Identification and Robust Control of DC/DC Converter Hammerstein Model

Francesco Alonge, Member, IEEE, Filippo D’Ippolito, Member, IEEE, and Tommaso Cangemi

Abstract—This paper deals with model-based robust control of dc/dc power electronic converters. The converter is described by means of a Hammerstein model consisting of the nonlinear static characteristics of the converter and a linear time-invariant (LTI) uncertain model whose parameters depend on the actual duty-cycle operating range. This suggests that the controller be designed using robust control techniques. In view of applying robust control, identification of the earlier LTI models is performed by means of simulation experiments, carried out on a converter switching model implemented on MATLAB/SIMULINK environment. Internal model control (IMC) structure is employed for the controller design, but its implementation is performed using the equivalent feedback control structure. Comparison with some controllers designed starting from models that do not require identification steps is performed with the aim of showing the advantages connected to the availability of a suitable model, which describes the essential aspects of the behavior of the system, for control purposes. Comparison with a PI controller designed by means of phase margin assignment is carried out with the aim of justifying the use of more sophisticated control methods. Experimental results are also shown that aimed to prove the validity of the whole approach. Comparison of experimental and simulation results is also performed.

Index Terms—Hammerstein model, model identification, power converters, robust control.

I. INTRODUCTION

MODELING and simulation are essential tools for developing a controller for a given system. Model-based control allows to obtain the best results compared to other control techniques, such as fuzzy and neural control. In fact, model-based control techniques bring to the inherent satisfaction of essential requisites of the whole control system such as stability, using, for example, the Lyapunov approach [1], robustness against disturbances and parameter variations [2]–[4], and adaptation to parameter variations [5], [6].

Simulation is essential for two reasons: analysis of the closed-loop control system consisting of the process and the controller models has to be performed before to carry out the practical realization of the system; synthesis of the controller requires the analysis of the earlier closed-loop control system in which the process is described by a model suitable for control. Obviously, the mathematical model used for the controller design has to be representative of the essential aspects of the behavior of the system, whereas the model used for the analysis of the overall system is usually more sophisticated than that required for controller design.

With reference to switching converters, switching models and circuit-oriented models are two alternative methods for simulation purposes. As is well known, switching models consist of as many continuous submodels as the converter conduction configurations are, and a switching logic that schedules the sub-model to be processed at the current instant of time [7], [8]. Circuit-oriented models consist of circuits that can be implemented in environments in which to each electronic component is associated a model whose complexity depends on the environment itself. As is well known, PSpice is a circuit-oriented simulation environment very popular for power electronic specialists [9]. Circuit-oriented simulation gives results more accurate than those obtained using switching models at the expenses of a much greater simulation time.

On the other hand, implementation of switching models can be carried out, for example, in MATLAB environment that is used by researchers and engineers in many fields of science. Moreover, using MATLAB, it is possible to process data obtained from simulation in the same environment, for analysis, identification, and control purposes, whereas data obtained by PSpice have to be imported in MATLAB for their processing.

As already said, for control purposes, a suitable model of the converter needs to describe the essential aspects of the converter behavior. To this regard, many models have been proposed such as small-signal models obtained either by averaging techniques [10]–[12], or by identification approaches [13], [14], nonlinear autoregressive moving average with exogenous input (NARMAX) models obtained by means of identification [15]–[17], and recently, Hammerstein models [18], [19].

With reference to the control techniques, classical techniques have been applied for designing either small-signal model-based analog linear controllers [20]–[22] or sliding-mode nonlinear controllers [22], [23]. Moreover, classical digital control techniques have been applied to discrete-time models obtained either by means of identification [19] or discretization of a continuous-time model [24], [25]. A nonlinear controller is described in [26] that requires measurement of the current in the inductor that, as is well known, is rich of harmonics. An example of application of no model-based control technique is shown in [27].

The aim of this paper is that of giving a systematic procedure for designing a controller for switching dc/dc converters operating in continuous conduction mode (CCM). The peculiarities of this procedure are: 1) control-oriented modeling of the converter is performed using the Hammerstein approach [28] that brings to a model that accurately describes both the steady-state
and transient behaviors; 2) an uncertain Hammerstein model is identified, by means of simulation experiments carried out in MATLAB/SIMULINK environment, consisting of the nonlinear static characteristics of the converter followed by an uncertain LTI model whose parameters depend on the actual duty cycle operating range; 3) a robust controller is designed for the uncertain model, which also allows to cope with uncertainties arising from model identification carried out as already said by means of simulation.

