and vice-versa.
- In practical, a dead time as shown below is required to avoid "shoot-through" faults.



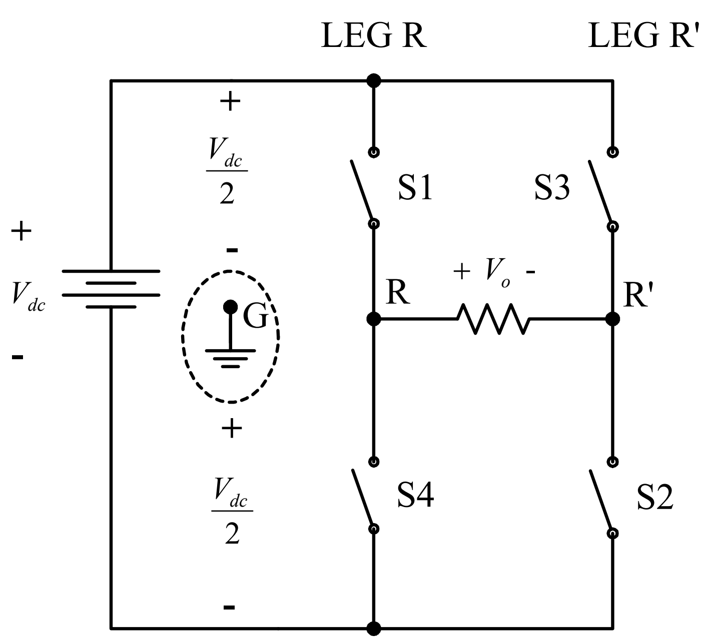
"Shoot through fault" .
I_{short} is very large



"Dead time' $= t_d$

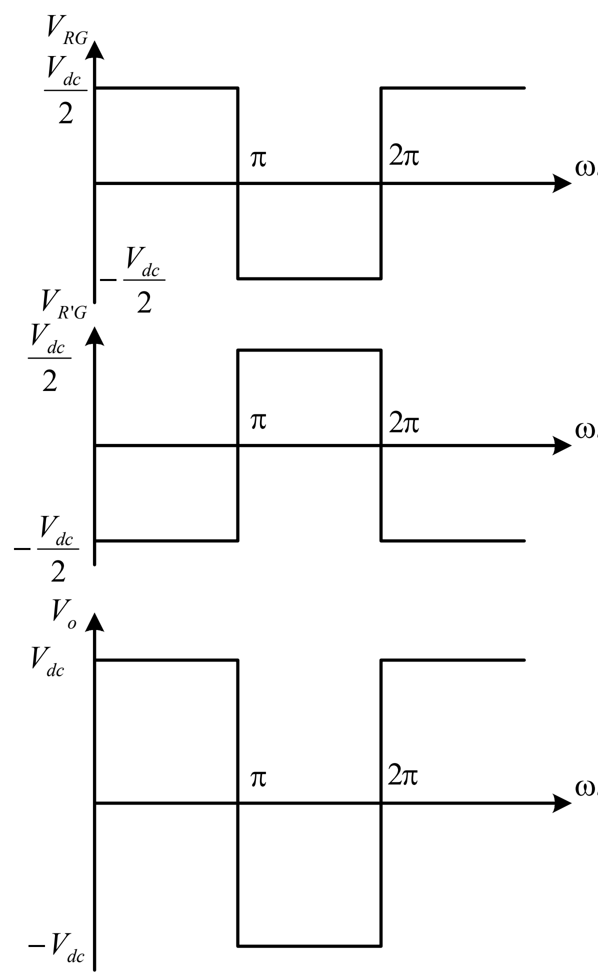
Single-phase, full-bridge (1)

- Full bridge (single phase) is built from two half-bridge leg.
- The switching in the second leg is "delayed by 180 degrees" from the first leg.



$V_o = V_{RG} - V_{R'G}$

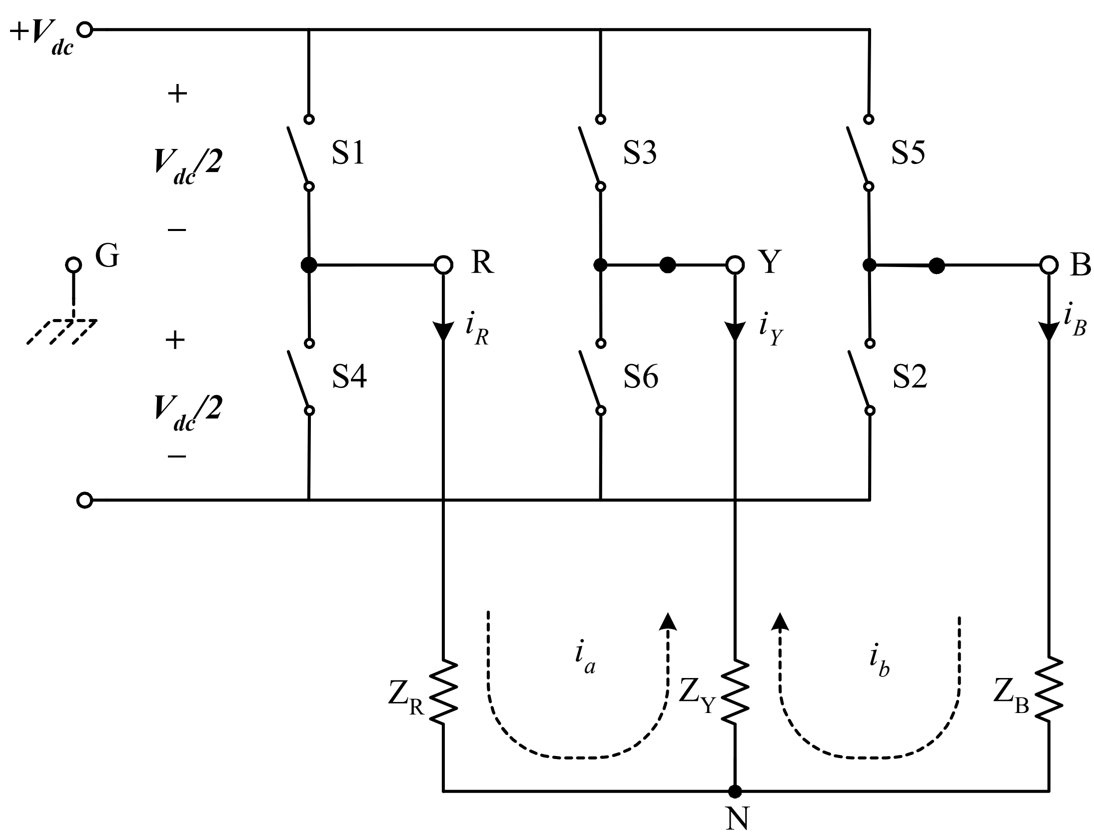
G is "virtual groumd"