In this paper, the described procedure is applied for designing a robust controller for a boost converter modeled according to the Hammerstein structure. For identification and validation purposes, a switched model [7], [25] is implemented in MATLAB/SIMULINK environment. The steady-state characteristic of the converter, i.e., the steady-state output voltage versus duty cycle, is then constructed and assumed as the static nonlinearity of the Hammerstein model. Few linear, time-invariant, discrete-time models are identified with the aim of describing the converter in some contiguous regions of interest. The corresponding continuous-time models are computed using the step-response invariance method, and among them, a nominal model is chosen. This nominal model and the remaining models are employed for defining a continuous-time uncertain process. Continuous-time robust control techniques are used for the controller design. Some tests are carried out by means of simulation with the aim of verifying the behavior of the closed-loop control system. Experimental results are then obtained and compared with simulation results with the aim of proving the validity of the whole approach.

Comparison is carried out with controllers designed starting from models that do not require identification step, like a direct sliding-mode controller, a sliding-mode controller based on integral reconstructors [29], and a passivity-based controller [30]. Finally, comparison is carried out with a classical PI controller designed using phase margin assignment that, as is well known, also represents a robustness measurement.

II. MODELING THE DC/DC BOOST CONVERTER USING THE HAMMERSTEIN APPROACH

The circuit model of the dc/dc boost converter is given in Fig. 1, where \( v_i \) and \( v_o \) are the input and the output, respectively. The static gain, i.e., the output to input voltage ratio for a constant value of \( v_i \), depends on the duty cycle of the pulse width modulation (PWM) signal supplying the MOSFET. It is assumed that the values of the circuit parameters are chosen so that the converter operates in CCM.

In this paper, only the loop voltage is considered. This is because the use of a current loop together with the voltage loop requires measurement of the inductor current, which is hard to be measured with accuracy, due to the high-pass filter nature of the input circuit and the high-frequency switching that affects its time derivative (see, for example, [29]). For these reasons, the current control loop is often not considered [19], [29], [30].

The Hammerstein model of the boost converter is shown in Fig. 2, where \( f(\cdot) \) is the nonlinear static characteristics of the converter, \( u \) is the duty cycle (control variable), \( x = f(u) \) is an intermediate and inaccessible variable, and LTI is a second-order continuous-time linear and time-invariant model given by [31]

\[
G_p(s) = \frac{Y(s)}{X(s)} = \frac{b_{c,2} s^2 + b_{c,1} s + b_{c,0}}{s^2 + a_{c,1} s + a_{c,0}}
\]

where \( Y(s) = V_o(s) \), with the gain given by

\[
\lim_{s \to 0} G_p(s) = \frac{b_{c,0}}{a_{c,0}} = 1.
\]

In order to identify \( G_p(s) \), the corresponding discrete-time transfer function \( G_{d,p}(z) \) is first of all identified starting from a suitable set of input–output data acquired at the instants of time \( kT_s \), with \( k \) a positive integer, where \( T_s \) is the sampling period. Then, \( G_p(s) \) is computed from \( G_{d,p}(z) \).

A. Modeling the Linear Part of the Boost Converter

The discrete-time transfer function corresponding to (1) is given by

\[
G_{d,p}(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}.
\]

Since the duty cycle is constant within a sampling period, the discrete-time model (3) could be derived from model (1) using the step-response invariance method; more precisely, \( G_{d,p}(z) \) could be obtained by means of discretization of the model depicted in Fig. 3, where

\[
G_{zo}(s) = \frac{1-e^{-sT_s}}{s}.
\]

It follows that, in order to obtain the continuous-time transfer function from the identified discrete-time transfer function, it is again convenient to use the step-response invariance method. Note that the final value of the step response of the discrete-time model coincides with that of step response of the continuous-time model, and consequently, it results

\[
\lim_{z \to 1} G_{d,p}(z) = \frac{b_0 + b_1 + b_2}{1 + a_1 + a_2} = 1.
\]
B. Switching Model of the Boost Converter

The switching model of the boost converter is given by

\[
\dot{x} = A_{ON} x + b_{ON} v_i, \quad \text{MOSFET ON, Diode ON} \tag{5}
\]

\[
\dot{x} = A_{OFF} x + b_{OFF} v_i, \quad \text{MOSFET OFF, Diode ON} \tag{6}
\]

\[
\dot{x} = A_{OFF,1} x + b_{OFF,1} v_i, \quad \text{MOSFET OFF, Diode OFF} \tag{7}
\]

where

\[
x = [i_v_o]^T \quad A_{ON} = \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \quad b_{ON} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

\[
A_{OFF} = \begin{bmatrix} -\frac{R_L}{L} & \frac{1}{C} \\ \frac{1}{L} & -\frac{1}{RC} \end{bmatrix} \quad b_{OFF} = b_{ON},
\]

\[
A_{OFF,1} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \quad b_{OFF,1} = 0
\]

and \(R_L\) takes into account the resistance of the inductor.

C. Identification of the Hammerstein Model

The first step for the identification of the Hammerstein model of the boost dc/dc converter is the determination of the nonlinear static characteristics. Due to the stability properties of the boost converter and condition (4), which assures that the linear part of the Hammerstein model has unitary gain, the nonlinear static characteristic can be obtained starting from open-loop simulation experiments carried out at constant duty cycle values, for a given input voltage \(v_i\). More precisely, for a given duty cycle \(d_i\), a simulation is started and the output voltage of the converter is monitored; at the steady state, this voltage becomes constant and coincides with the value of the intermediate variable \(x(d_i)\), according to (4). Obviously, during transients, the intermediate and the output voltages are different between them. The second step is to identify the linear part of the Hammerstein model. To this end, the procedure described in [19] and [31] can be used. In this paper, the procedure described in [31] is employed, and is carried out by means of simulation starting from a switching model of the converter.

The boost converter having the following parameters \(R = 100 \Omega, L = 67 \mu H, C = 200 \mu F,\) and \(R_L = 0.095 \Omega\) is considered. Putting \(v_i = 10 V,\) using a sampling frequency of 100 kHz and simulation step size of 0.1 \(\mu s,\) the static characteristic of Fig. 4, the discrete-time models of Table I, and the corresponding continuous-time models of Table II are obtained.

III. HAMMERSTEIN-MODEL-BASED ROBUST CONTROL

It is shown in [19] that the converter can be conveniently described by means of a Hammerstein model consisting of a nonlinear static characteristics followed by an LTI second-order model whose parameters depend on the actual duty cycle range of operation. In particular, it is shown that few LTI models need to describe the converter in a large range of operation. This suggests to describe the linear part as an uncertain process consisting of a well-defined nominal model, \(\tilde{G}_p(s),\) and a set of transfer functions (cf., Table II) that satisfy the following
inequality:

\[ P = \left\{ G_p(s) : \left| \frac{G_p(j\omega) - \tilde{G}_p(j\omega)}{\tilde{G}_p(j\omega)} \right| \leq \Delta_m(\omega) \right\} \quad (8) \]

where \( \Delta_m(\omega) \) is the multiplicative uncertainty bound; then, model-based robust control in the frequency domain for continuous-time models [2] can be employed.

The closed-loop control system to be designed is shown in Fig. 5; this scheme is equivalent to that of Fig. 6 because the plant is the one depicted in Fig. 2, where \( G_p(s) \) is the transfer function of the linear part of the Hammerstein model.

### A. Process Modeling

For modeling the process \( P \), it is sufficient to define the nominal transfer function and the uncertainty bound. The candidate nominal transfer function chosen is that corresponding to the nominal transfer function and the uncertainty bound. The candidate uncertainty bound is shown in Fig. 8. This location suggests the following Remarks.

**Remark 1:** The zero \( z_1 \) displays a dominant pole \( p_1 \); the effects of the pole-zero couple \((p_2, z_2)\) take place out of the bandwidth, and consequently, this couple can be canceled from the nominal transfer function \( G_{p,1}(s) \) considering it as unmodeled dynamics. It can be easily verified that the transfer function with or without the aforementioned zero-pole couple is practically superimposed if the gain of the new nominal transfer function remains equal to 1.

According to Remark 1, the new candidate nominal transfer function \( G_{p,2}(s) \) is given by

\[ G_{p,2}(s) = \frac{\tilde{K}_p_2(s - z_1)}{(s - p_1)} \quad \tilde{K}_p_2 = \frac{\tilde{K}_p}{-p_1} \quad \tau = \frac{1}{-p_1}. \quad (10) \]

**Remark 2:** The zero \( z_1 \) is far from the pole \( p_1 \), and consequently, it can be considered as a zero at the infinity. This suggests to choose the nominal transfer function \( G_p(s) \) as

\[ G_{p,1}(s) = \frac{\tilde{K}_p_1(s - z_1)}{(s - p_1)} \quad \tilde{K}_p_1 = \frac{\tilde{K}_p}{-p_1} \quad \tau = \frac{1}{-p_1}. \quad (11) \]

The Bode diagrams of the transfer functions \( G_{p,1}(s) \) and \( G_p(s) \) are given in Fig. 7. These diagrams show that the two functions differ in the phase for high values of the angular frequency when the amplitudes of the two transfer functions are less than about \(-25\) dB.

In order to complete modeling of process \( P \), it is necessary to compute the uncertainty bound \( \Delta_m(\omega) \). This is accomplished starting from the nominal transfer function \( G_{p,1}(s) \) [cf., (12)] and the remaining transfer functions displayed in Table II. The resulting uncertainty bound is shown in Fig. 8.