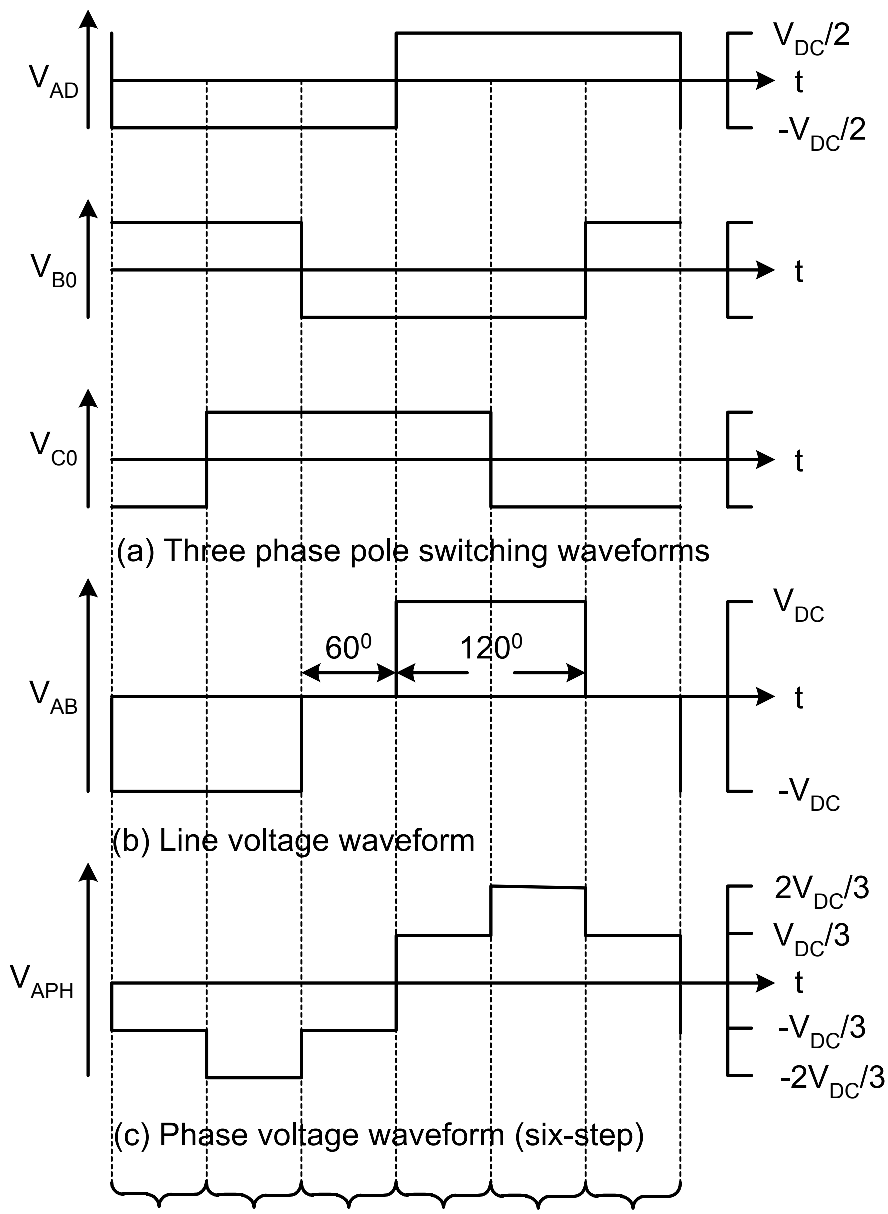
Three-phase inverter

- Each leg (Red, Yellow, Blue) is delayed by 120 degrees.

- A three-phase inverter with star connected load is shown below



Square-wave inverter waveforms



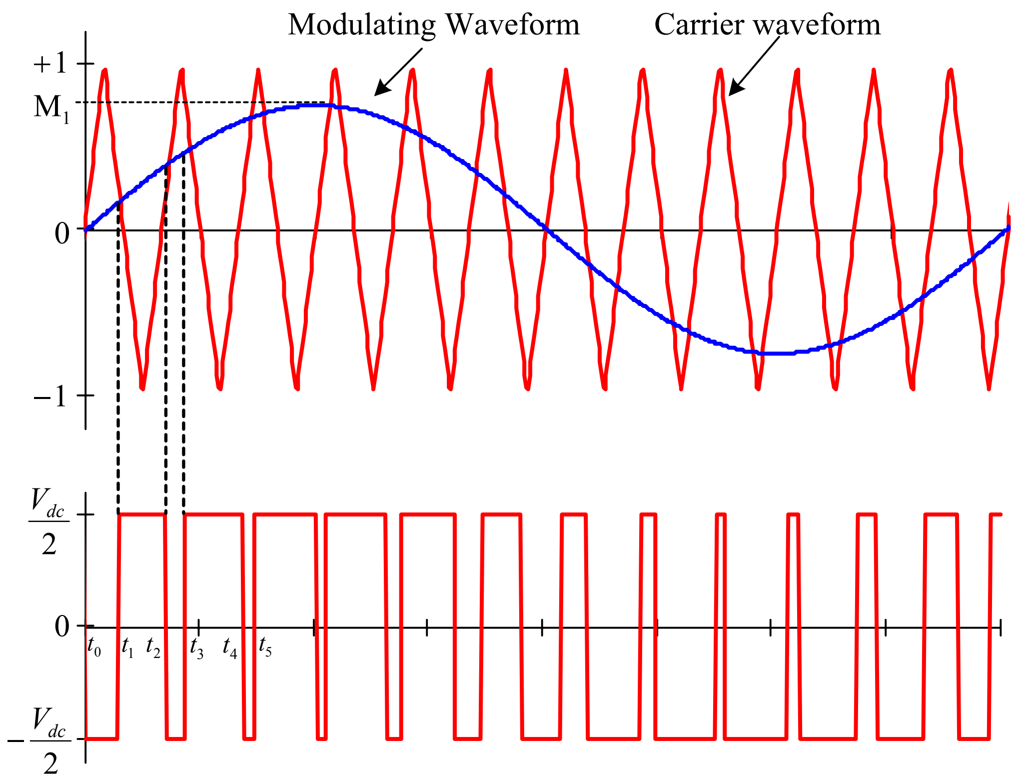
Interval	1	2	3	4	5	6
Positive device(s) on	3	3,5	5	1,5	1	1,3
Negative devise(s) on	2,4	4	4,6	6	2,6	2

Quasi-square wave operation voltage waveforms

Three-phase inverter waveform relationship

- V_{RG}, V_{YG}, V_{BG} are known as “pole switching waveform” or “inverter phase voltage”.
- V_{RY}, V_{RB}, V_{YB} are known as “line to line voltage” or simply “line voltage”.
- For a three-phase star-connected load, the load phase voltage with respect to the “N” (star-point) potential is known as V_{RN}, V_{YN}, V_{BN}. It is also popularly termed as “six-step” waveform

MODULATION: Pulse Width Modulation (PWM)



- Triangulation method (Natural sampling)
  - Amplitudes of the triangular wave (carrier) and sine wave (modulating) are compared to obtain PWM waveform. Simple analogue comparator can be used.
  - Basically an analogue method. Its digital version, known as REGULAR sampling is widely used in industry.

PWM types

- Natural (sinusoidal) sampling (as shown on previous slide)
  – Problems with analogue circuitry, e.g. Drift, sensitivity etc.
- Regular sampling
  – simplified version of natural sampling that results in simple digital implementation
- Optimised PWM
  – PWM waveform are constructed based on certain performance criteria, e.g. THD.
- Harmonic elimination/minimisation PWM
  – PWM waveforms are constructed to eliminate some undesirable harmonics from the output waveform spectra.
  – Highly mathematical in nature
- Space-vector modulation (SVM)
  – A simple technique based on volt-second that is normally used with three-phase inverter motor-drive

Natural/Regular sampling

MODULATION INDEX = M_I :

$$M_I = \frac{\text{Amplitude of the modulating waveform}}{\text{Amplitude of the carrier waveform}}$$

M_I is related to the fundamental (sine wave) output voltage magnitude. If M_I is high, then the sine wave output is high and vice versa.

If $0 < M_I < 1$, the linear relationship holds :

$$V_1 = M_I V_{in}$$

where V_1, V_{in} are fundamental of the output voltage and input (DC) voltage, respectively.

MODULATION RATIO = M_R (= p)

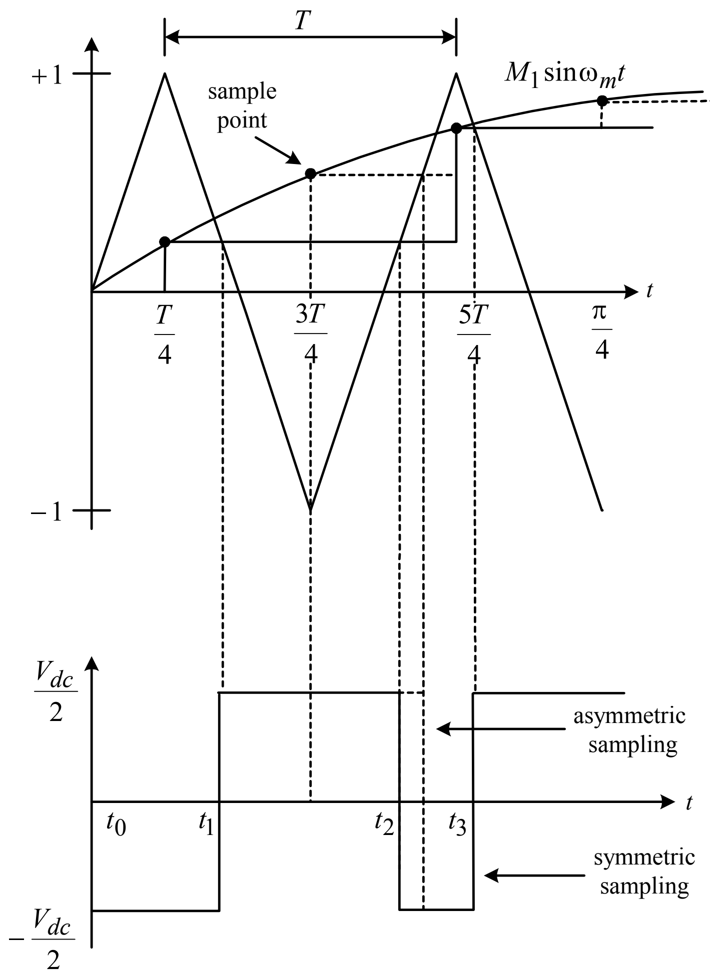
$$M_R = p = \frac{\text{Frequency of the carrier waveform}}{\text{Frequency of the modulating waveform}}$$

M_R is related to the "harmonic frequency". The harmonics are normally located at :

$$f = kM_R(f_m)$$

where f_m is the frequency of the modulating signal and k is an integer (1,2,3...)