### B. Controller Design via IMC Control Scheme

As is well known, the controller of the feedback control structure of Fig. 6 is conveniently designed using the IMC control structure of Fig. 9, where \( \xi \) is an equivalent disturbance equal...
to zero in this application, and \( G_q(s) \) is the transfer function of the IMC controller. As is easy to verify, the controller of the feedback structure of Fig. 6 is given by

\[
G_c(s) = \frac{G_q(s)}{1 - G_p(s)G_q(s)}. \tag{12}
\]

The IMC controller is given by \( G_q(s) = \tilde{G}_q(s)G_f(s) \), where \( G_f(s) \) is the transfer function of the IMC filter, and \( \tilde{G}_q(s) \) is obtained by minimizing the cost function

\[
J = ||e(\cdot)||_2^2 = \int_0^\infty e^2(t) \, dt \tag{13}
\]

where \( e(t) = v_{\text{ref}} - v_0 \) (cf., Fig. 6). In order to cope with overshoot problems in the step response, it is convenient to design \( \tilde{G}_q(s) \) for the smooth input given by

\[
v_{\text{ref}}(s) = \frac{\beta\gamma}{s(s + \beta)(s + \gamma)} \tag{14}
\]

which in the time domain has the properties \( v_{\text{ref}}(0) = v_{\text{ref}}(0) = 0, \lim_{t \to -\infty} v_{\text{ref}}(t) = 1 \).

For design purposes, it is convenient to consider this input generated at the output of a system having transfer function given by

\[
W(s) = \frac{\beta}{\gamma s^2 + 1},
\]

supplied by the input \( V'(s) = \sqrt{2\beta}/(s + \beta) \). Note that this input is bounded, i.e., it satisfies the condition

\[
||v'(t)||_2^2 = \int_0^\infty v^2(t) \, dt = 1.
\]

Since both \( \tilde{G}_p(s) \) and \( V_{\text{ref}}(s) \) are minimum-phase, the controller \( \tilde{G}_q(s) \) that minimizes (13) is given by [2]

\[
\tilde{G}_q(s) = \tilde{G}_p^{-1}(s) = \frac{1}{K(1 + s\tau)}. \tag{15}
\]

**Remark 3:** The designed continuous-time controller has to be implemented on a digital device, and consequently, it has to be discretized. In presence of real poles, the discretization is carried out choosing the discretization step that coincides with the sampling time of the digital device, ten times lower than the time constant relative to the higher pole of the transfer function of the controller. The presence of both the zeros in the nominal transfer \( \tilde{G}_{p,1} \) far from the imaginary axis brings to poles in the controller that require impractical sampling frequency. For this reason, it is considered \( \tilde{G}_p \) (cf., (12)) as the nominal transfer function instead of \( \tilde{G}_{p,1} \).

The IMC filter \( G_f(s) \) is chosen so that the IMC controller \( G_q(s) \) be proper and the whole system be type 1. The following \( G_f(s) \) satisfies the aforementioned requirements:

\[
G_f(s) = \frac{1}{1 + s\lambda} \tag{16}
\]

independently of the value of \( \lambda \) that will be chosen later in order to satisfy the robust behavior condition.

From (11), (15), and (16), the structure of the feedback controller \( G_c(s) \) is given by [cf., (12)]

\[
G_c(s) = K_p \left( 1 + \frac{1}{sT_i} \right), \quad K_p = \frac{\tau}{K\lambda}, \quad T_i = \tau. \tag{17}
\]

It follows that the robust control procedure employed in this paper brings to a PI-type controller able to assure the desired robustness properties to the whole system.

The free parameter \( \lambda \) is chosen so that the following robust behavior condition be satisfied:

\[
|\tilde{G}_p(j\omega)\tilde{G}_q(j\omega)G_f(j\omega)\Delta_m(\omega)|
\]

\[
+ |1 - \tilde{G}_p(j\omega)\tilde{G}_q(j\omega)G_f(j\omega)|W(j\omega)| < 1 \quad \forall \omega \tag{18}
\]

Obviously, if this condition is satisfied, the first term of (18) is less than 1, and consequently, the robust stability condition is satisfied.

As is easy to verify, choosing \( \beta = 5000 \) and \( \gamma = 10000 \) [cf., (14)], the minimum value of \( \lambda \) that satisfies (18) is given by \( \lambda = 0.0016 \). Moreover, for \( \omega = 0 \), (18) is satisfied because the first term is zero and the second term is equal to \( l\sqrt{\beta}/2 < 1 \).

**Remark 4:** Note that the choice of an input whose Laplace transform is minimum-phase affects the design of the robust controller only in the computation of \( \lambda \), because the structure of the input affects that of \( W(s) \).

**Remark 5:** According to (17), the aforementioned value chosen for \( \lambda \) brings to an upper bound for \( K_p \) whereas the value of \( T_i \) remains unchanged.

**IV. VALIDATION OF THE CONTROLLER**

The designed controller has been implemented in MATLAB/SIMULINK environment together with the switching model...
Fig. 10. Closed-loop control scheme.

(5)–(7) with the aim of validating it. To cope with windup problems, the implementation of the PI robust controller has been carried out using the antiwindup scheme described in [31] and shown in Fig. 10, where SAT denotes saturation of the duty cycle [0.05, 0.95], LUT denotes a look-up table that stores the inverse of the nonlinear characteristic of the Hammerstein model, and

\[ G_{ca}(s) = K_p, \quad G_{ch}(s) = \frac{1}{1 + sT_i}, \quad V_r(s) = \frac{V_{r1}}{s}, \]

\[ G_i(s) = \frac{\beta \gamma}{(s + \beta)(s + \gamma)}. \]

Note that \( G_i(s) [V_r(s)/V_{r1}] \) generates the previously defined input \( V_{ref}(s) \) [cf., (14)].

A. Robust Controller

The responses of the closed-loop control system of Fig. 10 are shown in Figs. 11 and 12, for two reference voltages belonging to two different ranges of duty cycle. Fig. 13 shows the results of tests carried out for an output voltage range ten times higher. The results show that for the use of the proposed control scheme in this new range, it is only sufficient to change few parameters, such as \( V_{r1} \), the supply voltage increased to \( v_i = 100 \) V, the levels of the SAT block and the LUT both computed for \( v_i = 100 \) V, whereas the controller remains unchanged. The results displayed in Figs. 11–13 show the robustness of the designed controller.

Remark 6: In the operating situation relative to Fig. 11(a), there are isolated instants or short intervals of time in which the inductor current is equal to zero. However, these intervals are so short that the operations of the converter can be considered CCM with a confidence interval of about 95%. This is also highlighted from the results obtained with the controller designed for a CCM behavior.

V. EXPERIMENTAL RESULTS

In order to prove the validity of the approach followed, entirely based on the simulation executed starting from a switching model of the converter, the experiments are carried out on a prototype of the converter itself. In particular, the designed robust controller is implemented on a DSpace 1103 microcontroller for controlling the dc/dc boost converter whose parameters are given at the end of Section II. The results of the experiments are displayed in Figs. 14–16. The value of \( K_p \) is chosen less than the maximum value allowed, due to the noise that affects the measured variables caused principally from the non-optimized layout of the prototype. In particular, it has been chosen \( K_p = 0.652 \). These results show the good behavior of the closed-loop control system.

VI. COMPARISON OF EXPERIMENTAL AND SIMULATION RESULTS

In order to compare simulation and experimental results, the tests of Figs. 14–16 are repeated in simulation. The corresponding results are displayed in Figs. 17–19. The comparison of Figs. 14–17, 15–18, and 16–19 shows a good concordance between the simulation and experimental results, except for the first 0.03 s during which simulation starting transients take place.
VII. COMPARISON WITH CONTROLLERS DESIGNED USING MODELS THAT DO NOT REQUIRE IDENTIFICATION STEP

A. Sliding-Mode Control (SMC)

As is well known, sliding-mode control is a powerful approach for designing robust controllers for linear and nonlinear systems. This control technique can be applied starting from the variable structure mathematical model of the dc–dc boost converter given by (cf., for example, [29] and [32])

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{L}ux_2 + \frac{1}{L}v_i \\
\dot{x}_2 &= \frac{1}{C}ux_1 - \frac{1}{RC}x_2
\end{align*}
\]  

where

\[ u = \begin{cases} 
1, & \text{MOSFET OFF} \\
0, & \text{MOSFET ON.}
\end{cases} \]
As is easy to verify (see also [29]), choosing the switching surface \( \sigma(x) = x_{2,\text{ref}} - x_2 \), where \( x_{2,\text{ref}} \) is the reference value of \( x_2 \), the equivalent control, obtained from \( \sigma(x) = \dot{\sigma}(x) = 0 \), is given by \( u_{2,\text{eq}} = x_{2,\text{ref}}/(Rx_1) \), the equilibrium state during the sliding motion is \( x_{1e} = x_{2,\text{ref}}/(Rv_i) \), and the corresponding zero dynamics relative to the variable \( x_1 \) results

\[
\dot{x}_1 = -\frac{1}{LR} \frac{x_{2,\text{ref}}^2}{x_{1e}^2} + \frac{1}{L} v_i. \tag{21}
\]

Linearizing (21) around \( x_{1e} \), the following equation is obtained:

\[
\Delta \dot{x}_1 = \frac{1}{LR} \frac{x_{2,\text{ref}}^2}{x_{1e}^2} \Delta x_1 \tag{22}
\]

where \( \Delta x_1 = x_1 - x_{1e} \). According to the first Lyapunov’s method, the zero dynamics (21) is unstable, and consequently, the switching surface based on the choice of the output voltage as output variable cannot be employed.