Asymmetric and symmetric regular sampling

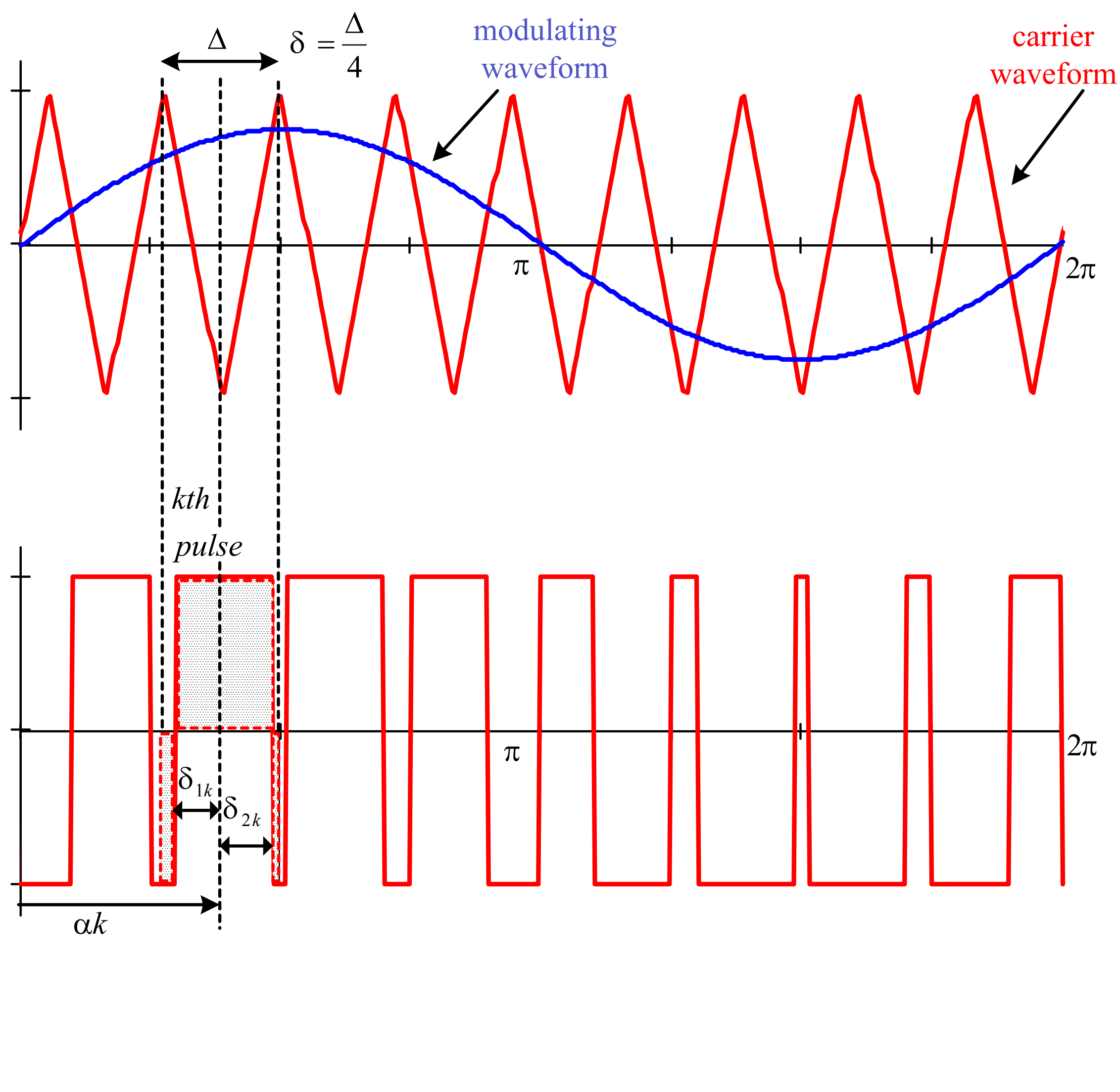


Generating of PWM waveform regular sampling

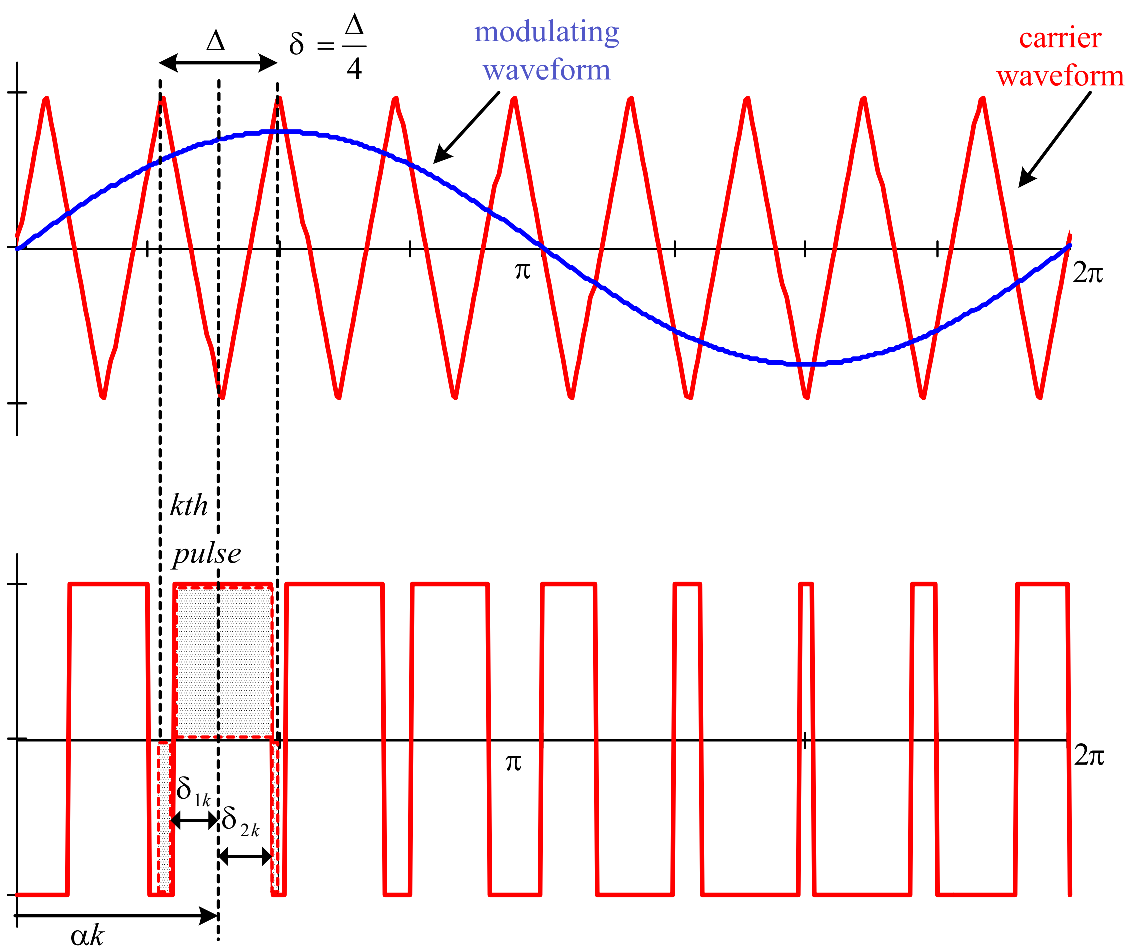
Bipolar and unipolar PWM switching scheme

- In many books, the term “bipolar” and “unipolar” PWM switching are often mentioned.
- The difference is in the way the sinusoidal (modulating) waveform is compared with the triangular.
- In general, unipolar switching scheme produces better harmonics. But it is more difficult to implement.
- In this class only bipolar PWM is considered.

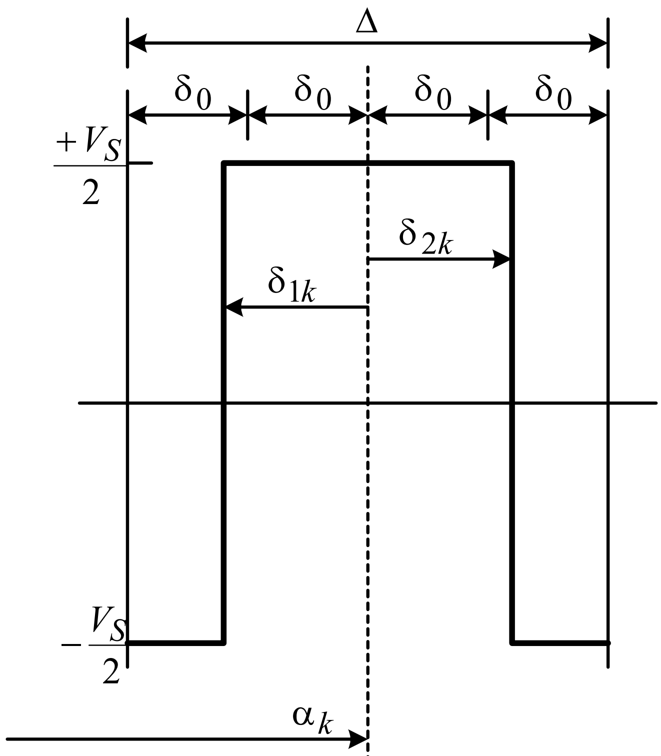
Bipolar PWM switching



Pulse width relationships

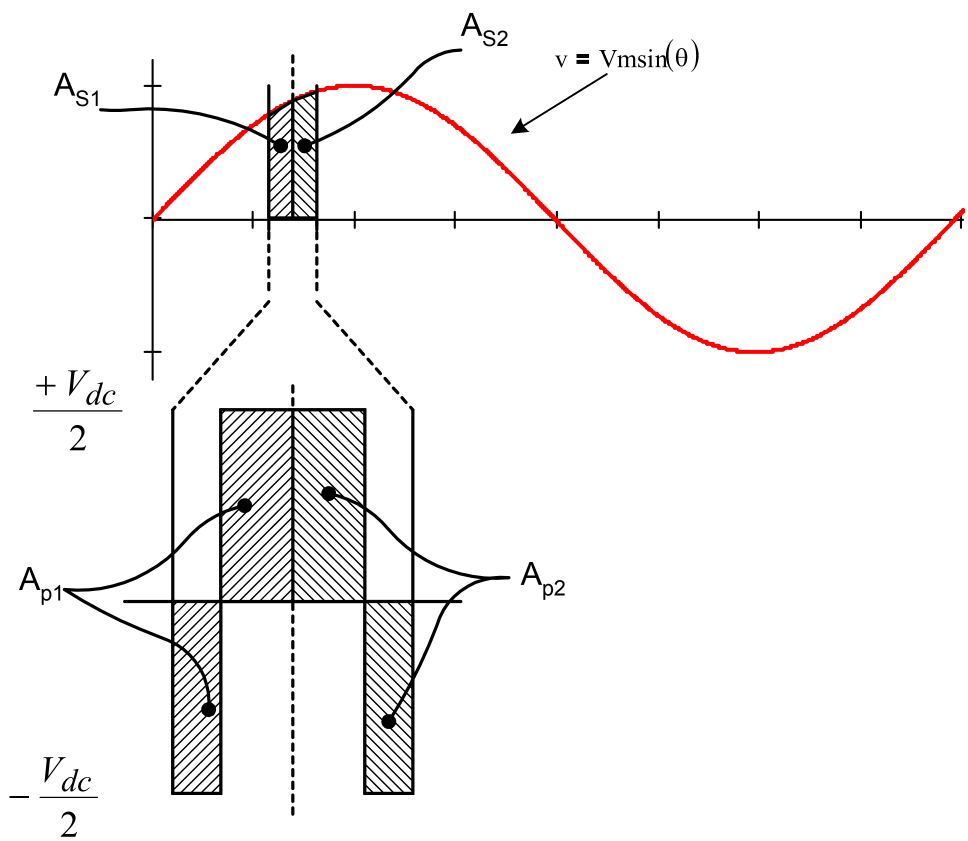


Characterisation of PWM pulses for bipolar switching



The kth PWM pulse

Determination of switching angles for kth PWM pulse (1)



Equating the volt - second,

$$A_{s1} = A_{p1}$$

$$A_{s2} = A_{p2}$$

PWM Switching angles (2)

The average voltage during each half cycle of the PWM pulse is given as :

$$\overline{V}_{1k} = \left(\frac{V_{dc}}{2}\right)\left(\frac{\delta_{1k} - (2\delta_o - \delta_{1k})}{2\delta_o}\right)$$

$$= \left(\frac{V_{dc}}{2}\right)\left(\frac{\delta_{1k} - \delta_o}{\delta_o}\right) = \beta_{1k}\left(\frac{V_s}{2}\right)$$

$$\text{where } \beta_{1k} = \left(\frac{\delta_{1k} - \delta_o}{\delta_o}\right)$$

Similarly,

$$\overline{V}_{2k} = \beta_{2k}\left(\frac{V_{dc}}{2}\right) \text{ ; where } \beta_{2k} = \left(\frac{\delta_{2k} - \delta_o}{\delta_o}\right)$$

The volt - second supplied by the sinusoid,

$$A_{s1} = \int_{\alpha_k - 2\delta_o}^{\alpha_k} V_m \sin\theta d\theta = V_m\left[\cos(\alpha_k - 2\delta_o) - \cos\alpha_k\right]$$

$$= 2V_m \sin\delta_o \sin(\alpha_k - \delta_o)$$

Switching angles (3)

Since,

$\sin\delta_o \rightarrow \delta_o$ for small δ_o,

$A_{s1} = 2\delta_o V_m \sin(\alpha_k - \delta_o)$

Similarly,

$A_{s2} = 2\delta_o V_m \sin(\alpha_k + \delta_o)$

The volt-seconds of the PWM waveforms,

$$A_{p1} = \beta_{1k}\left(\frac{V_{dc}}{2}\right)2\delta_o; \qquad A_{p2} = \beta_{21k}\left(\frac{V_{dc}}{2}\right)2\delta_o$$

To derive the modulation strategy,

$A_{p1} = A_{s1}; \quad A_{p2} = A_{s2}$

Hence, for the leading edge

$$\beta_{1k}\left(\frac{V_{dc}}{2}\right)2\delta_o = 2\delta_o V_m \sin(\alpha_k - \delta_o)$$

$$\Rightarrow \beta_{1k} = \frac{V_m}{(V_{dc}/2)}\sin(\alpha_k - \delta_o)$$

PWM switching angles (4)

The voltage ratio,

$$M_I = \frac{V_m}{(V_{dc}/2)}$$ is known as modulation index or depth. It varies from 0 to 1.

Thus,

$$\beta_{1k} = M_I \sin(\alpha_k - \delta_o)$$

Using similar method, the trailing edge can be derived :

$$\beta_{2k} = M_I \sin(\alpha_k - \delta_o)$$

Substituting to solve for the pulse - width,

$$\beta_{1k} = \frac{\delta_{1k} - \delta_o}{\delta_o}$$

$$\Rightarrow \delta_{1k} = \delta_o[1 + M_I \sin(\alpha_k - \delta_o)]$$

and

$$\delta_{2k} = \delta_o[1 + M_I \sin(\alpha_k + \delta_o)]$$

PWM Pulse width

Thus the switching angles of the kth pulse is :

Leading edge : $\alpha_k - \delta_{1k}$

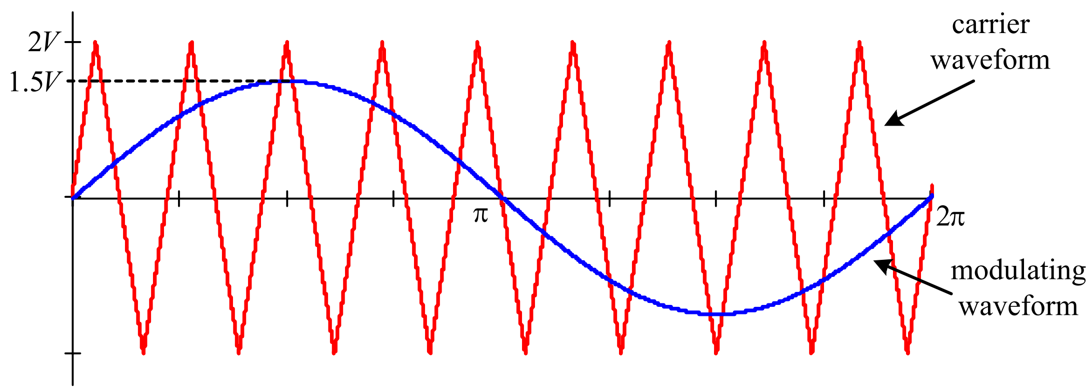
Trailing edge : $\alpha_k + \delta_{1k}$