Choosing the switching surface \( \sigma(x) = x_{1,\text{ref}} - x_1 \), where \( x_{1,\text{ref}} \) is the reference value of \( x_1 \), the equivalent control is given by \( u_{1,\text{eq}} = v_i/x_2 \), the equilibrium state during the sliding
Fig. 16. Responses of the closed-loop robust control system for sudden variation of the load resistance of the converter from 100-to-50-to-100 $\Omega$. (a) Output voltage versus time. (b) Input voltage $v_i$ versus time. (c) Duty cycle versus time.

Fig. 17. Simulation results of the closed-loop robust control system for sudden variation of $V_r$ from 10 to 20 V. (a) Output voltage versus time. (b) Duty cycle versus time.

motion is $x_{1,ref} = x_{1e} = x_{2,ref}^2/(Rv_i)$ and $x_{2e} = x_{2,ref}$, and the corresponding zero dynamics relative to the variable $x_2$ results [29]

$$\dot{x}_2 = \frac{v_i}{C} x_{1e} - \frac{1}{RC} x_2^2. \quad (23)$$

Assuming $V(x_2) = 0.5x_2^2$ as a candidate Lyapunov’s function, it can be easily found [29] that $V(x_2)$ exponentially converges to $V(x_{2,ref})$, which implies that $x_{2,ref}$ exponentially converges to $x_2$. This shows that the choice of the current as output variable brings to a stable sliding motion. Unfortunately, in this case, the output voltage largely varies with the variation of the load resistance [29].

It follows that the conventional sliding-mode control technique applied to the dc–dc boost converter fails to obtain performance robustness, whereas the method proposed in this paper allows to obtain this property.

B. Sliding Control With Integral Reconstructors [29]

According to the approach described in [29] for the model of the boost converter normalized both in variables and time, the following modified sliding surface is proposed:

$$\sigma(x) = \frac{1}{L} \int_0^t [v_i - u(\tau)y(\tau)]d\tau - x_{1e} + \frac{1}{L} k_0 \xi(t) \quad (24)$$

$$\dot{\xi}(t) = y(t) - x_{2,ref} \quad (25)$$

where $y = x_2$, the first term of (24) represents an estimation, $\dot{x}_1(t)$, of $x_1(t)$ [cf., (21)] and differs from it by the initial state $x_1(0)$ ($\dot{x}_1(t) = x_1(t) - x_1(0)$).

The equivalent control is given by

$$u_{eq} = \frac{v_i + k_0(y - x_{2,ref})}{y}. \quad (26)$$
Fig. 18. Simulation results of the closed-loop robust control system for sudden variation of $V_r$ from 20 to 15 V. (a) Output voltage versus time. (b) Duty cycle versus time.

The condition $u_{eq} \in (0, 1)$ [33] allows to obtain

$$ u_{eq} > 0 \Rightarrow v_i + k_0(y - x_{2,ref}) > 0 \quad (27) $$

$$ u_{eq} < 1 \Rightarrow v_i + k_0(y - x_{2,ref}) < y. \quad (28) $$

Since $y > 0$, (27) is always satisfied choosing $k_0 > 0$ and $v_i > k_0 x_{2,ref}$. Since for the boost $x_{2,ref} > v_i$, it follows that $k_0 < 1$. Consequently, $k_0 \in (0, 1)$. From (28), the following equation is obtained:

$$ \frac{y}{v_i} > \frac{(1 - k_0 x_{2,ref} / v_i)}{(1 - k_0)}. \quad (29) $$

Since $k_0 \in (0, 1)$ and $v_i > k_0 x_{2,ref}$, the right side of (29) is less than 1 whereas for the boost converter $y > v_i$. Consequently, (29) is satisfied. It can be easily verified that the equilibrium state during the sliding motion is given by

$$ x_{1e} = \frac{x_{2,ref}^2}{(Rv_i)} \quad x_{2e} = x_{2,ref} \quad \xi_e = \frac{L x_1(0)}{k_0}. \quad (30) $$

It follows that the equilibrium state for the inductor current and voltage output is unchanged, whereas the variable $\xi$ converges to a value that compensates the unknown initial state.

The results obtained by means of simulation of the closed-loop system employing the sliding-mode controller with integral reconstructor (SMIR) as shown in Fig. 20. The sampling frequency is 200 kHz, twice of that employed for the proposed robust controller.

The comparison of Figs. 12 and 20 shows that the proposed controller works better than the sliding-mode controller with integral reconstructor. In fact, the last controller produces higher ripple in the current and worst transients in the output voltage in correspondence to load variations.

C. Passivity-based Approach [30]

This approach gives results similar to those obtained using sliding-mode techniques.