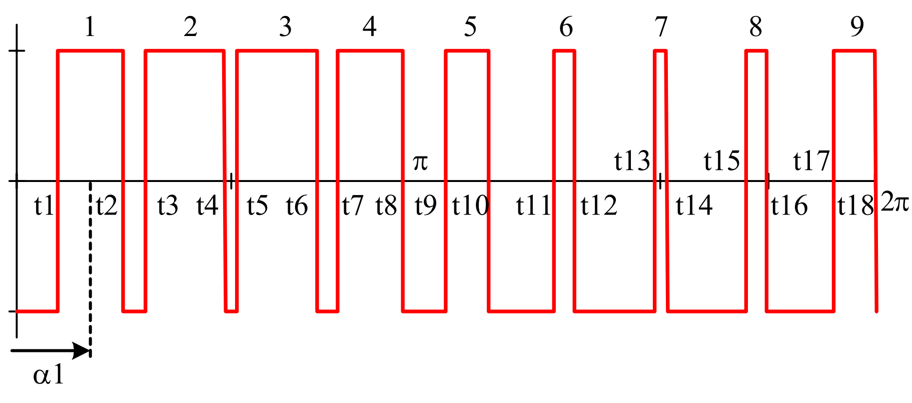
The above equation is valid for Asymmetric Modulation, i.e $\delta_{1\mathrm{k}}$ and $\delta_{2\mathrm{k}}$ are different.
For Symmetric Modulation, $\delta_{1\mathrm{k}} = \delta_{2\mathrm{k}} = \delta_{\mathrm{k}}$

$$\Rightarrow \delta_k = \delta_o \left[1 + M_I \sin\alpha_k\right]$$

Example

- For the PWM shown below, calculate the switching angles for all the pulses.





Harmonics of bipolar PWM

Assuming the PWM waveform is half - wave symmetry, harmonic content of each (kth) PWM pulse can be computed as :

$$b_{nk} = 2\left(\frac{1}{\pi} \int_0^T f(v) \sin n\theta d\theta \right)$$

$$= \frac{2}{\pi} \left\{ \int_{\alpha_k - 2\delta_o}^{\alpha_k - \delta_{1k}} \left(-\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

$$+ \frac{2}{\pi} \left\{ \int_{\alpha_k - \delta_{1k}}^{\alpha_k + \delta_{2k}} \left(\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

$$+ \frac{2}{\pi} \left\{ \int_{\alpha_k + \delta_{2k}}^{\alpha_k + 2\delta_o} \left(-\frac{V_{dc}}{2} \right) \sin n\theta d\theta \right\}$$

Which can be reduced to :

$$\begin{aligned} b_{nk} = -\frac{V_{dc}}{n\pi} \{ & \cos n(\alpha_k - 2\delta_o) - \cos n(\alpha_k - \delta_{1k}) \\ & + \cos n(\alpha_k + \delta_{2k}) - \cos n(\alpha_k - \delta_{1k}) \\ & + \cos n(\alpha_k + \delta_{2k}) - \cos n(\alpha_k + 2\delta_o) \} \end{aligned}$$

Harmonics of PWM

Yeilding,

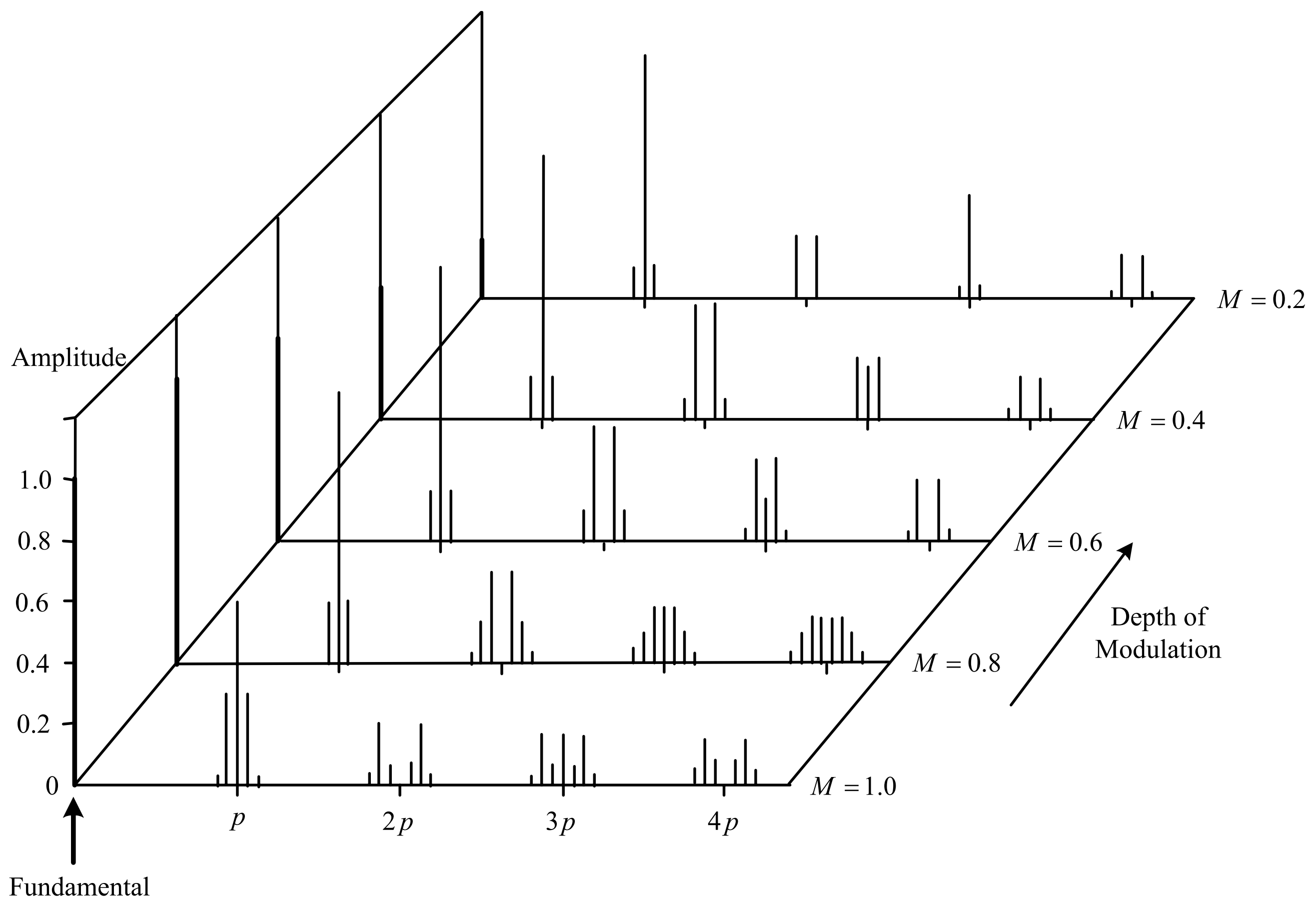
$$b_{nk} = \frac{2V_{dc}}{n\pi}\left[\cos n(\alpha_k - \delta_{1k}) - \cos n(\alpha_k - 2_{1k}) + 2\cos n\alpha_k \cos n2\delta_o\right]$$

This equation cannot be simplified productively.The Fourier coefficent for the PWM waveform isthe sum of b_{nk} for the p pulses over one period, i.e.:

$$b_n = \sum_{k=1}^{p} b_{nk}$$

The slide on the next page shows the computation of this equation.

PWM Spectra



NORMALISED HARMONIC AMPLITUDES FOR
SINUSOIDAL PULSE-WITDH MODULATION

PWM spectra observations

- The amplitude of the fundamental decreases or increases linearly in proportion to the depth of modulation (modulation index). The relation ship is given as: $V_1 = M_I V_{in}$

- The harmonics appear in "clusters" with main components at frequencies of :
  $f = kp\,(f_m)$; k=1,2,3....
  where f_m is the frequency of the modulation (sine) waveform. This also equal to the multiple of the carrier frequencies. There also exist "side-bands" around the main harmonic frequencies.

- The amplitude of the harmonic changes with M_I. Its incidence (location on spectra) is not.

- When p>10, or so, the harmonics can be normalised as shown in the Figure. For lower values of p, the side-bands clusters overlap, and the normalised results no longer apply.

Bipolar PWM Harmonics

h \ M_I	0.2	0.4	0.6	0.8	1.0
1	0.2	0.4	0.6	0.8	1.0
M_R	1.242	1.15	1.006	0.818	0.601
$M_R \pm 2$	0.016	0.061	0.131	0.220	0.318
$M_R \pm 4$					0.018
$2M_R \pm 1$	0.190	0.326	0.370	0.314	0.181
$2M_R \pm 3$		0.024	0.071	0.139	0.212
$2M_R \pm 5$				0.013	0.033
$3M_R$	0.335	0.123	0.083	0.171	0.113
$3M_R \pm 2$	0.044	0.139	0.203	0.716	0.062
$3M_R \pm 4$		0.012	0.047	0.104	0.157
$3M_R \pm 6$				0.016	0.044
$4M_R \pm 1$	0.163	0.157	0.008	0.105	0.068
$4M_R \pm 3$	0.012	0.070	0.132	0.115	0.009
$4M_R \pm 5$			0.034	0.084	0.119
$4M_R \pm 7$				0.017	0.050

Bipolar PWM harmonics calculation example

Note : for full bridge single - phase bipolar PWM,

$$v_o = v_{RR'} = v_{RG} - v_{R'G}$$
$$= 2v_{RG}$$

The harmonics are computed from :

$$\left(\hat{V}_{RG}\right)_n \Big/ \left(\frac{V_{DC}}{2}\right)$$

as a function of M_I

Example :

In the full - bridge single phase PWM inverter, $V_{DC} = 100V$, $M_I = 0.8$, $M_R = 39$. The fundamentalfrequency is 47Hz. Calculate the values of the fundamental - frequency voltage and some of the dominant harmonics.

Three-phase harmonics: "Effect of odd triplens"

- For three-phase inverters, there is significant advantage if *p* is chosen to be:
  - **odd and multiple of three (triplens)** (e.g. 3,9,15,21, 27..)
  - the waveform and harmonics and shown on the next two slides. Notice the difference?
- By observing the waveform, it can be seen that with odd *p*, the line voltage shape looks more "sinusoidal".
- The even harmonics are all absent in the phase voltage (pole switching waveform). This is due to the *p* chosen to be odd.

Spectra observations

- Note the absence of harmonics no. 21, 63 in the inverter line voltage. This is due to *p* which is multiple of three.

- In overall, the spectra of the line voltage is more “clean”. This implies that the THD is less and the line voltage is more sinusoidal.

- It is important to recall that it is the line voltage that is of the most interest.

- Also can be noted from the spectra that the phase voltage amplitude is 0.8 (normalised). This is because the modulation index is 0.8. The line voltage amplitude is square root three of phase voltage due to the three-phase relationship.

Waveform: effect of "triplens"