From the average Eulero–Lagrange parameters (electric field energy, magnetic coenergy, Raleigh dissipation cofunction, and generalized forcing functions) [34], the following PWM average model is obtained

$$ L \dot{z}_1(t) = -(1 - \mu) z_2(t) + v_i \quad (31) $$

$$ C \dot{z}_2(t) = (1 - \mu) z_1(t) - \frac{1}{R z_2(t)} \quad (32) $$

where $\mu$ is the duty cycle and $z_1(t)$ and $z_2(t)$ are the average inductor current and output voltage, respectively. Moreover, the following model can be derived that describes the dynamics...
Fig. 20. Responses of the closed-loop system: \( V_{r1} = 30 \text{ V}, R = 50 \Omega \) for \( t \in [0.05, 0.1) \), \( R = 100 \Omega \) for \( t \in [0, 0.05) \) and \( t \in [0.1, 0.15) \), SMIR. (a) Inductor current. (b) Output voltage.

of the desired values \( z_{1d}(t) \) and \( z_{2d}(t) \) of \( z_1(t) \) and \( z_2(t) \), respectively:

\[
L \dot{z}_{1d}(t) + (1 - \mu) z_{2d}(t) - (z_1(t) - z_{1d}(t)) R_1 = v_i \tag{33}
\]

\[
C \dot{z}_{2d}(t) - (1 - \mu) z_{1d}(t) + \frac{1}{R z_{2d}(t)} = 0. \tag{34}
\]

Assuming \( z_{2d}(t) = V_{r1} \) as output, from (34), the following expression can be found:

\[
\dot{z}_{1d}(t) = \frac{1}{R} \frac{V_{r1}}{1 - \mu}. \tag{35}
\]

From (33) and (35), the following expression is obtained for the control \( \mu \):

\[
\dot{\mu} = \frac{R(1 - \mu)^2}{LV_{r1}} \left[ v_i - (1 - \mu) V_{r1} + R_1 \left( z_1(t) - \frac{V_{r1}}{R(1 - \mu)} \right) \right]. \tag{36}
\]

It can be shown that this control generates an unstable zero dynamics.

Assuming the average current as output variable, \( z_1(t) = z_{1d}(t) = I_d = v_{ref}^2/(R v_i) \), from (33), the desired output voltage becomes

\[
z_{2d}(t) = \frac{v_i + (z_1(t) - z_{1d}(t)) R_1}{1 - \mu}. \tag{37}
\]

From (34) and (37), the following control law is obtained:

\[
\dot{\mu} = \frac{(1 - \mu)}{C[v_i + (z_1(t) - I_d) R_1]} \times \left[ (1 - \mu)^2 I_d - \frac{R_1 C}{L} [v_i - (1 - \mu) z_2(t)] - \frac{1}{R} [v_i + (z_1(t) - I_d) R_1] \right]. \tag{38}
\]

The zero dynamics corresponding to this control is stable.

The implementation of the control law (38) requires that of the average model (31)–(32). In fact, the use of instantaneous values of inductor current and output voltage, stated in [30], does not give good results because the output voltage converges toward a value that depends on the instantaneous value of the inductor current acquired for the computation of the control law itself.

The simulation results obtained by applying the control law (31)–(32) and (38) to the boost converter are given in Fig. 21.

Comparison with the results displayed in Fig. 12(a) and (b) shows that the mean values of the current obtained using the passivity-based controller are greater than those produced by
the robust controller; moreover, the closed-loop control system with the passivity-based controller displays transients, due to load variations, more evident compared to the robust controller. Finally, a steady-state error appear in Fig. 21(b) due to the fact that it is necessary to consider the inductor current as output, and consequently, it is necessary to specify the reference current $I_d$ that varies with the load.

The design of the passivity-based control law is carried out starting from the nonlinear average model of the converter. The implementation of the control law is more complicated because it requires the implementation of both the average dynamics (31)–(32) and the control law (38).

D. PI Versus Robust Control

Finally, it is convenient to compare the robust controller developed in this paper with a PI controller derived using classical robust measures such as gain and phase margins. More precisely, comparison is carried out between the robust controller and a classical PI controller derived by means of phase margin assignment. The convenience of this comparison is justified because of: 1) the approach described in this paper brings to a robust controller with PI structure and 2) the described approach seems to be too much complicated with respect to the easy structure of the designed robust controller. To this regard, the following observations can be made.

1) For the models common in industrial practice (first- and second-order models), the described synthesis approach brings to a robust controller consisting of a PID controller, eventually followed from a first-order low-pass filter [2, p. 114].

2) Even in simple cases leading to PID controllers, the described approach can be considered as a systematic procedure for designing these controllers so as to satisfy robustness measures more sophisticated than the classical ones, i.e., phase and gain margins.

3) It is common practice to change the gain of the proportional part of a PID controller when passing from simulation to practical implementation of the controller. This, obviously, modifies the characteristics of the overall control system such as, for example, phase and gain margins. For controllers designed using these robustness measures, it results that the design requirements could not be satisfied, whereas for controllers designed using the described approach, the design requirements continue to be satisfied. In particular, in this paper, it is shown that for gaining robust performance, an upper bound for the gain of the $P$ action has to be imposed. Consequently, diminishing the gain for practical implementation does not cause the loss of the performance robustness.