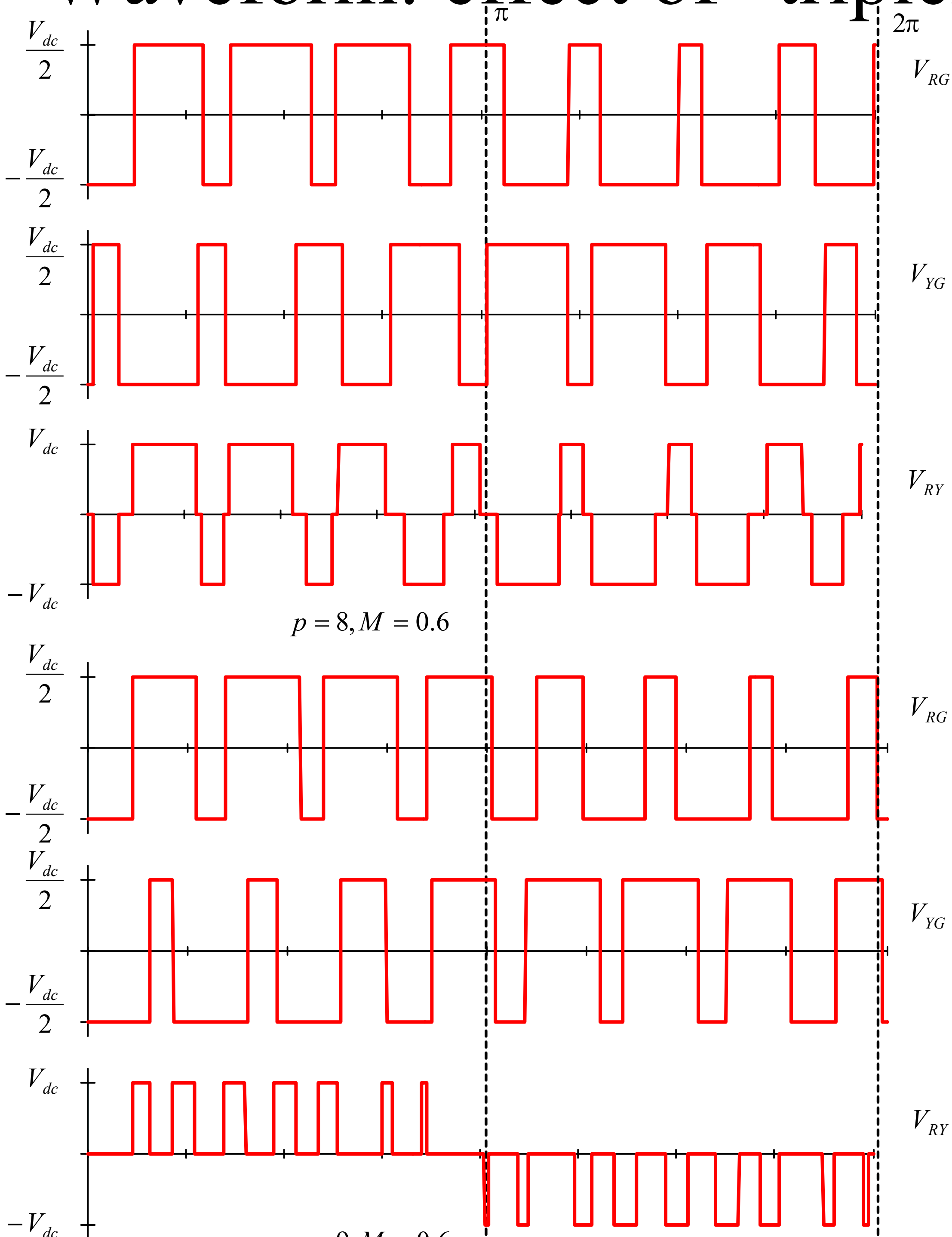
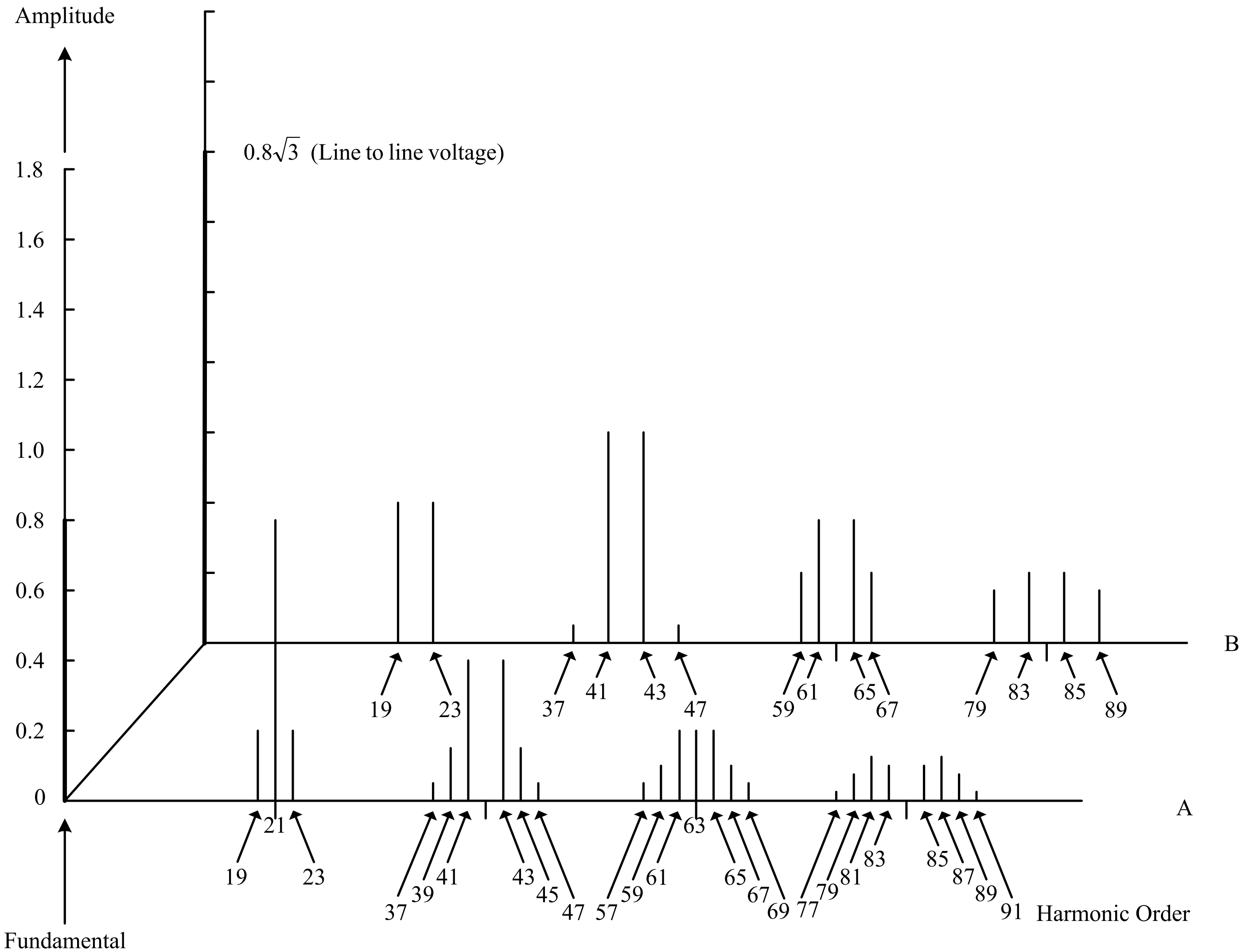


ILLUSTRATION OF BENEFITS OF USING A FREQUENCY RATIO THAT IS A MULTIPLE OF THREE IN A THREE PHASE INVERTER

Harmonics: effect of "triplens"



COMPARISON OF INVERTER PHASE VOLTAGE (A) & INVERTER LINE VOLTAGE (B) HARMONIC (P=21, M=0.8)

Comments on PWM scheme

- It is desirable to push *p* to as large as possible.
- The main impetus for that when p is high, then the harmonics will be at higher frequencies because frequencies of harmonics are related to: $f = kp(f_m)$, where f_m is the frequency of the modulating signal.
- Although the voltage THD improvement is not significant, but the current THD will improve greatly because the load normally has some current filtering effect.
- In any case, if a low pass filter is to be fitted at the inverter output to improve the voltage THD, higher harmonic frequencies is desirable because it makes smaller filter component.

Example

The amplitudes of the pole switching waveform harmonics of the red phase of a three-phase inverter is shown in Table below. The inverter uses a symmetric regular sampling PWM scheme. The carrier frequency is 1050Hz and the modulating frequency is 50Hz. The modulation index is 0.8. Calculate the harmonic amplitudes of the line-to-voltage (i.e. red to blue phase) and complete the table.

Harmonic number	Amplitude (pole switching waveform)	Amplitude (line-to line voltage)
1	1	
19	0.3	
21	0.8	
23	0.3	
37	0.1	
39	0.2	
41	0.25	
43	0.25	
45	0.2	
47	0.1	
57	0.05	
59	0.1	
61	0.15	
63	0.2	
65	0.15	
67	0.1	
69	0.05	