That being stated, the comparison of a classical and robust PI controller is carried out in simulation considering a PI classical controller designed so as to obtain a phase margin of $45^\circ$ in correspondence to a crossover frequency of $800$ rad/s. More precisely, the same tests considered in Figs. 11 and 12 are carried out computing the mean square error at the steady state using the last 30,000 values of the output voltage error corresponding to $R = 100$ W and the last 30,000 values corresponding to $R = 50$ W. The corresponding results, given in Tables III and IV, show that the robust controller displays the best behavior. Note that the duty cycle range corresponding to reference voltage equal to 15 V is near to that corresponding to the nominal transfer function $[0.4, 0.45]$, whereas the range corresponding to 30 V is far from the range corresponding to the nominal transfer function. This justifies the increasing of the mean square error when the reference output voltage increases.

<table>
<thead>
<tr>
<th>Reference output voltage</th>
<th>Classical PI</th>
<th>ROBUST PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>$2.21 \times 10^{-6}$</td>
<td>$2.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>30</td>
<td>$2.45 \times 10^{-3}$</td>
<td>$1.13 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reference output voltage</th>
<th>Classical PI</th>
<th>ROBUST PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>$7.72 \times 10^{-4}$</td>
<td>$9 \times 10^{-5}$</td>
</tr>
<tr>
<td>30</td>
<td>$5.1 \times 10^{-3}$</td>
<td>$8.4 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Fig. 22. Output voltage error versus time $V_r = 30$ V. (a) Classical PI. (b) Robust PI.
In Fig. 22, the output voltage is error for classical and robust PI corresponding to a reference output voltage equal to 30 V is shown. Note that the waveform of the output voltage error is the same as that of Fig. 13(c) corresponding to $V_c = 300$ V.

VIII. CONCLUSION

In this paper, a method is illustrated for designing a robust controller for a boost dc/dc converter described by means of a Hammerstein model. The robust controller is designed using an IMC control structure that allows to obtain the set of all the stabilizing feedback controllers for the system in question. The design in question allows to incorporate the reference input into the design requirements. This input can be chosen so as to obtain a better behavior during the transients occurring just after the application of the reference voltage. Several simulation experiments show that the procedure based on the simulation is suitable for modeling purposes of dc/dc converters by means of Hammerstein models, whereas the use of robust control techniques is suitable for designing controllers based on these models. The comparison of simulation and experimental results is satisfactory and shows the validity of the proposed approach. The comparison with nonlinear control techniques, such as sliding mode with integral reconfigurators and passivity-based control, shows that the proposed approach is more time-consuming, but it can be carried out practically by means of simulation, can be applied using simple concepts, such as least-square techniques for identification and robust control techniques in the frequency domain for controller design, and, which is more important, it gives the best results. Comparison with classical PI controller shows that the proposed robust controller allows to obtain better results.

Finally, the proposed method, based on an identified model that captures the low-frequency behavior of the converter, allows to satisfy slow-scale stability property. However, several simulation tests carried out using the switching model revealed that chaotic behaviors do not appear. The oscillations present in the inductor current are due to the high-frequency commutations of the MOSFET, and could be reduced by increasing the inductance of the circuit.

REFERENCES

Francesco Alonge (M’02) was born in Agrigento, Italy, in 1946. He received the Laurea degree in electronic engineering from the University of Palermo, Palermo, Italy, in 1972. Since 1972, he has been with the University of Palermo, where he is currently a Full Professor of automatic control in the Department of Systems and Control Engineering. His current research interests include electrical drive control (also including linear and nonlinear observers, stochastic observers, parametric identification), robot control, parametric identification and control in power electronics, and unmanned aerial vehicle (UAV) motion control in aeronautics.

Filippo D’Ippolito (M’00) was born in Palermo, Italy, in 1966. He received the Laurea degree in electronic engineering and the Research Doctorate degree in systems and control engineering from the University of Palermo, Palermo, in 1991 and 1996, respectively. He is currently a Research Associate in the Department of Systems and Control Engineering, University of Palermo. His current research interests include control of electrical drives, adaptive and visual/force control of robot manipulators, and control of electrical power converters.

Dr. D’Ippolito received the 2000 Kelvin Premium from the Institution of Electrical Engineers (IEE) for the paper “Parameter identification of induction motor model using genetic algorithms.”

Tommaso Cangemi was born in Palermo, Italy, in 1974. He received the Master’s degree in computer science engineering from the University of Palermo, Palermo, in 2003, and the Ph.D. degree in systems and control engineering in 2007. Since 2007, he has been a Full-Time Postdoctoral Researcher in the Department of Systems and Control Engineering, University of Palermo. His current research interests include control of electrical drives, optimal control theory, and Markovian jump system, and their applications on networked control systems and control of electrical power converters.